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THE ROLE OF CONSTRUCTIVISM-BASED LEARNING IN IMPROVING MATHEMATICAL HIGH ORDER THINKING SKILLS OF INDONESIAN STUDENTS

Ani Minarni*[1], E. Elvis Napitupulu[2]
[1,2]Universitas Negeri Medan

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ABSTRACT

To make students actively involved in learning to grasp mathematical higher-order thinking skills (MHOTS) is not easy. Meanwhile, the ability is so important for students to master for it takes place when students continue their studies to a higher level as well as work within a variety of professions, especially in the era of the industrial revolution such nowadays. Many factors affect students' thinking abilities, including learning factors. This study, which implemented constructivism-based learning, aims to investigate the role and contribution of constructivism-based learning approaches as well as mathematical prior knowledge (MPK) to the achievement of MHOTS of middle secondary school students. The data tested through Multivariate Analysis at the 0.05 significance level. In general, this study found that: (1) In the experimental class, the learning approach plays an important role in the way it increased students' MHOTS significantly. (2) The average contribution of constructivism-based learning to MHOTS was at the range of 18% to 57%. (3) Student activity in learning increased significantly. (4) In some cases, there is an effect of interaction between learning factors and MPK towards the achievement of MHOTS. The study recommended the teachers to have courageous in implementing constructivism-based teaching and learning to improve students' MHOTS.

Corresponding Author:

Ani Minarni,
Departement of Mathematics Education,
Universitas Negeri Medan
Jl. William Iskandar Ps. V, Kenangan Baru, Deli Serdang, North Sumatera 20221, Indonesia.
Email: animinarni10@gmail.com

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1. INTRODUCTION

Parents keep some will for their children, for example they want their children to be useful people for themselves, family & society, and to serve their parents, country and religion. God has the will of His creatures; God wants His creatures to be on a straight path in terms of ways that can make His creatures feel peace in living. The teachers want their students to have thinking competencies, life skills and useful characters. The teacher wants students to have the habit of lifelong learning and succeed in learning according to what NCTM document emphasized (NCTM, 2000).

Many factors influence student achievement, but the point is there are two, namely factors within themselves and external factors (Brown, 1990). What can be considered as internal factors include intellectual level (intelligence level), learning ability, learning motivation, learning independence (self-regulated learning), attitudes, feelings, interests, psychological conditions, and due to socio-culture. Meanwhile, external factors include the attitude of parents and teachers to students, learning factors applied in schools, curriculum, school discipline, teachers themselves, learning facilities, grouping students, social systems, student social status, teacher and student interaction, political economy conditions, circumstances time and place or climate.

By paying attention to a series of influential factors, one of the things teachers can try in schools is to improve learning outcomes through the implementation of appropriate learning. Learning should be empowering students to think and construct their knowledge, arise students' interest in learning and make students understand the topics they learn. Learning that has characteristics like this is constructivism-based learning (Resnick, 1987), such as Problem-based learning, Discovery learning, Cooperative learning (Arends, 2012; Ronis, 2008), even contextual learning and Open-ended approaches, and mathematical realistic approach.

The learning achievement will be even better if parents help encouraging their children to understand what is learned in school through often asking what they learned, whether they understand the subject matter learned today, or trigger the student to do homework. If students understand they learned and can construct their knowledge for themselves, they will have the opportunity to gain understanding skills at the HOTS (high order thinking skills) level, the level that reaches the ability to apply knowledge to solve problems. In other words, the understanding ability will bring up the ability of problem solving. On the other side, problem-solving ability will build up HOTS as well.

HOTS always plays an important role from time to time, especially in the present era that has entered the era of industrial revolution 4.0. HOTS, moreover MHOTS, has proven to be the basis/foundation in the development of that era (Formaggia, 2017). To achieve MHOTS is not enough just to rely on learning factors, but the mathematical prior knowledge (MPK) also needs to consider since its also holds an important role in problem solving process. This is because according to the results of the research, the MPK factor contributes to the achievement of mathematical problem solving abilities (Minarni, 2017). Reasoning ability, connection ability & mathematical representation, even the interactions between the two factors can also affect student' learning outcomes. Therefore, it is interesting to investigate how is the role, especially the contribution, of constructivism-based learning approaches to the achievement of mathematical thinking skills of middle secondary school students. The findings of this study could be triggered the teachers to grasp the courage in implementing constructivism-based learning approaches that they considered difficult to implement.

Mathematical High Order Thinking Skills (MHOTS)

High Order Thinking Skills (HOTS) is the concept of education reform based on the taxonomy of learning objectives from Bloom and its revisions in Marzano & Kendall (2007). The idea is that some types of learning not only require higher-level thinking skills but also require ways to teach it differently from other types of learning so there is the term HOTS. In Bloom's taxonomy, for example, skills that involve analysis, evaluation and synthesis (the creation of new knowledge) are considered abilities at the highest level that require learning and teaching methods different from learning or teaching methods that require students to master facts and concepts (Anderson et al., 2001). Whereas in Marzano & Kendall (2007) the level of high-order thinking is self-system & metacognition.

HOTS generally consist of include critical, logical, reflective, metacognitive thinking, problem solving, and creative thinking. The ability to think has the potential to develop and increase when a person faces a problem that is not familiar to him, uncertain, raises a dilemma or invites questions. HOTS that runs successfully produces explanations, decisions, a series of performance, and products that are valid in the context of existing experience and knowledge and it fosters the growth of other intellectual skills in a sustainable manner. HOTS is rooted in the skills of simply applying and analyzing knowledge and cognitive strategies that intertwined with prior knowledge. The appropriate learning strategy and learning environment is a facility for the growth of HOTS along with the growth of accuracy, self-supervision, openness, and flexibility in students. This explaination is in line with the theory related to how HOTS is learned and developed in students' cognitive structure (Kulm, 1990).

One kind of the HOTS is critical thinking that has many different definitions. Scriven & Paul (1987) stated that critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. In its exemplary form, it is based on universal intellectual values that transcend subject matter divisions: clarity, accuracy, precision, consistency, relevance, sound evidence, good reasons, depth, breadth, and fairness. Therefore, a good critical thinker generally needs to be able to both analyze and synthesize information. Another definition of critical thinking comes from Cottrell (2005) that stated critical thinking is a cognitive activity that involves mental processes such as attention, categorization, selection, and judgment.

Thus, based on the definition of critical thinking, it can be said that mathematical creative thinking is the abilities that require elements to test, question, connect, evaluate, all aspects that exist in mathematical problems. All of the areas of mathematics requires critical thinking, including algebra and geometry. Algebra work requires analysis, which is the ability to break apart the pieces of a problem to solve while doing geometry involves more synthesis than analysis, in that we take all the elements of geometry and combine them to solve problems (do geometric proofs).

