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UTILIZING LESSON STUDY IN TEACHING SYNTHETIC DIVISION FOR PROCEDURAL FLUENCY IN A POST-PANDEMIC CLASSROOM

Jessa Christine Dedal Agsalon, Joy Meribeles Anore, Hanna Arnedo Salinas, Princess Pera Dipasupil, Minie Rose Caramoan Lapinid*
De La Salle University - Manila, Philippines

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ABSTRACT

Rigid planning, implementation, and evaluation of the learning activities has proved crucial in reflective teaching practice, especially in collaboration through a lesson study. This study was conducted in the Philippine post-pandemic context with the aim of using lesson study to improve lesson delivery in a HyFlex classroom setup in teaching synthetic division for procedural fluency. Participants included four collaborating full-time teachers and an intact class of twenty-two online and seventeen in-person learners. Research instruments were a self-assessment tool, classroom observation, and a focus group discussion. Most students could perform synthetic division but some failed to achieve procedural fluency due to poor prior knowledge in performing operations on real numbers and arranging terms in descending order of degree, and inadequate understanding of the concepts behind the algorithm. These findings underscore the importance of striking a balance between procedural fluency and conceptual understanding in a lesson. The challenges in conducting lesson study were difficulty in scheduling and conducting online meetings. The challenges in implementing the research lesson were intermittent and weak internet connection, HyFlex learning classroom management, and getting students to express their mathematical ideas. On the basis of these findings, the research lesson is then revised and improved for future implementation.

Corresponding Author:

Minie Rose Caramoan Lapinid,
Faculty of the Science Education Department,
De La Salle University - Manila
2401 Taft Ave, Malate, Manila, 1004 Metro Manila, Philippines.
Email: minie.lapinid@dlsu.edu.ph

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1. INTRODUCTION

The COVID-19 pandemic has drastically impacted the education sector (Keshavarz, 2020) and the sudden transition to online learning has paved the way for online synchronous learning to become mainstream (Chellathurai, 2020). It has forced schools to a new method of teaching and learning, which is foreign to teachers and learners (Miller et al., 2021) as there has been a shift from the pre-pandemic face-to-face to online or modular distance learning during the pandemic and subsequently to post-pandemic hybrid learning (UNESCO, 2021). Schools in the Philippines post-pandemic education have been considering different modalities of instruction deliveries for the gradual transition to face-to-face modality (Estrellado, 2021). A post-pandemic classroom (Chellathurai, 2020) would combine online synchronous learning - through Zoom, Google Meet, and other online meeting platforms and in-person campus learning.

The participation of the Philippines in large-scale international assessments pre-pandemic allows us to understand how our students perform with global benchmarks. For example, the Philippines scored 249 in TIMSS (Trends in International Mathematics and Science Study, 2020) for Grade eight mathematics, while PISA (Organization for Economic Co-operation and Development, 2018) results show that average 15-year old students in the Philippines scored 353, the second lowest score in Mathematics compared to an average of 489 points in Organization for Economic Co-operation and Development (OECD) countries.

The COVID-19 pandemic has worsened the learning crisis in the Philippines (Cho et al., 2021). In effect, learning losses caused by the closure of schools may lead to significant adverse long-term impacts on learning-adjusted years of schooling, learning outcomes, income in the long run, and the basic proficiency level. It was suggested that post-COVID-19 schools should adapt to the learning needs of every child and should continue to blur the walls to allow children to still learn both in school and at home. These changes have forced the education system to adopt a modality that is new to the stakeholders – the hybrid learning modality (Nelson et al., 2022).

Hyflex (Hybrid-Flexible) learning is a modality that allows learners both in school and those who cannot come to school to learn skills and concepts equally with equivalent tasks, the same learning objects, and similar, if the same authentic assessments (Beatty, 2019). Some of the advantages of HyFlex learning include flexibility for both learners and teachers (Beatty, 2019), while some of the disadvantages include lack of student focus, the teacher's tendency to overwhelm the students with tasks and not provide similar activities that would be engaging for both the online learners and face-to-face learners (College of Dupage, 2016), and the lack of gadgets and poor internet connectivity (UNESCO, 2021). The difficulty of student feedback is also one of the challenges in a HyFlex learning environment, which is rooted in communication (Kohnke & Moorhouse, 2021).

