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MATHEMATICAL PROBABILITY: LEARNER'S MISCONCEPTION IN A SELECTED SOUTH AFRICAN SCHOOL

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ABSTRACT

Mathematics plays an essential role in developing human thought, particularly in developing problem-solving and reasoning. While mathematics has become a problem-solving tool in various fields, including science, it has distinct qualities known as probability and, more specifically, probability theory. For most learners, the probability is difficult to learn and conceptualize. Hence, the present study investigates learners' misconceptions in the teaching and learning of probability in a selected school in the Eastern Cape Province, South Africa. Underpinned by a Post-positivist paradigm, the study employed a quantitative research approach and a survey design in which data were gathered from mathematics learners from grades 10-12. Findings revealed that although the frequency of misconceptions varied across grade levels, it was difficult to describe how misconceptions about probability changed. As such, while learners progressed through the grades, some misconceptions faded with age, others remained stable, and others grew in power. The findings also revealed that the types of probability misconceptions did not differ significantly by gender, and male learners tend to have more misconceptions about probability than female learners.

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1. INTRODUCTION

Student misconceptions have been a concern for many educators, researchers, and mathematics teachers. As a result, an increasing number of researches in mathematics education focus on students' misconceptions in various mathematical domains (Mut, 2003; Santos-Trigo, 2020; Stylianides et al., 2016). Several studies that deal with probability and probabilistic thinking have often been categorised into two types. While the first type focuses on how people think, the second focuses on influencing how people think, both of which are investigated by psychologists and mathematics educators, respectively (Mut, 2003;

Shaughnessy, 1992). There is also substantial evidence that traditional probability instruction, which consists mostly of formal definitions, rules, and procedures, does not eliminate misconceptions about probability (Khazanov, 2005; Mutara & Makonye, 2016; Shaughnessy, 1992).

The above assertion is true, given that misconceptions can coexist perfectly with correct conceptions, thereby interfering with learners' ability to apply them regularly and confidently (Khazanov & Gourgey, 2009; Mutara & Makonye, 2016). As a result, while learners may master probability rules and processes and can calculate accurate answers on mathematics tests, these same learners often misunderstand essential principles and concepts, frequently disregarding the rules when making decisions regarding uncertain situations. However, numerous studies from various theoretical vantage points appeared to support the idea that students frequently hold beliefs that impede their understanding of ideas in probability (Ang & Shahrill, 2014; Batanero et al., 2016). As a result, it is widely agreed upon that Representativeness, Equiprobability bias, Beliefs, and Human control are a few frequent ways of thinking that prevent learners from learning about probability (Ang & Shahrill, 2014).

For instance, the representativeness misconception concerns students' erroneous belief that samples that match the population distribution are more likely to be accurate than those that do not (Kahneman & Tversky, 1973). As such, students with this misconception, for instance, will believe that a series of coin tosses with roughly equal numbers of heads and tails is more likely than a series with significantly more tails than heads (Ang & Shahrill, 2014; Kahneman & Tversky, 1973). However, the probability for both series is the same. Regarding equiprobability bias, students who hold this false belief typically believe that random events are equally likely by their very nature (Ang & Shahrill, 2014; Khazanov & Prado, 2010). Or, to put it another way, they see the odds of achieving diverse results as equally likely occurrences. According to Lecoutre (1992), the propensity of learners to see various experiment results as equally likely is equiprobability. For instance, pupils who believe in the equiprobability bias mistakenly believe that all possible sums are equally likely when two dice are rolled. However, students are unaware that the sum of the two dice is more likely to be 6 or 7 than 2 or 12. In terms of beliefs, many young learners believe that a force outside of their control determines the final result of an event. Sometimes this power is God, other forces like the wind, and other times its luck or wishes (Truran, 1994). Regarding human control, Nicolson (2005) argues that some learners believe that the outcomes depend on how one throws or manipulates these various tools.

It has long been understood that one of the most crucial educational objectives of stochastic instruction is to rectify students' misconceptions about probability (Ang & Shahrill, 2014; Gallagher, 2023). Shaughnessy, a prominent researcher in the field of probability and statistics education, argues that one of the critical objectives of stochastic instruction should be to show students "how misconceptions of probability can lead to erroneous decisions" (Shaughnessy, 1992, p. 482). Thus, addressing students' incorrect intuitions and assumptions requires a significant shift in focus from merely supplying formulas, rules, and calculational procedures when teaching probability for conceptual understanding (Khazanov & Gourgey, 2009). While probability provides a tool for modelling and producing reality, misconceptions about probability can influence people's judgment in crucial situations, such as investing, jury verdicts, and medical tests (Sharna et al., 2021).