The term of mathematical problem solving skills (MPSS) referred to the definition of problem solving defined by Anderson et al. (2001), which is the process of applying mathematical knowledge in new and unfamiliar situations (problems). In the process of solving mathematical problems one will go through Polya's footsteps (Polya, 2004) which include understanding the problem, device a plan, carry out the plan and looking back. Understand the problem is the ability to represent the problem in any other form that makes one easier to attain the solution. Understanding skills also enable one to demonstrate mathematical connection skills (the ability to connect among mathematical knowledge/ideas/ procedure/concepts) (NCTM, 2000). The stage of looking back or reflection can be interpreted as draw conclusions for the solutions.

In solving mathematical problems, one should have creativity. The ability to think creatively in math problem solving is the ability to solve mathematical problems flexibly, involving convergent thinking and divergent thinking. Mathematical creative thinking enable one to make connections between problems under consideration, mathematical knowledge, variety of strategies for possible solutions, variety solutions, models and related questions, evaluate the problem solving process and not just at the end, communicate with peers, teachers, and other interested adults while working on the problem as well as following its solution (Jensen, 1976).

Torrance (1960) had studied for long time to reveal an understanding of students' mathematical creative thinking skills. Just like other skills, mathematical creative thinking can be learned through the teaching and learning process that required students to construct their knowledge via problem solving. In addition, Sheffield (2013) stated that encouraging and supporting the development of mathematical creativity have the added benefit to increasing students enjoyment of and engagement in mathematical reasoning, sense making, problem solving, and problem posing. Mathematical creative thinking skills is consisted the aspects of to think flexibly, original (proposed new idea), fluency, detailed and depth explanation to the solution.

Constructivism-based learning

For a long time, the learning approach teachers used in Indonesian schools is dominated by direct learning or direct instructions and demonstrations to resolve routine problems with communication tend to be one-way from teacher to student (Minarni, Napitupulu, & Husein, 2016). This learning approach can indeed foster mathematical problem solving abilities but the problem that can be solved is the problem that exists in the textbook which is often not related to real life problems (Silver, 2013), does not have the characteristics of a problem that serves to increase mathematical high order thinking skills (HOTS) such as problem solving (Ronis, 2008).

The learning approach aimed at reaching HOTS requires clarity of communication to avoid ambiguity and confusion and to increase students' positive attitudes towards tasks that require them to think and as a way out to address the diverse needs of students. This kind of learning needs scaffolding techniques, i.e., the support and assistance as needed in students at the beginning of their problem solving. Scaffolding should gradually reduce until finally the students are left to work independently. Excessive or too little scaffolding can hinder a student's development or progress in reaching HOTS.

At present, learning that is expected to improve HOTS and MHOTS is constructivism-based learning because this learning carries the principle that knowledge is the result of human construction (Widodo, 2004), knowledge is the result of social construction (Vygotsky, 1980). Social interaction participates in giving an important role in the process of knowledge construction (Phillips, 1997). Knowledge is constructed in a particular social context and influenced by a variety of 'strengths', including ideology, religion, politics, economics, human interest, and group dynamics. Therefore, individuals must construct their own knowledge due to knowledge cannot be simply transferred directly from the teacher to students or from the book to the readers.

Constructivism-based learning such as problem-based learning (PBL), discovery learning (DL), cooperative learning (Arends, 2012), realistic mathematics education (RME) (Gravemeijer & Doorman, 1999), contextual teaching-learning (CTL), and the Open-ended approach (Becker & Shimada, 1997) is designed by considering the factors that influence the learning outcomes. For example, PBL carries interdisciplinary learning, considers local culture and places emphasis on social interaction to foster students' problem solving skills and social skills (Arends, 2012). Meanwhile, social skills allow the improvement of academic achievement (Minarni, 2013).

Instead of starting the learning process by presenting content for students to memorize and understand, PBL emphasizes the process of how humans learn naturally, that is, learning occurs when there are problems (Hmelo-Silver, 2004). To obtain a solution to the problem, people will be motivated to learn the skills and knowledge related to the problems they face, learn or recall the contextual knowledge related to the problem. PBL relies on problems that integrate useful knowledge for students in their personal lives or in facing their professional careers later on. Problems are designed to be authentic, unstructured, and sufficiently challenging students to become active and reliable problem solvers. It can be inferred from Savin-Baden & Major (2004) that the goal of PBL is to guide students to construct meaning rather than gathering facts and to become collaborative learner. Those characteristics open the opportunities for the achievement of HOTS.

Another learning approach based on constructivism is Realistic Mathematics Education (RME). RME is a teaching and learning theory in mathematics education that was first introduced and developed by the Freudenthal Institute in the Netherlands (de Lange, 1996). RME is an approach that insisted mathematics should be connected to reality and human activity, close to children and be relevant to everyday life situations (Gravemeijer & Doorman, 1999). However, the word ‘realistic’, refers not just to the connection with the real-world, but also refers to problem situations which real in students' mind. The context of the problems presented to the students can be a real-world one but this is not always necessary. De Lange (1996) stated that problem situations can also be seen as applications or modeling.

There are two types of mathematization in RME formulated explicitly in an educational context (Treffers, 1991). These are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools that can help to organize and solve a problem located in a real-life situation. Examples of horizontal mathematization: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical problem. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. Examples of vertical mathematization: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing (Gravemeijer, 1994).

The learning process starts from contextual problems. Using activities in the horizontal mathematization, for instance, the student gains an informal or a formal mathematical model. By implementing activities such as solving, comparing and discussing, the student deals with vertical mathematization and ends up with the mathematical solution. Then, the student interprets the solution as well as the strategy used to another contextual problem.

RME is closely related to socio-constructivism (de Lange, 1996; Gravenmeijer, 1994). In both approaches, students are offered opportunities to share their experiences with others. In addition, de Lange (1996) stated that the compatibilities of socio-constructivist and RME are based on a large part or similar characterizations of mathematics and mathematics learning. Those are: (1) both struggle with the idea that mathematics is a creative human activity; (2) that mathematical learning occurs as students develop effective ways to solve problems (Streefland, 1991; Treffers, 1991); and (3) both aim at mathematical actions that are transformed into mathematical objects (Freudenthal, 2006).