Lesson Study (LS) is a professional development approach from Japan that follows a four-step process: (1) investigation; (2) planning; (3) research lesson; and (4) reflection (Lewis, 2015). It aims to improve the lesson delivery (Nelson et al., 2022), which could also help students understand the lesson and enhance their procedural fluency. LS is a platform for teachers to collaborate and improve their teaching practice such as to become more technologically literate (Mullis et al., 2020) through hybrid learning classes. Moreover, LS helps teachers to be well-rounded mathematically (Lewis, 2015), be able to utilize school and classroom contexts as venues of inquiry (Elipane, 2012), create challenging and appropriate tasks and activities that promote student engagement (Ferrer & Lapinid, 2017), and make effective changes to the lesson that show noticeable improvements in student learning (Appova, 2018).

One of the strands of mathematical proficiency is procedural fluency. Professional fluency (PF) is the skill of an individual to carry out procedures flexibly, accurately, efficiently, appropriately, and correctly (Kilpatrick et al., 2001). High PF tends to provide learners to gain confidence in task performance (Williams et al., 2020). Moreover, procedural fluency is one of the five strands of mathematical proficiency. It does not only refer to knowing how to apply a particular formula but knowing how to apply the procedure in problem-solving (Kilpatrick et al., 2001). Knowing how to use a procedure efficiently is also part of a learner's procedural fluency (Laswadi et al., 2016). Student's procedural fluency may refer to students' knowledge about procedures, knowledge of when to use and how to use each procedure correctly, and the ability to use these procedures effectively and accurately (Laswadi et al., 2016).

This study was conducted in the Philippine post-pandemic context the overarching aim of which is to use lesson study to improve lesson delivery in a HyFlex classroom setup by looking at students' procedural fluency in synthetic division. This aim prompted the researchers to answer the following questions: (1) What discussion episodes target procedural fluency in synthetic division? (2) What is the students' procedural fluency in synthetic division? (3) How do students assess their procedural fluency in synthetic division? (4) What challenges are encountered in conducting the lesson study? (5) What challenges are encountered in implementing the research lesson in the HyFlex modality?

2. METHOD

The lesson study process followed a three-stage procedure: the pre-lesson implementation stage, the lesson implementation stage, and the post-lesson implementation stage. The pre-lesson implementation stage constitutes planning the research lesson by the LS group, securing the school's permission to conduct the research lesson implementation in the class, and securing students' and guardians' informed consent. The post-lesson implementation stage consists of the focus group discussion (FGD) to discuss areas the research lesson needs to improve based on class observation field notes. The LS process underwent only one cycle due to the limitated availability of the classes.

The lesson study consisted of four professional teachers from different schools in the Philippine National Capital Region who were likewise graduate students of the Teaching of Mathematics course. Their professor randomly chose the four professional teachers in their graduate class to form the LS group. They met four times, two to four hours each time, to decide the lesson topic, the schedule of implementation, the teacher implementor, and what to focus on in the lesson study, to plan and develop the research lesson, and to prepare the materials for lesson implementation. One of the LS group members was selected to teach the lesson while the other LS group members observed the class. All LS group members participated in the post-lesson FGD for one hour.

The research lesson was implemented to an intact class of twenty-two (22) students who participated virtually and seventeen (17) students who participated physically, following Beatty's model of Hyflex learning (Beatty, 2019) that allows a mix of both face-to-face and online students be in the class at the same time (Detyna et al., 2023). The class belongs to a private school in a city in the National Capital Region of the Philippines. The school is following an odd/even scheme of assigning students' mode of attendance wherein students with odd and even ID numbers attend the classes alternately in person or online. During this research study, students with odd ID numbers attended the class online, while the rest attended the class in person. The choice for the class was based on the availability of the LS group members and the schedule assigned to the class by the school to avoid disruption. All these student participants were allowed to use their gadgets in classroom activities per school regulations. After the HyFlex class instruction, the teacher administered a self-assessment tool to students and assigned them an error analysis asynchronous task as homework.