Given the importance of probability, new mathematics curricula for schools are being established in countries worldwide. For example, since gaining democracy in 1994, South Africa has implemented several educational reforms (Olawale et al., 2021), particularly in the mathematics curriculum, with the probability topic becoming compulsory for the first time in grades 10 to 12 in 2012 (Khazanov & Prado, 2010; Makhubele, 2015). The topic of probability in the South African Curriculum Assessment Policy Statements (CAPS) entails knowing how to determine the likelihood of events occurring, and this topic is ranked sixth in importance in the mathematics curriculum for Further Education and Training (FET) (Department of Basic Education, 2011). While topics such as theoretical and experimental probability, dependent and independent events, simple and compound events, and the generalisation of the fundamental counting principle are covered, the probability is given an 18 percent weighting in Grades 10, 11, and 12 mathematics curricula due to its relevance and importance (Mutara & Makonye, 2016).

However, despite the importance of probability and its weighting in the South African Curriculum, one of the major challenges faced by high school teachers saddled with the responsibilities of teaching this topic include incoherent probability content knowledge which became an immediate issue after the reintroduction and compulsion of this topic (Chernoff, 2012). It could also be argued that most South African mathematics teachers face numerous difficulties in teaching probability because teaching probability for conceptual understanding requires a considerable shift in focus from just providing formulas, rules, and methods for computations to addressing students' erroneous intuitions and prejudices (Khazanov & Gourgey, 2009; Khazanov & Prado, 2010; Sharma, 2006). In addition, there were also insufficient teaching and learning support materials available to deal with this new issue (Mutara & Makonye, 2016). In light of these circumstances, we investigated learners' errors and misconceptions related to the solutions of probability problems amongst learners in grades 10-12. Thus, the main objective of this study was to investigate how the five common misconceptions of mathematical probability differ in relation to grade level and gender among high school learners.

2. METHOD

This study is guided by a post-positivist paradigm. According to Creswell (2014), post-positivists are deterministic, reductionists interested in identifying the reasons that impact specific outcomes. This paradigm shifts away from the solely objective perspective taken by the logical positivists and is concerned with the subjectivity of reality. A post-positivist paradigm was found suitable for this study because it prioritises creating numerical measures of observations and researching human behaviour. Given that paradigm selection determines the research approach, this study employed a quantitative research approach. To investigate phenomena and their interactions in a methodical way, quantitative research methods use numbers and anything that can be measured to explain, predict, and control a phenomenon; the quantitative approach seeks to provide answers to queries about correlations among quantifiable variables (Creswell, 2014; Mohajan, 2020).

For this study, three groups of learners were investigated which are five learners in grade 10 (ages 15-16), 12 learners in grade 11 (ages 16-17), and seven learners in grade 12 (ages 17-18), making a total of 24 learners (see Table 1). The statistical age standard per grade is the grade number + 6 according to the South African Schools Act, 84 of 1996 (Department of Basic Education, 2011). The convenience sampling technique was considered appropriate for the study given that the sample represented a range of learners from different socio-economic and cultural backgrounds. For the present study, female learners constituted 54% of the sample size, while male learners were only 46% (see Table 2).

Table 1. Learner’s distribution according to grade level

Grade	Number(s)	Percentage
Grade 10	5	21%
Grade 11	12	50%
Grade 12	7	29%
Total	24	100