Like wise RME, discovery learning (DL) is also based on constructivism. DL is also referred to problem-based learning, experiential learning and 21st century learning. Discovery learning is the work of learning theorists and psychologists Jean Piaget, Jerome Bruner, and Seymour Papert (Arends, 2012). Jerome Bruner is often credited to the origin of DL in the 1960s, but his ideas are very similar to those of earlier writers such as John Dewey. Bruner argues "Practice in discovering for oneself teaches one to acquire information in a way that makes that information more readily viable in problem solving".

This philosophy later became the discovery learning movement of the 1960s. This philosophical movement suggests that people should "learn by doing".

The label of DL can cover a variety of instructional techniques. A discovery-learning task can range from implicit pattern detection, to the elicitation of explanations and working through manuals to conducting simulations. DL can occur whenever they do not provide the student with an exact answer but rather the materials in order to find the answer. Discovery learning takes place in problem solving situations where the learner draws on his own experience and prior knowledge and is a method of instruction through which students interact with their environment by exploring and manipulating objects, wrestling with questions and controversies, or performing experiments.

It has been suggested that effective teaching using discovery techniques requires teachers to do one or more of the following: 1) Provide guided tasks leveraging a variety of instructional techniques, 2) Students should explain their own ideas and teachers should assess the accuracy of the idea and provide feedback, 3) Teachers should provide examples of how to complete the tasks. A critical success factor to discovery learning is the teacher assistance. On the other hand, DL potentially make the students feel confused and frustrated. Silver (2013) argued that pure unassisted discovery should be eliminated due to the lack of evidence that it improves learning outcomes. Bruner (1961) who was one of the early pioneers of discovery learning cautioned that discovery could not happen without some basic knowledge (mathematical prior knowledge).

In summary, the teachers' role in discovery learning is critical to the success of learning outcomes. Students must build foundational knowledge through examples, practice, and feedback. This can provide a foundation for students to integrate additional information and build upon problem solving and critical thinking skills. Early research demonstrated that guided discovery had positive effects on retention of information at six weeks after instruction versus that of traditional direct instruction. It is believed that the outcome of discovery-based learning is the development of inquiring minds and the potential for life-long learning. Discovery learning promotes student exploration and collaboration with teachers and peers to solve problems. Children are also able to direct their own inquiry and be actively involved in the learning process with the support of sufficient motivation (Reid, 2007).

The next one is contextual teaching and learning (CTL). CTL involves making learning meaningful to students by connecting to the real world (Johnson, 2002). It draws upon students' diverse skills, interests, experiences, and cultures and integrates these into what and how students learn and how they are assessed. In other words, contextual teaching situates learning and learning activities in real-life and vocational contexts to which students can relate, incorporating not only content, the "what," of learning but the reasons why that learning is important.

Some examples of contextual teaching and learning are interdisciplinary activities across content areas, classrooms, and grade levels; or among students, classrooms, and communities. Problem-based learning strategies, for instance, can situate student learning in the context of students' communities. Many skills learned as parts of contextual learning activities are transferable skills, can be used not only for successful completion of a current project but also in other content areas to prepare a student for success in later vocational endeavors. Contextual learning, then, engages students in meaningful, interactive, and collaborative activities that support them in becoming self-regulated learners. Additionally, these learning experiences foster interdependence among students and their learning groups. Complementary outcomes assessments for contextual student learning are authentic assessment strategies, i.e., the assessment is not only limited to the results of the written test but also based on the students' performance in doing the assignments.

Another constructivism-based learning model or approach is cooperative learning, an educational approach aimed to organize classroom activities into academic and social learning experiences (Arends, 2012). Cooperative learning is actually not merely arranging students into groups; it is characterized as "structuring positive interdependence." Students must work in groups to complete tasks collectively toward academic goals. Unlike individual learning, which can be competitive in nature, students learning cooperatively can capitalize on one another's resources and skills (asking one another for information, evaluating one another's ideas, monitoring one another's work, etc.). Furthermore, the teacher's role changes from giving information to facilitating students' learning. *Everyone succeeds when the group succeeds*. Ross & Smyth (1995) describe successful cooperative learning tasks as intellectually demanding, creative, open-ended, and involve higher order thinking tasks. Cooperative learning has also been linked to increase levels of student satisfaction. In cooperative and individualistic learning, student efforts are evaluated on a criteria-referenced base while in competitive learning teachers grade in a norm-referenced base.

Five essential elements are identified for the successful incorporation of cooperative learning in the classroom i.e:

i. Positive interdependence
ii. Individual and group accountability
iii. Promotes interaction (face to face)
iv. Teaching the students the required interpersonal and small group skills
v. Group processing.

Students in cooperative learning settings compared to those in individualistic or competitive learning settings, achieve more, reason better, and gain higher self-esteem. The next constructivism based learning approach is Open-ended approach. Open-ended approach provides students with experience in finding something new in the process of open problem solving (Becker & Shimada, 1997), while open problem solving is based on open-ended problems. It can be concluded that Open-ended problems used in mathematics lessons from elementary through high school grades. These problems proposed have several or many correct answer, and several ways to get the correct answer. There are five advantages of Open-ended approach:

a. Students participate more actively in lessons and express their ideas more frequently because Open-ended approach provides free, responsive, and supportive learning environment. The problem has many different correct solutions, so each student has opportunities to get his own unique answer. Hence, students are curious about other solutions and they can compare on and discuss their solutions. Those activities bring a lot of interesting conversation to the classroom.

b. Students have more opportunities to make comprehensive use of their mathematical knowledge and skills. Since there are many different solutions, students can choose their favorite ways toward the answer and create their unique solution. Activities can be the opportunities to make comprehensive use of their mathematical knowledge and skills.

c. Every student can respond to the problem in some significant ways of his/her own. Therefore, it is very important for every student to be involved into the classroom activities, and the lessons should be understandable for every student. The open-ended problems provide every student with the opportunities to find his/her own answer.

d. The lesson can provide students with a reasoning experience. Through comparing and discussing in the classroom, students are intrinsically motivated to give reasons of their solutions to other students. It is a great opportunity for students to develop their mathematical thinking.

There are rich experiences for students to have the pleasure of discovery and to receive approval from fellow students. Since every student has each solution based on each unique thinking, every student is interested in fellow students' solutions. There are also some disadvantages of the Open-ended approach (Sawada, 1997), such as the difficulty of posing problems successfully, the difficulty of developing meaningful problem situations, and the difficulty of summarizing the lesson.

Besides learning approach, ICT also influences learning outcomes (Agyei & Voogt, 2011). Moreover, affective aspects also affect learning achievement (Minarni, Napitupulu, Lubis, & Annajmi, 2018). Therefore, this paper focuses more on the description of the contribution of learning approach as well as the influence of interactions between the learning approach and the MPK on the achievement of MHOTS. However, the MPK factor plays an important role as well since it is needed to be recalled previously learned or provide the results of a calculation, which were considered lower cognitive questions in previous studies, played key stages at introducing new mathematical content as well as in the stage of solving mathematical problems.