Learners' procedural fluency self-assessment responses were analyzed using descriptive statistics, while learners' responses to the open-ended questions were analyzed using thematic analysis (Braun & Clarke, 2012). The study adopted the use of the self-assessment form by the school with a 4-point Likert scale where responses 1 and 4 correspond to 'strongly disagree' and 'strongly agree', respectively. Cronbach's alpha was determined ($\alpha = 0.995$), which indicates the instrument is deemed reliable. The open-ended questions are: (1) How did you find/understand your lessons? (2) Which part of the topic/lesson was easy? Explain why. (3) Which part of the topic/lesson was difficult? Explain why. The research lesson class was video recorded. Other pertinent data constitute students' written works and observation field notes of the LS implementers.

3. RESULT AND DISCUSSION

3.1. Discussion Episodes Targeting Procedural Fluency in Synthetic Division

Three notable episodes took place during the implementation of the lesson. In an attempt to connect the current with the previous meeting's lesson on long division of polynomials, the first episode was the comparison and contrast of synthetic and long divisions.

Episode 1: Comparison and Contrast of Synthetic and Long Division

Teacher : *What are the similarities and differences between synthetic and long division?*
Teacher : *Let's talk about similarities first.*
Student A : *Both are methods of division.*

The lesson was the first time the students encountered synthetic division as a topic to study in Mathematics 10. The teacher needed to repeat the question several times and wait for the responses until Student A raised his hand and answered the question.

Episode 2: Definition of Numerical Coefficients

Teacher : *So, when we say coefficient, what does it mean? For example, 3x2. What is the coefficient of that term?*
Student B : *3*
Teacher : *So, what is the definition of a coefficient? In your own words. Yes, student C?*
Student C : *Same [The class laughed]*
Teacher : *What do you mean the same? Anyone who can help with the class?*
Student D : *Value of the variable.*
Teacher : *It's the numerical part of the expression. Thus, the coefficient in 3x2 is 3. So, don't be confused. When I say "coefficient", I refer to the numerical coefficient.*

The teacher asked the students to define coefficients based on their understanding by giving the mathematical expression 3x2. The teacher posed another question to verify if they understood numerical coefficients. The students could hardly express in words the concept of numerical coefficients. However, they can quickly identify it when given a mathematical expression. The teacher gave an idea of numerical coefficients. However, a better terminology or description could have been used, such as the numerical factor of a term in an algebraic expression. Nonetheless, the teacher underscored the numerical coefficient concept's importance as a prerequisite to synthetic division.

Episode 3: The Implication of a nonzero Remainder

Student E : *Miss, does the remainder always have to be 0?*

Teacher : *[It] doesn't have to be 0. When you divide numbers, is the remainder always 0? Is it always like that?*
Class : *No*
Teacher : *No, right? It is similar to polynomial expressions.*
Student E : *What if the remainder isn't 0? How will we interpret kung hindi siya [if it is not] zero?*
Teacher : *The teacher introduced an example.*

In Episode 3, the student was very inquisitive and used critical thinking, as all the given examples had zero as the remainder. Although the primary goal of the examples was to check how students understood the procedures of synthetic division, without this student raising this question, students might misconstrue synthetic division always yields a zero remainder. This incident happened just about the time the teacher was to give an example with a nonzero remainder. The teacher should have directly answered the question. Instead, she gave the next example. The situation allowed students to use critical thinking, collaborate, and interpret observations based on the examples.

3.2. Students' Procedural Fluency in Synthetic Division

The instruction given in the homework was for the students to identify the error in a given solution and correct it. Please see Figure 1. Students who identified the mistake correctly got one point, provided the correct solution, received two points, and answered correctly, received 1 point, for a total of four points.

$$\frac{-7x - x^2 + 14 + 2x^3}{x + 2}$$

$$\begin{array}{r|rrrr} -2 & -7 & -1 & 14 & 2 \\ & & 14 & -26 & 24 \\ \hline & -7 & 13 & -12 & 26 \end{array}$$

Figure 1. Error analysis homework

Table 1 shows the mean and standard deviation of students' scores in the asynchronous task. The average score of thirty (30) students who submitted their works on the 4-item asynchronous task is 3.47 (*SD = 1.14*) out of 4 points signifying a good turnout of results. This result means students generally could discern which aspects of the solution went wrong, and provided correct solutions and answers. Consequently, implying most students have acquired procedural fluency.