Table 2. Learners’ distribution according to gender

Grade	Number(s)	Percentage
Females	13	54%
Males	11	46%
Total	24	100

The Probabilistic Misconception Test (PMT) was developed to collect data for the study. The PMT test consisted of five well-known probability questions administered to the participants. The test was related to five different types of probability misconceptions such as:

a. Simple and Compound Events: for example, “The letters in the word “CICEK” are written one by one on the cards, and then these cards are placed in a bag. What is the probability of getting the letter “C” from this box at random?” (İlgün, 2013; Mut, 2003).
b. Representativeness: for example, “Say you flip an ordinary quarter several times in successions with H representing a Head coming up and T representing a Tail. The notation HT means that in two successive flips, a Head occurred, followed by a Tail. If you flip a quarter 5 times in succession, which of the following sequences are you most likely to observe” (İlgün, 2013; Mut, 2003).
c. Positive and Negative Recency Effects: for example, “When tossing a coin, there are two possible outcomes: either heads or tails. Adu flipped a fair coin three times, and in all cases, tails came up. Adu intends to flip the coin again. What is the chance of getting heads at the fourth time?” (İlgün, 2013; Mut, 2003).
d. Effect of sample size: for instance, “A doctor keeps the records of newborn babies. According to his records, the probability of which of both gender (male & female) is higher?” (İlgün, 2013; Mut, 2003).
e. Equiprobability Bias: e.g., “There are six fair dies, each of which an ordinary cube with one face is painted white, and the other faces painted black. If these dies are tossed, which would be more likely?” (İlgün, 2013; Mut, 2003).

As such, one question each was raised based on the different types making a total of five (5) questions. The items in the PMT test were gathered from relevant literature. Two mathematics teachers from the selected school and one lecturer from the mathematics education department revised and controlled the question in terms of mathematical structure to ensure the instrument's content validity. The data collected were analysed descriptively. The levels of all the independent variables utilised in this study were used to compute the frequency of dependent variables. Frequency tables were used to tabulate the dependent variables in relation to the independent variables. The university's ethics committee approved this study, and formal approval letters were sent to the participating schools' students, lecturers, and principals to request their consent.

3. RESULT AND DISCUSSION

The present study sought to investigate learners' misconceptions with regard to probability based on grade level and gender.

3.1. Types of Misconceptions in Relation to Grade Level

3.1.1. Misconceptions in relation to Simple and Compound Events

For this study, the first question in relation to "Simple and Compound Events" with respect to grade levels was presented as shown in Table 3.

Table 3. Number and percentages of learners' responses to question 1 by grade level

Answers	Grade Levels			Total
	Grade 10	Grade 11	Grade 12	
Incorrect Answers	1	2	0	3
	20%	17%	0%	13%
Correct Answers	2	10	7	19
	40%	83%	100%	79%
Misconceptions	2	0	0	2
	40%	0%	0%	8%

Table 3 shows that 79 percent of learners answered the question correctly, and it could be concluded that learners better understand the simple and compound events in probability. In grades 11 and 12, the percentages of correct answers were higher. However, only 8 percent of learners had misconceptions in relation to this topic. From the above table, one could conclude that there was an effect of grade level on this misconception type as learners in grades 11 and 12 had not reported any form of misconception which could be because of their exposure to this topic during their entrance into the Further Education and Training (FET) phase, that is, grade level 10.

3.1.2. Misconceptions in relation to Representativeness

Table 4. Number and percentages of learners' responses to question 2 by grade level

Answers	Grade Levels			Total
	Grade 10	Grade 11	Grade 12	
Incorrect Answers	2	7	4	13
	40%	58%	57%	54%
Correct Answers	0	3	2	5
	0%	25%	29%	21%
Misconceptions	3	2	1	6
	60%	17%	14%	25%

In the second question, which investigated the misconceptions in relation to representativeness, 25 percent of the learners had misconceptions in this question. However, misconceptions varied across the grade level. As shown in Table 4, a larger percentage of learners in grade 10 showed misconception at 60 percent, while 17 percent and 14 percent of misconception were recorded as grade 11 and 12, respectively. This finding is similar to that of Ang and Shahrill (2014), who argued that due to the false belief that samples that match the population distribution are more likely to occur than samples that do not, learners frequently hold misconceptions about samples that are representative of the population. This is mostly due to the widespread perception that outcomes or consequences are determined by natural forces that impact an event’s course of action (Ang & Shahrill, 2014). From the above findings, although students showed a significant number of misconceptions about representativeness, these misconceptions become less common as students move up the grade levels.