The study first aimed to seek the answer on how the contribution degree of the constructivism-based learning to the achievement or improvement of students' MHOTS is, and second to reveal if there exists an interaction between the learning approach and the mathematical prior knowledge on the achievement of MHOTS.

2. METHOD

The population of this study was junior high school (PJHS) students in Medan, Deli Serdang, Binjai, and Padang Sidempuan in the Province of North Sumatera, and Banda Aceh in the Province of Nanggroe Aceh Darussalam. Because the school does not allow students to take randomly from each class, samples are taken per class. Classes are taken through simple random sampling because the students at all classes assumed homogenous mathematical prior ability, two classes from each district. One class is used as the experimental class, the other one is the control class. The experimental class applied constructivism-based learning, while the control class applies conventional learning. This research runs in the odd semester in the 2017/2018 and 2018/2019 academic years.

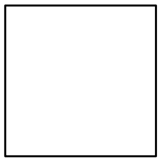
The instrument used in this study was an essay test of high-order mathematical thinking skills (MHOTS), which included tests of mathematical problem solving skills (MPSS). The MPSS indicators used in this study are modifications of Polya (2004) and NCTM (2000), including:

1. The ability of mathematical understanding shown by external representations and connections between ideas/facts/concepts/mathematical procedures.
2. The ability to propose problem solving strategies that are demonstrated by the existence of techniques/methods of problem solving in student worksheets, whether in the form of mathematical models, graphs, tables, diagrams, or others.
3. The ability to execute the proposed problem solving strategy, shown by calculations and mathematical manipulations to obtain a solution.
4. Summing up the solution obtained by following the initial problem.

Problem 1 below is an example of a question for developing mathematical creative thinking skills (Maharani, 2014) Grade VIII junior high school students.

Problem 1

Look at the quadrilateral model below.

10 cm


i. Draw a quadrilateral that has same area with the image above.
ii. Create at least two different questions related to square and solve it.

The following problem is an example of a problem that teacher proposed in the Open-Ended class (Problem 2).

Problem 2

The average of mathematics score of students from a junior high school is 65. What is the additional score if the average score of the exam becomes 68. Write down the steps you do to get a solution.

Implementation of Constructivism-based Learning

After the learning tools are validated, the next stages of research are as follows:

i. Conduct the test of mathematical prior knowledge (MPK).
ii. Implement constructivism-based learning.
iii. Organizing post-tests
iv. Analyzing research data
v. Discuss the results of the research
vi. Conclude

As long as the learning program took place, students are directed to solve mathematical questions contained in student worksheets (SWS). Each SWS, which consists of three to four problems, is designed based on the aspects of MHOTS the students must achieve. The teacher directs students to work cooperatively and collaboratively in groups to solve them. In general, this is how the learning process takes place in the classroom, whatever type of constructivism-based learning approach used. Of course, there are syntaxes differences between one learning approaches to the other, for example in the Open-ended approach, the questions contained in the SWS are open, that is, have various ways to get the solution and diverse solutions. In the RME approach, the questions are required to be contextual, and in the discovery learning approach, the question requires enough teacher's guidance to enable the student in getting the solution. In general, the questions contained in the SWS are designed based on the MPSA indicators modified from Polya and NCTM as mentioned in the introduction to this article.

In the control classroom, the teacher implemented direct teaching to the whole class. In this case, the teacher is considered as an essential role model and is expected to be an expert or learned figure. A good grasp of the subject matter is more important and serves as a prerequisite for this kind of pedagogy.

Data Analysis

Univariate and multivariate analysis is used as a statistical tool to analyze the contribution of treatment towards mathematical high order thinking skills (MHOTS) achievement, while t-Students is used as a statistical tool to determine the significant improvement of MHOTS (Glass & Hopkins, 1996). All analyses use a 0.05 level of significance. The role of the learning approach is elaborated through linking MHOTS achievements with the steps of the learning approach applied in the classroom based on the output of the regression analysis.

3. RESULTS AND DISCUSSION

The research took place in six different schools in the provinces of North Sumatra and Nanggroe Aceh Darussalam continuously from 2017 to 2019. The results of the study are presented in the following order.

3.1. Mathematical Communication achievement in PBL Classroom

The first study was conducted at public junior high school (PJHS) Muara Batu, Aceh. The purpose of this study is to improve mathematical communication skills (MCS) as one of the mathematical high-order thinking skills (MHOTS). For the sake of this matter, we implemented instructional materials that integrated Acehnese cultural context to problem-based learning (PBL). Instructional materials based on PBL is designed so that they meet valid, practical and effective criteria. Table 1 show the data on mathematical communication skills at trial I and II as a result of the research.

Table 1. Students MCS achievement at PBL classroom

Category	Trial I	Trial II
Highest	87.5	95.8
Lowest	50.0	68.8
Average	74.3	80.3

Every aspect of average MCS scores in experiments I as well as experiment II is presented in Table 2. There was an increase in mathematical communication skills after PBL implementation. This supports the results of previous studies that PBL can improve the ability of mathematical high-order thinking skills (MHOTS), where mathematical communication is one of the MHOTS. The implementation of PBL also allows the development of social skills (Arends, 2012) where one of the benefits of social skills is increased academic achievement (Minarni, 2013). Whether academic achievements in the field of mathematics or social fields, this requires separate research.

Table 2. Average score of students MCS at each aspect

Aspect	Trial I	Trial II
Explain the idea or situation of an image in his own words	10.3	11.2
Describe a situation in image	13.1	13.9
Describe the situation in mathematical equation	12.2	13.0

Overall, instructional materials based on PBL that integrate Aceh culture have fulfilled the criteria valid, practical and effective in accordance with the objectives of this study. The meaning of these criteria is:

a. Validity and Practicality:
   1) The average validity of RPP, Student Book, and Students Work Sheets (SWS) given by five validators is 4.60.
   2) In trial I, this instructional material only requires a slight revision. In trial II, the validators stated that this instructional material was valid.
   3) Based on the interview and questionnaire, the teachers and the students stated that there were no obstacles in using this instructional material.

b. Effectivity:
   1) More than 75% of students involved in this study have achieved minimum learning completeness requirements, namely achieving test scores more than 65 (in accordance with what was agreed by the ministry of education) (Table 1 and Table 2).
   2) Time provided is sufficient for learning implementation.
   3) Both the teachers and the students respond positively to the instructional materials.
   4) The mathematical communication skills of the students in the experimental classroom increased with average N-Gain 0.61 (calculated based on Table 1).