Table 1. Descriptive statistics of students' scores in homework

N	Mean	Standard Deviation
30	3.467	1.137

Table 2. Distribution of students' scores in homework

Scores	Counts	% of Total	Cumulative %
0	2	6.7 %	6.7 %
1	0	0%	6.7%
2	3	10.0 %	16.7 %
3	2	6.7 %	23.3 %
4	23	76.7 %	100.0 %

Table 2 shows the distribution of students' scores. Specifically, 23 (76.7%) students received a perfect score, while 5 (16.7%) students received a score of 2 or below. Only two students scored 0 points. These results show that most learners know how to find the error in the given solution. Afterward, they provided the correct solution by uploading a photo or screenshot of their work. The following were examples of students' responses to the given questions.

Item #1. What error did Joshua make in his solution?

Student C : *He did not arrange the dividend in descending order.*
Student D : *The error Joshua made in the solution was the arrangement of the polynomial, which was not in descending order.*
Student E : *Joshua made an error in the solution where he should first rearrange the polynomial's terms in descending form. The proper arrangement should be 2, -1, -7, and 14 for the coefficients.*
Student F : *The coefficients 14 and – 26 were not combined correctly. It should be 14-(-26) = 40.*

Both students C and D identified the error, albeit they did not specify the terms in the dividend needed to be arranged in descending order of their degree. Student E identified and determined that the terms must be arranged and, at the same time, listed down the correct arrangement of numerical coefficients of the dividend, albeit the degree of the term as the basis of the descending order arrangement was left out. Student F should have recognized the error and misconstrued the correct operation (addition) as erroneous since the additive inverse has been taken care of in the divisor.

Item #2. Divide using the Synthetic Division: $\frac{-7x - x^2 + 14 + 2x^3}{x+2}$. Show your complete solution and final answer.

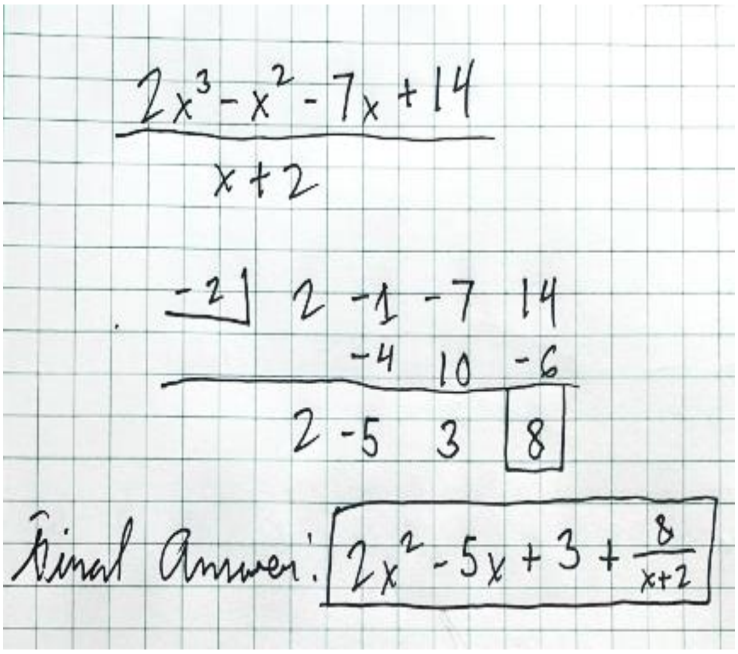

Figure 2. Solution of student H

Figure 2 and Figure 3 show the correct solution by students H and I for the given item. On the other hand, in Figure 4, Student J failed to recognize the error in the solution and performed the operations incorrectly. Student J did not arrange the terms in descending order, but the student could determine the correct divisor. Student J was also able to follow the algorithm of synthetic division since the latter steps of his solution were correct. In Figure 5, student K identified the error but used an incorrect value for the divisor. It can be observed that both students J and K were able to follow the algorithm of procedural fluency but tend to forget the initial and most crucial steps. These mistakes necessitate students to clearly understand the concepts behind each procedure in the synthetic division: why it is necessary to convert the binomial divisor into the form (x - c) and use the numerical value c as the divisor in the synthetic division.