3.1.3. Misconception in relation to Positive and Negative Recency Effects

Table 5. Number and percentages of learners' responses to question 3 by grade level

Answers	Grade Levels			Total
	Grade 10	Grade 11	Grade 12	
Incorrect Answers	2 40%	6 50%	5 71%	13 54%
Correct Answers	1 20%	1 8%	0 0%	2 8%
Misconceptions	2 40%	5 42%	2 29%	9 38%

In this type of misconception, learners assume that independent events' outcome depends on the previous outcomes. Learners believe that the probability of obtaining a head is increased on the next toss after a run of five tails with a fair coin. As such, the result in Table 5 shows that only 8% of the learners were able to provide correct answers to the 3rd question. However, 38 percent of the learners had a main misconception, while 54 percent provided an incorrect answer. Thus, in terms of grade level, positive and negative recency effect misconception types did not change across grade levels, as shown in Table 5, given that 40 percent demonstrated this misconception at grade level 10. In comparison, 42 percent and 29 percent showed this misconception in grades 11 and 12, respectively. This stresses the importance of effective teaching of probability in schools. Thus, Mut (2003) argues that there are two approaches to effectively teaching probability. As such, while some learners believe they must estimate a specific conclusion, others think they must evaluate the probability of a series of outcomes. Even for the same issue, conflicts may result in discrepancies between the two approaches.

3.1.4. Misconception in relation to Effect of Sample size

Table 6. Number and percentages of learners' responses to question 4 by grade level

Answers	Grade Levels			Total
	Grade 10	Grade 11	Grade 12	
Incorrect Answers	3 60%	3 25%	0 0%	6 25%
Correct Answers	1 20%	1 8%	0 0%	2 8%
Misconceptions	1 20%	8 67%	7 100%	16 67%

In question 4, only 8 percent of the learners answered the question correctly, whereas 67 percent had a misconception about this question. This percentage is high, showing that learners are confused between ratio and proportion, as well as probability subjects. As shown in Table 6, none of the learners in grade 12 solved the question correctly. As such, the frequency of the misconception increased with grade level. With high expectations that this concept would be easier for learners in the higher-grade level, the causes of this misconception type are, therefore, difficult and complex to explain. Therefore, Kaplar et al. (2021) argue that learners often commit this type of error because they think that the probability of the judged sample statistic is independent of the sample size. As such, they become insensitive to sample size. In fact, a lot of learners think that the sample size has no bearing on how closely the sample statistic and population parameter resemble each other. As a result, they tend to commit errors. The findings of this study refute those of other studies, which revealed that most learners often answered probability questions that are related to the effect of sample size correctly (Kang & Park, 2019; Kaplar et al., 2021; Kustos & Zelkowski, 2013).

3.1.5. Misconception in relation to Equiprobability Bias

Table 7. Number and percentages of learners' responses to question 5 by grade level

Answers	Grade Levels			Total
	Grade 10	**Grade 11**	**Grade 12**	
Incorrect Answers	1	3	1	5
	20%	25%	14%	21%
Correct Answers	4	8	4	16
	80%	67%	57%	66%
Misconceptions	0	1	2	3
	0%	8%	29%	13%

In Table 7, it is clear that learners do not have much difficulties solving the question, as 66 percent answered it correctly. As such, only 13 percent of the learners had misconceptions. However, the frequency of equiprobability bias misconception increases from grade level 10 to grade level 12. On the other hand, the frequencies of correct answers decrease across grade levels. Although the question seems easy, learners in high-grade levels, such as grades 11 and 12, had difficulty in solving this question correctly. Surprisingly, the percentage of the correct answer for this question was the highest in grade 10. One could conclude that the equiprobability bias amongst these learners is because they see various experiment results as equally plausible. As a result, learners who suffer from equiprobability bias, for instance, often believe that when two dice are rolled, all possible sums are equally likely. The fact that the sum of 6 for the two dice is more likely than the sum of 2 is not apparent to them. This finding is consistent with that of Gauvrit and Morsanyi (2014), who argued that equiprobability bias is not exclusive to any class or grade. As a result, learners are prone to this misconception because of their belief that all outcomes have the same probability, and in such situations, the base set is always neglected. Hence, researchers such as (Ang & Shahrill, 2014; Gauvrit & Morsanyi, 2014; Kaplar et al., 2021) adds that learners who hold this false belief typically believe that random events are equally likely by their very nature. Or, to put it another way, they see the odds of achieving diverse results as equally likely occurrences (Gauvrit & Morsanyi, 2014). Hence, a need for teachers and researchers to better understand the topics which are perceived as important so that they may better evaluate the conceptions of probability that learners have at different ages and how these conceptions can be changed.