These findings show that integrated PBL in the instructional materials affects significantly the achievement of MCS. The learning approach gives contribution to students learning outcomes.The study implies that if the teacher has the opportunity to design appropriate instructional materials based on the constructivism learning approach for developing MHOTS, then teacher's desire to improve students HOTS will be viable. Furthermore, Indonesia is a country with rich types of local culture; it should be easier to enrich the repertoire of cultural-based mathematical knowledge. Another idea conveyed based on this research is that schools can ask the government to provoke the implementation of research results such as learning materials developed based on constructivism. In addition, based on an interview the students give positive responses to the implementation of Problem-based learning, so it makes sense that PBL gives contribution to the improvement of students' MCS.

3.2. Mathematical Understanding Achievement in Cooperative Learning Classroom

The second study was carried out to investigate the effect of cooperative learning assisted mapping concept and Microsoft Visio (CLMV) towards mathematical understanding concepts (MUC). The sample consisted of 34 students of eighth-grade Al-Ulum Islamic Middle Secondary School, Medan, in the Academic Year 2017/2018. CLMV was used in the experiment classroom, while HOTS to be developed was MUC. Previous research (Arslan & Altun, 2007) revealed mathematical prior knowledge (MPK) does not affect the achievement of mathematics learning outcomes, but many other researchers confirmed that it is influential. MUC test scores from the experimental class and the control class are shown in Table 3.

Table 3. MUC score test of the students

MPK	Learning Approach	
	CLMV	Conventional
High	82	56
Medium	62	42
Low	63	44

Table 3 shows that there is a difference in the MUC test score between the experimental class and the control class. This suggests that there is a significant contribution of CLMV to the achievements of the MUC. Thus, we used ANOVA to test the contribution. The result of the test is presented in Table 4.

Table 4. Contribution of learning approach to MUC

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	8065.8[a]	5	1613.2	8.5	0.000
Intercept	108817.9	1	108817.9	571.4	0.000
MPK	960.3	2	480.2	2.5	0.088
Learning Approach	4303.3	1	4303.3	22.6	0.000
Learning App.*MPK	416.2	2	208.1	1.1	0.342
Error	11997.0	63	190.4		
Total	223107.0	69			
Corrected Total	20062.8	68			

a. R Squared = 0.40 (Adjusted R Squared = 0.36)

The test results in Table 4 interpreted as follows:

a. There is an effect of the learning model on the ability of MUC.
b. The contribution of learning factors to the achievement of MUC is 40%.
c. There is no effect of interaction between the learning approach and MPK factors on MUC achievement.
d. MUC of the students taught through cooperative learning assisted mapping concepts and Microsoft Visio software is better than MUC of the students taught through direct instruction.

Indeed, the involvement of software as part of ICT undeniably gives a positive impact on student learning outcomes (Agyei & Voogt, 2011). Furthermore, Indonesia is a country that is responsive to the development of ICT. Almost all PJHS students have smartphones that can make it easier for them to download software or other applications needed in the learning process. Therefore, the readiness of teachers is needed to integrate ICT in mathematics learning.

Besides, the results of the observation indicate that learning in the experimental class is in line with the stages specified in the cooperative learning approach. Activities to solve problems in the class that is done cooperatively give results in the form of increasing student MUC achievements. This is the important role of Cooperative learning in improving mathematical high order thinking of the students. This research is in line with the theory of cooperative learning, which states that learning through small groups enables increased learning achievement because in cooperative learning; tasks are designed so that they meet intellectually demanding, creative, open-ended, and involve higher-order thinking tasks (Ross & Smyth, 1995) which allow the growth of MUC as one of MHOTS.

The weakness found in this study is that the teacher is a little excessive in assisting because some students experience dead ends in solving problems. This needs to get the attention of policymakers so that teachers do not give up in applying this innovative learning.

3.3. The Achievement of MPSS in Contextual and Cooperative Learning Classroom

Subsequent research was carried out at Medan Budi Agung Middle Secondary School. Mathematical high order thinking skills (MHOTS) is investigated is mathematical problem solving skill (MPSS). Through the implementation of Cooperative learning and Contextual Teaching Learning (CTL) Geogebra-assisted, this study aimed to investigate the difference between students' MPSS in Cooperative learning classroom (Experimental class I) and contextual classroom (Experimental class II). In both experimental classes, learning was implemented with the help of Geogebra software. Student MPSS test scores are presented in Table 5.

Table 5. Statistic of students MPSS score

Learning Approach	Statistic	
	Average	SD
Cooperative	52.79	17.059
CTL	53.45	15.999

Table 5 shows that there is a difference in the MPSS score test between the two experimental classrooms. Geogebra-assisted contextual learning (CTL-G) is superior in improving student MPSS compared to Geogebra-assisted cooperative learning. The difference in MPSS achievements shows that there is an influence or contribution of the learning approach to the MPSS. Thus, we do the test of difference achievement of the MPSS through two-way ANOVA at 0.05 significance level. The test result is presented in Table 6.

Table 6. Test of the Effect of Learning Approach to MPSS

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	1206.4[a]	3	402.1	4.2	0.009
Intercept	326446.6	1	326446.6	3.4E3	0.000
MPK	178.2	1	178.2	1.9	0.177
Learning Approach	571.4	1	571.4	5.9	0.018
Learning App. * MPK	402.2	1	402.2	4.2	0.045
Error	5340.2	56	95.4		
Total	332448.0	60			
Corrected Total	6546.6	59			

a. R Squared = 0.18 (Adjusted R Squared = 0.14)

Based on data in Table 6, it can be concluded that the test result is significant, this means that:

a. There is the MPSS difference between students taught by Geogebra-assisted cooperative learning and students taught Geogebra-assisted CTL. This means, there is an effect of learning factor on the students' MPSS achievement. The effect is measured by the degree of contribution. The contribution of learning factors to MPSS achievements is around 18%. Both Geogebra-assisted CTL and Geogebra-assisted Cooperative learning plays a substantial role in achieving MPSS.
b. There is an interaction effect between the learning approach and MPK factors on MPSS achievement. It means the students from low and medium MPK get benefit from this kind of learning approach.

The advantages of contextual learning that make it possible to play an important role in improving the MPSS of the students are characteristics of problems that are designed to connect with the context of students' daily lives. Based on interviews, students acknowledge that the problems given by the teacher are quite interesting and easier to understand because they are familiar with the theme of the problem. Understanding the problem is the first and foremost thing in solving problems and according to the CTL theory, making learning meaningful to students by connecting to the real world is the core element in CTL (Johnson, 2002).