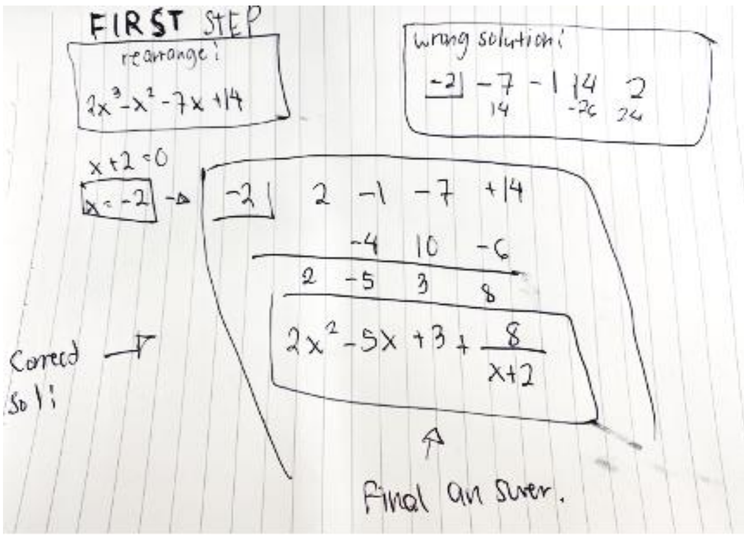

Figure 3. Solution of student I

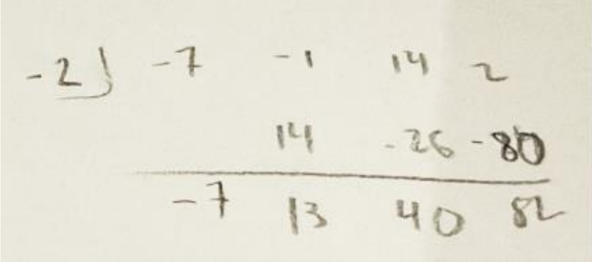

Figure 4. Solution of student J

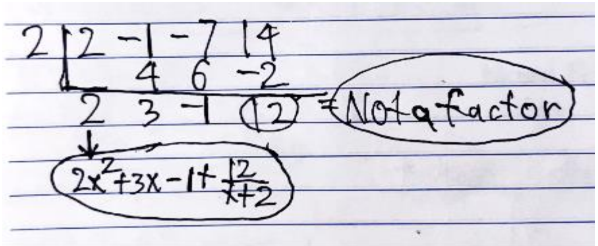

Figure 5. Solution of student K

3.3. Students' Self-Assessment on Procedural Fluency in Synthetic Division

Only twenty-five (25) of the thirty-nine (39) enrolled learners answered a self-assessment questionnaire through Google forms. Table 3 shows students' perceived procedural fluency in synthetic division.

The mean for statements 1 and 2 is 3.72 (*SD = 0.54*), the highest mean score. The skills in identifying the dividend and the divisor and rewriting expressions in descending order are essential before performing the process of synthetic division. Statement 4 has a mean of 3.64 (*SD = 0.57*), showing that students can determine and differentiate quotient from remainder.

Statements 3 and 5 have a mean of 3.60 (*SD = 0.65* and *0.58*, respectively). Performing operations on real numbers got the lowest mean among the prerequisite skills. Overall, students think they can perform division of polynomials using synthetic division.

Table 3. Descriptive statistics of students' self-assessment of their procedural fluency in synthetic division

	N	**Mean**	**Standard Deviation**
1. I can identify the dividend and divisor in a given expression	25	3.72	0.54
2. I can rewrite expressions in descending order.	25	3.72	0.54
3. I can perform operations on real numbers.	25	3.60	0.65
4. I can determine the quotient and remainder when dividing polynomials by binomials using synthetic division.	25	3.64	0.57
5. I can perform division of polynomials using synthetic division.	25	3.60	0.58

Students' responses to the open-ended questions: "Which part of the topic/lesson was easy?" and "Which part of the topic/lesson was difficult?" were analyzed using thematic analysis. One student responded that, "they [synthetic division and long division] were equally okay." Some students (n=5) found the solving part of synthetic division easy. Three students stated that following the steps was easy. Most of the students (n=9) classified identifying the degree of the polynomial and the coefficients to be an easy task. Three students responded that everything in the lesson was easy, and two students answered that none was easy. There was one student who did not answer any of the two questions.