3.2. Types of Misconceptions in Relation to Gender

Table 8. Number and percentages of learners' responses to questions 1-5 based on grade level

QUESTIONS	GENDER		GL 10 I	GL 10 C	GL 10 M	GL 11 I	GL 11 C	GL 11 M	GL 12 I	GL 12 C	GL 12 M
Question 1	FEMALE	n	-	-	2	-	7	-	-	4	-
		%	-	-	100%	-	100%	-	-	100%	-
	MALE	n	1	2	-	2	3	-	-	3	-
		%	33%	67%	-	40%	60%	-	-	100%	-
Question 2	FEMALE	n	2	-	-	3	2	2	3	1	-
		%	100%	-	-	43%	28.5%	28,5%	75%	25%	-
	MALE	n	-	-	3	4	1	-	1	1	1
		%	-	-	100%	80%	20%	-	33.3%	33.3%	33.3%
Question 3	FEMALE	n	1	-	1	3	1	3	2	-	2
		%	50%	-	50%	43%	14%	43%	50%	-	50%
	MALE	n	1	1	1	3	-	2	3	-	-
		%	33.3%	33.3%	33.3%	60%	-	40%	100%	-	-
Question 4	FEMALE	n	1	-	1	2	1	4	-	-	4
		%	50%	-	50%	29%	14%	57%	-	-	100%
	MALE	n	2	-	1	1	-	4	-	-	3
		%	67%	-	33%	20%	-	80%	-	-	100%
Question 5	FEMALE	n	-	2	-	1	5	1	1	3	-
		%	-	100%	-	14%	72%	14%	25%	75%	-
	MALE	n	1	2	-	1	3	-	-	1	2
		%	33%	67%	-	25%	75%	-	-	33%	67%

3.2.1. Misconceptions on Simple and Compound Events in relation to Gender

Question 1 examines the “simple and compound events” misconception. The misconceptions were more frequent among females than males in grade 10 as there were more correct answers from male learners. This finding is consistent with that of Mut (2003), who investigated students’ probabilistic misconceptions and found that the percentage of females who had the simple and compound event misconception was higher than males. This may be explained by the fact that the majority of learners who had this kind of misconception were unable to distinguish between these two types of events (İlgün, 2013; Mut, 2003). Kennis (2006) also added that this misconception may have its roots in the fact that some learners fail to take into account the sequence in which the outcomes of a compound event will occur, which leads them to determine the sample size for this event wrongly. However, in grades 11 and 12, there were no misconceptions from both genders. This finding is in line with that of İlgün (2013), who examined the reasons underlying probabilistic misconceptions in relation to gender, findings revealed that, in terms of simple and compound events, males and females do not differ from each other. İlgün (2013) claimed that the underlying reason for no significant difference could be attributed to the fact that both genders have been exposed to the idea at the higher grades and now have a sufficient grasp of the concept. In conclusion, one could argue that misconceptions in relation to simple and compound events varied across gender at the lower grade.

3.2.2. Misconceptions on Representativeness in relation to Gender

While question 2 investigated misconceptions in relation to gender, findings in Table 8 revealed that this misconception was frequent amongst male learners in grades 10 and 12.

In contrast, female learners tend to have this type of misconception in grade 11. Thus, one could conclude that the misconception regarding representativeness varies across gender. This finding corroborates that of Mut (2003) who argued that although misconception type in probability in relation to representativeness may not vary across gender in different grade level, female learners had more tendencies in misconceptions than their male counterparts. However, Kustos (2010), argued that regardless of gender, insensitivity to sample size impacts on predictive accuracy, inappropriate confidence in prediction based on false input data, misunderstandings of chance, the illusion of validity, and misconceptions of regression are some of the ways that learners maintain a representative misconception.

3.2.3. Misconceptions on Positive and Negative Recency Effects in relation to Gender

In this study, the third question examined the misconception f positive and negative recency effects concerning gender. While females had a high misconception rate in question 3, male learners deferred a little in grades 10 and 11. As shown in Table 8, in grade levels 10, 11, and 12, the misconception was stronger among female learners. For grade 12 learners, while females showed a high percentage of misconception of 50%, male learners had no misconceptions with respect to the question. One could conclude that positive and negative recency effects were more frequent amongst female than male learners. The finding of this study is in line with Mut (2003), who investigated the distribution of misconception type 'positive and negative regency effect' with respect to gender at all grade levels and found out that the Positive-Negative Regency Effect was more frequent among female learners than male (Kustos, 2010; Mut, 2003). According to the data gathered, Mut (2003) also discovered that the misconception type for the Positive-Negative Regency Effect remained constant across grade levels. However, it was shown that learners were less likely to fall victim to this fallacy, also known as the "Gambler Fallacy," in higher grade levels where probability is taught than in lower grade levels. This result emphasizes how crucial it is to teach probability adequately in the classroom.