Meanwhile, the weakness of this study mainly lies in the weakness of student MPK, in line with the results of other studies (Minarni et al., 2016), such that the teacher is forced to remind students of mathematical knowledge that is not well stored in the cognitive structure of students. Based on the results of this study, it is suggestions that:

a. The teacher is advised to use Cooperative learning and CTL to enhance student achievement in mathematical problem solving skills.
b. In implementing constructivism-based learning such as Cooperative learning and CTL, the teacher is advised to involve information and computer technology (ICT) such as Geogebra software, especially for generating student's interest in studying geometry. Because through the help of the software, the display of geometric forms can be visualized more accurately and more 'eye-catching' which increases students' enthusiasm for learning and challenging them to explore other problems related to geometry problems. From this activity, it is hoped that the student's perseverance and life-long learning will be grown.
c. The teacher is advised to strengthen students' comprehension of mathematical knowledge and mathematical concepts, as well as MPK.

If MPK becomes an obstacle in achieving MPSS then the implementation of innovative learning such as CTL and cooperative learning becomes increasingly important because the main advantage provided by these two constructivism-based learning is that learners will be able to store knowledge in long-term memory to guarantee the availability of MPK. This again shows that self-constructed knowledge can make a person firm in storing his knowledge. The following explanation is an example of the problem used in research and alternative solutions.

Problem:

"A cube-shaped aquarium with a length of 85 cm is filled with water. If a decorative stone with a volume of 125 litres is put into the tank, determine the volume of water left in the tank."

This problem is closely related to other disciplines, namely Physics. Moreover, this question is also related to the context of students' daily life where students are very familiar with aquariums as a container for keeping fish that require ornamental stones to mimic original fish habitat.

Solution:

$$V_{aq} = \text{s} \times \text{s} \times \text{s cm}^3 = 85 \times 85 \times 85 \text{ cm}^3$$
$$= 614.125 \text{ cm}^3 = 614.125 \text{ dm}^3$$
$$= 614.125 \text{ liter}$$

$$V_{water} = V_{aq} - V_{stone} = 614.125 - 125.000$$
$$= 489.125 \text{ liter.}$$

To solve the problem, students should execute four steps, i.e.:

a. Calculate the initial volume of water in the aquarium.
b. Convert water volume units (from cm^3 to liters) to equal the volume units of ornamental stones.
c. Find the reduction of the initial volume because of the insistence of ornamental stones.
d. Conclude the volume of water left in the aquarium after ornamental stones press some water out.

Thus, this mathematical problem has characteristics as a good question, that is contextual, interesting, related to other disciplines, and requires multi-step to get a solution. All of these characteristics are in line with HOTS proposed by Resnick (1987).

3.4. MPSS Achievement in Realistic Mathematics Education Classroom

The fourth study was carried out at PJHS 2 Beringin, District of Deli Serdang. This research is an effort to improve students' mathematical problem solving skills (MPSS) through a realistic mathematical education (RME) approach assisted by Autograph. The MPSS score test of the students is presented in Table 7.

Table 7. Statistic of students MPSS

Learning Approach	MPK	Statistic	
		Average	SD
RME	High	19.02	2.452
	Medium	18.02	2,706
	Low	17.20	2,680
Conventional	High	17.03	1.150
	Medium	15.90	2.850
	Low	12.80	2.030

Data from the research was analyzed through two-way ANOVA. The result of the analysis is presented in Table 8. It can be seen in Table 8 that there are differences in MPSS scores between students in the experimental class and students in the control class. The difference in achievement is quite large, this means that there is an influence or contribution of the treatment to MPSS achievements (Glass & Hopkins, 1996). The significance level test of the contributions is carried out through the Analysis of Variance. The output of the test is displayed in Table 8.

Table 8. The Contribution of RME towards MPSS

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	175.9[a)]	5	35.2	3.3	0.012
Intercept	12886.6	1	12886.6	1193.9	0.000
Learning Approach [b)]	59.5	1	59.5	5.5	0.022
MPK	98.5	2	49.3	4.6	0.014
Learning App. *MPK	2.2	2	1.1	0.1	0.902
Error	625.9	58	10.8		
Total	18292.0	64			
Corrected Total	801.9	63			

a) R squared = 0.22 (Adjusted R Squared = 0.15)
b) Learning Approach: RME

Based on the result of the analysis in Table 8, the research findings are:

a. The enhancement of the student MPSS in the experiment classroom is higher than the in the conventional classroom.
b. The learning factor has a significant influence on the achievement of MPSS.
c. The adjusted R squared is 0.15. This means the contribution of the learning approach to MPSS is 22%.
d. There is no interaction effect between the learning approach and MPK factor on the MPSS achievement.
e. The process of solving mathematical problems shown by students in the experimental classroom is better (in terms of more MPSS indicators that are met, systematic and directed).

In this study, the teacher has implement RME properly, that is, the learning is conducted so that the students construct knowledge by themselves, in line with the socio-constructivism as one principle of RME (de Lange, 1996; Gravemeijer, 1994) and students are offered opportunities to share their experiences with others. Besides, de Lange (1996) stated that the compatibilities of socio-constructivist and RME are based on a large part or similar characterizations of mathematics and mathematics learning because they are struggling with the idea that mathematics is a creative human activity and mathematical learning occurs as students develop effective ways to solve problems (Streefland, 1991; Treffers, 1991). So, if this research is not successful enough in developing student MPSS, it is shown by the low MPSS score and the low contribution of RME to MPSS achievements (only 15%), then that becomes a problem that we must think of a solution.

The most striking obstacle in this study is the weakness of students in representing problems into various forms of representation such that they have trouble at the stage of ‘model of’ or mathematical horizontal stage. Although students have trouble in the horizontal mathematical stage and vertical mathematical in the RME, or the ‘model of’ and ‘model for’ stages make students feel helped in solving problems. It seems that this gives a role in increasing student MPSS.

One thing could be suggested from the study is time allocation. As in other studies, the time available is always insufficient to conduct the learning process that aims in enhancing high-order thinking skills. For this reason, it is recommended that the teacher prevent the debate or a prolonged argument among students. The teacher must immediately take over and decide firmly which correct solution is for a certain problem, and which solution still have shortcomings or mistakes.

3.5. Mathematical Creative Thinking Achievement in Open-Ended Classroom

The fifth study took place in the Public Middle Secondary School Number 2 at Padangsidempuan. The constructivism-based learning applied here is the Open-ended approach. The research objectives are:

a. To investigate the influence of the Open-ended approach integrated Batak Angkola culture (OEBC) towards students' mathematical creative thinking skills (MCTS).
b. To investigate the effect of the interactions on MCTS.