On the other hand, some students (n=2) responded that they have a hard time solving when the divisor is a fraction. Two students had difficulty in the actual computation, and two students found it challenging to identify the remainder and the quotient since there were terms that they needed help understanding. A total of 18 students wrote "none" as their answers meaning they did not find any part of the lesson difficult to do.

The students did not include their learning modality in answering the forms, hence we are not able to compare which set of learners had difficulty following the lesson instruction. But, majority (n=18) of the students claimed no part of the lesson was difficult to do.

3.4. Challenges Encountered in Conducting the Lesson Study

Due to different schedules and workplaces, the members of the lesson study group struggled to find a common time during the planning stage. The group separated the tasks into those that could be done asynchronously and those that needed to be done synchronously as a way to go about this. The approval of the study's research ethics delayed the research lesson class implementation. Consequently, we changed the lesson topic to avoid disrupting the classes. Likewise, there was an unforeseen class suspension before the lesson implementation, so the researchers had to change the topic - from long division of polynomials to synthetic division - and make necessary adjustments in the research lesson, instructional materials, and assessment tools.

In the lesson planning during the lesson study, the teachers had a difficult time thinking of strategies to make the lesson more engaging and with a deeper conceptual understanding. Thus, the original lesson plan did not show the connectivity between long and synthetic division.

Furthermore, the LS group conducted collaborative research lesson planning through an online platform meeting that was new for them. In the next cycle, we highly recommend that the teachers schedule a weekly appointment to develop the lesson plan to maximize available resources and exhaust the best strategies to use to maximize learning results.

3.5. Challenges Encountered in Implementing the Research Lesson in HyFlex Modality

Since the study was conducted in a HyFlex learning environment, intermittent and weak internet connection became a challenge during implementation. Additionally, there was an unforeseen electrical problem at the beginning of the lesson, so the class had to transfer to another room with a more stable connection, which led to a reduced time for discussion.

The HyFlex setup itself was a challenge. During the class, the teacher was confronted with so many things that needed her attention - delivering the lesson, interacting with the students in the physical and virtual classroom, or achieving interaction between the two groups of students. In addition, In addition, the teacher barely used the physical whiteboard.

Instead, she used the annotation feature in PowerPoint so that both groups could see the solutions. Moreover, it was difficult for the teacher to write on the actual board because the teacher needed to be close to the virtual classroom equipment in order to get a stable audio reception. Roaming around the classroom to have more active interaction also became a struggle because the audio quality for the virtual group would be compromised. On the other hand, the planned error analysis for more in-depth procedural fluency was not executed in class as originally planned due to time constraints. Instead, this activity was assigned as an asynchronous homework to the students.

Lastly, the students practically had no idea about the lesson. Had the LS group thought of an activity integrated into the research lesson plan and implementation to link concepts in long division to the synthetic division algorithm, it could have been less challenging for the teacher to interact with students who have become passive.

3.6. Discussion

Most of the students could easily follow the given procedure in synthetic division. However, this raises questions about whether they understand the concepts behind each step of the process. After the lesson implementation in the class, the lesson study group reflected and realized that we could improve the research lesson by interweaving the concepts behind the algorithm and the algorithm per se. Since the students already took up long division of numbers and long division of polynomials in previous lessons, the teacher may direct them to observe the similarities and differences between the long division method and the synthetic division method. Consequently, students could see that the synthetic division is a shorter version derived from the long division algorithm. The teacher may prompt students to think about why synthetic division works and why it is a shorthand of long division method. The teacher may also ask why a crucial synthetic division step is to equate the divisor by zero and solving for x. Another way to deliberately show a more vital link between the two methods of division is through a brief recall on long division, such as watching a video available on the internet with focus questions to answer, preferably before the class. Such activities may be included for the students to discover by themselves and make sense of why synthetic division is a shorthand method of long division, that the two approaches are not different except we have foregone changing of signs in the partial subtrahends with a one-time change of sign in the divisor for the synthetic division.