3.2.4. Misconceptions on the Effect of Sample size in relation to Gender

The distribution of the misconception-type Effect of Sample Size in relation to gender at all grade levels is shown in Table 8. Question 4 examined the effect of sample size as a misconception type. In grade 10, the misconception was higher amongst female learners than male learners. However, in grade 11, the frequency was higher amongst male learners than female learners. Lastly, in grade 12, both genders reported a high frequency of misconception.

In conclusion, although misconception varied across gender, it appears in Table 8 that the misconception type was more frequent amongst male than female learners. This result is comparable to that of Kennis (2006), who looked at probabilistic misconception across age and gender and discovered that females outperform males in tasks, involving the effect of sample size. This is due to the fact that knowledge of equal fractions, percentages, or fractions outweighs the use of common sense amongst female learners. However, both genders are more inclined to ignore sample space while making a probabilistic decision (Kennis, 2006).

3.2.5. Misconceptions on Equiprobability Bias in relation to Gender

The distribution of the equiprobability bias misconception in relation to gender at all three-grade levels is shown in Table 8. In grade 10, while there were no misconceptions by participating learners, the frequency of this misconception in grade 11 was higher among females than males. However, in grade 12, males showed a high level of misconception than females.

Based on the above frequency shown in Table 8, it could be stated that equiprobability bias was frequent among male learners. The finding of this study refutes that of Mut (2003), who stated that the frequency of Equiprobability Bias among females is often higher than among males across all grade levels. The findings further revealed that males had much more tendency to have this misconception than females. In the literature, we were unable to locate any recent and relevant studies that looked at gender-related misconceptions about equiprobability bias. As a result, we were unable to compare the current study's findings further. However, we decided to indirectly compare the findings of this study with research on the gender gap in mathematical achievement, some in probability. Although each area had mixed results, that is, while some studies such as (Bottia et al., 2015; Fortin et al., 2015; Li et al., 2018; Marcenaro–Gutierrez et al., 2018) showed that females had a higher achievement in mathematics and probability, others such as (Bottia et al., 2015; Innabi & Dodeen, 2018; Niederle & Vesterlund, 2010) showed that males had higher achievement, and some found no significant difference (Guo et al., 2015; Reilly et al., 2015).

4. CONCLUSION

The study's findings indicate that the prevalent assumption about intuition's stability is incorrect. To put it another way, the frequency of misconceptions varied by grade level. It was difficult to describe how misconceptions about probability changed. As learners progressed through the grades, some misconceptions faded with age, others remained stable, and others grew in power. However, it should be noted that, in comparison to previous grades, learners in grades 10 and 12 show minimal tendency against misconceptions. According to the study, the curriculum program contains probability subjects at all three grade levels.

Also, another goal of the study was to see how different types of misconceptions differ by gender. According to the findings, describing the change in the type of misconception in probability with respect to gender is fairly tough and complex. Despite the fact that the types of probability misconceptions did not differ significantly by gender, male learners tended to have more misconceptions about probability than female learners. Thus, given that probability as a subject necessitates a method of thinking that is not solely based on technical information and actions that lead to solutions, mathematics teachers should strive to encourage learners to develop new intuitions when teaching probability. Furthermore, probability instruction should enable the learners to experience conflicts between their intuition and specific sorts of reasoning in stochastic settings. Lastly, these probability misconceptions should be taken into account by programs/policymakers in the development of mathematics curricula in schools. Also, teacher training colleges should also incorporate several ways of teaching probability and statistics in classrooms to aspiring mathematics teachers. However, while the sample size for this study comprises a very small number with three grade levels at the Further and Education and Training Phase, further studies may consider a big sample size and other educational phases such as the senior phase. Also, while this study relies solely on quantitative data, other studies may consider using a mixed methods approach as this may provide more accurate and robust information.

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