The average score of MCTS of the students is presented in Table 9.

Table 9. Statistic of students MCTS

Learning Approach	MPK	Statistic	
		Average	SD
OEBC	Low	61.99	8.39
	Medium	69,70	16.30
	High	90.70	7.00
Conventional	Low	32.30	5.40
	Medium	48.70	13.00
	High	68.15	8.40

Data from the research was analyzed through two-way ANOVA. The ANOVA output is presented in Table 10.

Table 10. Contribution of open-ended approach towards MCTS

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	15436.534[a]	5	3087.307	15.350	0.000
Intercept	195925.534	1	195925.534	974.151	0.000
MPK	9121.630	2	4560.815	22.677	0.019
Learning App.	5829.018	1	5829.018	28.982	0.012
Learning App.*MPK	499.494	2	249.747	1.242	0.296
Error	11665.216	58	201.124		
Total	258944.000	64			

a) R Squared = 0.57 (Adjusted R Squared = 0.53)

The results of the study indicate that: (a) There is an influence of the Open-ended approach integrated Batak Angkola culture towards the achievement of students' mathematical creative thinking skills (MCTS); (b) R Squared = 0.57. This means the contribution of learning factor to the achievement of MCTS is 57%; (c) There is no common influence (interaction) between the Open-ended approach integrated Batak Angkola culture and MPK towards the achievement of students' mathematical creative thinking skills.

The MCTS score test of the students at the experiment classroom is high enough, so does the contribution of learning approach towards MCTS. It is possible since the Open-ended approach implemented in the classroom provides students with experience in finding something new in the process of open problem solving (Becker & Shimada, 1997), while open problem solving is based on open-ended problems. These characteristics enable the students to participate more actively in lessons and express their ideas more frequently; have opportunities to get a unique answer; develop curiosity about other solutions and they can compare on and discuss their solutions; make comprehensive use of their mathematical knowledge and skills. Since there are many different solutions, students can choose their favorite ways toward the answer and create their unique solution. The study also shows that the students involved in the classroom activities and enable the students to have reasoning experience and build intrinsic motivation to give reasons for their solutions to other students. It is a great opportunity for students to develop their mathematical thinking. All of these characteristics meet the demands of the Open-ended approach.

The weakness of this study is the students are less courageous in conveying ideas. This may be due to Indonesian culture that children are generally educated not to argue with their parents or other family members. They are usually educated to obedient Children.

Overall, based on the results of this study, the following suggestions are offered.

a. The teacher is suggested to be creative in creating a learning atmosphere that allows students to express mathematical ideas in their language such that the students have self-confidence, creativity, and courage to argue with their classmates.
b. The teacher is suggested to provide a variety of mathematical problems that are in line with the context of the local culture and lure students to relate them to the subject matter or other mathematical problems. If this is done, it will build students' perception that mathematics is useful in their daily lives.
c. The teacher should allocate a more accurate time so that this constructivist-based learning activity can run smoothly.

Preparation of discussion groups is only carried out once at the beginning of learning such that there is more time available for group discussion activities in the next session.

3.6. MPSS Achievement in Discovery Learning Classroom

The final research was conducted to develop students' achievement in Mathematical Problem Solving Skills (MPSS) through the implementation of instructional materials based on discovery learning approach. There are 40 students included in the experiment class and 40 students in the control class. All of the students are from Public Junior High School (PJHS) 17 Medan. An essay test is used to collect data of the students' MPSS. Students' MPSS score test is presented in Table 11.

Table 11. Statistic of students MPSS

Learning Approach	Statistic	
	Average	SD
Discovery	15.03	2.282
Conventional	10.33	1.493

Note: Ideal score = 20

Based on Table 11, the MPSS achievement of students in the experimental class (Discovery learning class) is better than the conventional class. This shows that there is an influence/contribution of learning to student MPSS achievements. The results of this study support the theory that through the discovery learning approach, students' trust in efforts to solve problems increases because they are accustomed to conducting investigations to find information needed to solve problems. According to Dewey's opinion in finding such knowledge a person unknowingly stores information in ways that make information easier to use in solving new problems. (Arends, 2012). Thus, the achievement of problem solving skills become significant. The time limitation to implement discovery learning is a major obstacle in the completion of complete learning. The enhancement of MPSS performance was presented in Table 12.

Table 12. The enhancement of MPSS

	F	T	Df	Sig.	Mean Diff.	Lower	Upper
Eq. var.	0.003	3.557	78	0.001	3.800	1.673	5.927

This significant increase in MPSS encouraged researchers to statistically test the contribution of discovery learning to MPSS achievements. Test results are presented in Table 13.

Table 13. Test of learning approach effect towards MPSS

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
1. Corrected model	308.112	1	308.112	19.852	0.000
2. Intercept	11640.313	1	11640.313	750.011	0.000
3. App	308.112	1	308.112	19.852	0.000
4. Error	1210.575	78	15.520		
5. Total	13159.000	80			

R Squared = 0.203

Results of the research: (a) The enhancement of students' MPSS is significant with a mean difference of 3.80 (MPSS ideal total score was 20) (Table 12); (b) The contribution of the developed instructional materials based on discovery learning towards MPSS is 20.3% (Table 13).

The results of this study indicate a high increase in MPSS (3.8 points), but this research found that the increase is not due to the contribution of the learning approach because of its small contribution, which is only 20%. There may be other factors contribute more significantly to students' mathematical problem solving abilities. It is common in the world of education that there are indeed many factors that contribute to student learning outcomes in matthematics, including teacher factors, school environment, friends, and affective aspects such as mathematical disposition, social skills, self-confidence, self-regulated learning, and others (Minarni et al., 2018), and others. Finding out the dominant factors contribute to learning outcomes is the attention and interest of educational researchers.

4. CONCLUSION

Based on the result of the research then the conclusion are, firstly, constructivism-based learning can improve mathematical high order thinking skills (MHOTS) such as mathematical connection, mathematical understanding, mathematical problem solving, and mathematical creative thinking skills/ability. In the experimental classroom, students'

MHOTS increased significantly. The contribution of constructivism-based learning to MHOTS is in the range of 18% to 57%. Secondly, based on the results of observations made by the observer, the activity of students in the learning process increases significantly. Third, in some cases, there is an influence of interaction between the learning approach and students' mathematical prior knowledge towards the achievement of MHOTS. Fourth, based on observations and interview results, the integration of ICT to the learning approach increases students' enthusiasm in solving mathematical problems. Maybe there are other factors besides learning that contributes more to higher-order mathematical thinking skills (MHOTS), for example, affective factors such as mathematical disposition, social skills, and learning motivation. To approve this allegation, of course, requires special research.

Some suggestions can be drawn from the study are (1) We suggest that the teacher dares to implement constructivism-based teaching-learning approach to improve HOTS; (2) The teachers are advised to have the willingness to integrate local culture in the learning process to improve students' interest in solving mathematical problems; (3) The teachers are advised to integrate ICT in explaining subject matter and describing the solution the student gets so that the explanation is easier for students to understand.

REFERENCES

Agyei, D. D., & Voogt, J. (2011). ICT use in the teaching of mathematics: Implications for professional development of pre-service teachers in Ghana. *Education and information technologies*, *16*(4), 423-439. https://doi.org/10.1007/s10639-010-9141-9

Anderson, L. W., Krathwohl, D. R., Airasian, P. W., Cruikshank, K. A., Mayer, R. E., Pintrich, P. R., & Raths, R. (2001). M. C Wittrock. A Taxonomy for Learning, Teaching and Assessing. *New York: David McKay Company*.

Arends, R. I. (2012). *Learning to teach*. McGraw-Hill Companies.

Arslan, Ç., & Altun, M. (2007). Learning to solve non-routine mathematical problems. *Elementary Education Online*, *6*(1), 50-61.

Becker, J. P., & Shimada, S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics*. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1593.

Brown, A. L. (1990). Domain-specific principles affect learning and transfer in children. *Cognitive science*, *14*(1), 107-133.

Bruner, J. S. (1961). The act of discovery. *Harvard educational review, 31*, 21-32.

Cottrell, S. (2005). *Critical Thinking Skills: Developing Effective Analysis and Argument.* New York: Palgrave Macmillan.

De Lange, J. (1996). Using and applying mathematics in education. In *International handbook of mathematics education*, 49-97. Springer, Dordrecht. https://doi.org/10.1007/978-94-009-1465-0_3

Freudenthal, H. (2006). *Revisiting mathematics education: China lectures* (Vol. 9). Springer Science & Business Media.

Formaggia, L. (2017). *Mathematics and Industry 4.0*. International CAE Conference, Vicenza, November 6-7, 2017.

Glass, G., & Hopkins, K. (1996). Statistical methods in education and psychology. *Psyccritiques*, *41*(12).

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. Utrecht: Freudenthal Institute.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in mathematics*, *39*(1-3), 111-129. https://doi.org/10.1023/A:1003749919816

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn?. *Educational psychology review*, *16*(3), 235-266. https://doi.org/10.1023/B:EDPR.0000034022.16470.f3

Jensen, I., R. (1976). *Five-point Progress for the Gifted*. Poster Presentation. International Congress on Mathematical Education, Karlsruhe, Germany.

Johnson, E. B. (2002). *Contextual teaching and learning: What it is and why it's here to stay*. California: Corwin Press.

Kulm, G. (1990). *Assessing Higher Order Thinking in Mathematics*. American Association for the Advancement of Science Books.

Maharani, H. R. (2014). Creative thinking in mathematics: Are we able to solve mathematical problems in a variety of way. In *International Conference on Mathematics, Science, and Education*, 120-125.

Marzano, R. J., & Kendall, J. S. (2007). *The new taxonomy of educational objectives*. Corwin Press.

Minarni, A. (2013). Pengaruh pembelajaran berbasis masalah terhadap kemampuan pemahaman matematis dan keterampilan sosial siswa SMP Negeri di Kota Bandung. *Jurnal Paradikma*, *6*(02), 162-174.

Minarni, A., Napitupulu, E., & Husein, R. (2016). Mathematical understanding and representation ability of public junior high school in north sumatra. *Journal on Mathematics Education*, *7*(1), 43-56. https://doi.org/10.22342/jme.7.1.2816.43-56

Minarni, A. (2017). On Eight Grade Students Understanding in Solving Mathematical Problems. *Asian Social Science*, *13*(12), 86-96. https://doi.org/10.5539/ass.v13n12p86

Minarni, A., Napitupulu, E. E., Lubis, S. D., & Annajmi (2018). *Kemampuan Berpikir Matematis dan Aspek Afektif Siswa.* Medan: Harapan Cerdas Publisher.

NCTM (2000). *Principle and Standards for School Mathematics*. Reston: VA.

Phillips, D. C. (1997). How, why, what, when, and where: Perspectives on constructivism in psychology and education. *Issues in Education*, *3*(2), 151-194.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (No. 246). Princeton university press.

Reid, G. (2007). *Motivating learners in the classroom: ideas and strategies*. Sage.

Resnick, L. B. (1987). *Education and learning to think*. Washington DC: National Academies.

Ronis, D. L. (2008). *Problem-based learning for math & science: Integrating inquiry and the internet*. Corwin Press.

Ross, J. A., & Smyth, E. (1995). Differentiating cooperative learning to meet the needs of gifted learners: A case for transformational leadership. *Talents and Gifts*, *19*(1), 63-82.

Savin-Baden, M., & Major, C. H. (2004). *Foundations of problem-based learning*. McGraw-Hill Education (UK).

Sawada, T. (1997). Developing Lesson Plans. In: J. Becker, & S. Shimada (Eds.). *The Open-Ended Approach: A New Proposal for Teaching Mathematics* (pp. 1-9). Reston, VA: National Council of Teachers of Mathematics.

Scriven, M., & Paul, R. (1987). Defining critical thinking. In *8th Annual International Conference on Critical Thinking and Education Reform, Summer*.

Sheffield, L. J. (2013). Creativity and school mathematics: Some modest observations. *ZDM*, *45*(2), 325-332. https://doi.org/10.1007/s11858-013-0484-8

Silver, E. A. (2013). *Teaching and learning mathematical problem solving: Multiple research perspectives*. Routledge.

Streefland, L. (1991). *Fractions in realistic mathematics education: A paradigm of developmental research* (Vol. 8). Springer Science & Business Media.

Torrance, E., P. (1960). *Educational Achievement of the Highly Intelligent and Highly Creative: Eight Partial Replications of the Getzels and Jackson Study*. Research Memorandum. BER 60-68, Bureau of Educational Research: University of Minnesota.

Treffers, A. (1991). Realistic mathematics education in the Netherlands 1980-1990. *Realistic mathematics education in primary school*, 11-20.

Widodo, A. (2004). *Constructivist oriented lessons: The learning environments and the teaching sequences*. Frankfurt: Peter Lang GmbH.

Vygotsky, L. S. (1980). *Mind in society: The development of higher psychological processes*. Harvard university press.