The researchers also realized that foreseeing or anticipating learners' possible responses and thinking processes could be helpful in the development of any lesson plan, as it would create a structure for the effective delivery of the lesson. The members of the lesson study group realized that to implement the lesson better, the teacher needs to anticipate the possible challenges of recalling previous skills and concepts to prepare appropriate courses of action in case there are no responses from the students.

Some students could not completely follow the algorithm as they made mistakes in the first few steps in rearranging the terms in descending order of their degree, identifying the correct numerical divisor, and getting confused about whether to add or subtract the coefficient and the partial product in each term. These imply some learners still need to achieve procedural fluency, partly because they lack conceptual understanding. This tendency to be confused and make mistakes supports Hiebert and Lefevre's (1986) argument that students failing to attain procedural fluency may be due to their lack of conceptual understanding.

As this study was conducted in a HyFlex setup, internet connectivity due to the sudden power outage hindered the continuous flow or delivery of the lesson. This challenge is consistent with teachers' experiences in the study of Rodriguez (2022). Online synchronous learners were not able to maximize mathematical discourse due to limited access to the internet. The whiteboard was also not used in the lesson proper since no cameras were pointing to the board, nor were there drawing tablets/iPad for the teacher to use in the annotation. Nonetheless, there currently needs to be more studies on the integration of LS in HyFlex learning. Thus, we recommend teachers conduct similar studies utilizing a HyFlex learning environment since it makes learning more accessible to students as they have a choice to be physically or virtually present in the class. HyFlex learning is a way to cater to students' different needs (Rodriguez, 2022) as the locus of control lies on the student (Beatty, 2019) because it allows students who cannot attend the class physically to participate online real-time (Romero-Hall & Ripine, 2021), thus, making education more accessible and equitable.

Lohmann et al. (2021) suggest that teachers utilize their best practices in face-to-face classroom management as HyFlex learning poses unique challenges in terms of expectations and managing classroom behaviors. Teachers must lay down expectations explicitly, model desired behavior, and provide timely precise feedback to support students' learning. The lesson study group members realized that HyFlex learning demands more for teachers to be at their best when it comes to classroom management than in either face-to-face or online learning environments alone.

Similar to the experience of Cheng and Yee (2013), one of the constraints in doing LS is time. Teaching full-time in different schools with conflicting schedules and taking up graduate studies in the evenings became a challenge for the lesson study group members to regularly meet as LS requires considerable time and commitment (Rock & Wilson, 2005). Nonetheless, this did not hinder them from drafting and preparing a research lesson plan. The lesson in its initial cycle definitely needed much improvement to maximize learning. Areas for improvement were identified among collaborators of the LS through collegial and friendly feedback, peer learning, and reflections on actual classroom practice (Lee, 2008).

4. CONCLUSION

A lesson study in a HyFlex learning environment was conducted to improve lesson delivery by looking at students' procedural fluency in synthetic division. The episodes in the implementation of the lesson revealed some learners have forgotten the term 'coefficient'. Moreover, we noted that some students were very attentive and were thinking critically since somebody asked the question of the possibility of a nonzero remainder.

The majority of the learners claimed they did not have a hard time following the lesson. Most students got a perfect score, while only two got 0 in the error analysis asynchronous task. Nonetheless, some students failed to achieve procedural fluency because of a lack of understanding of the concepts behind the procedure and prior knowledge/skill in performing operations on real numbers. Their solutions indicate some computational errors, failure to arrange terms in descending order of degree, incorrect sign in the divisor, and subtracting partial product from the dividend term instead of adding them.

The conduct of a lesson study has helped the researchers to reflect on how vital conceptual understanding is to procedural fluency for students' meaningful learning and retention. Lesson study allowed the proponents to realize aspects of their teaching practice that need improvement. Through this lesson study, we found out that although it is beneficial for the learners to be algorithmically adept in manipulating expressions and following procedures, understanding the underlying concepts proved valuable so they could make sense of the solutions they create.

The challenges encountered in planning the lesson study were unforeseen circumstances that contributed to the delays in the class implementation, difficulty in scheduling, and conducting meetings on online platforms. These delays made the researchers realize the importance of time and commitment in conducting a lesson study.

The challenges in implementing the research lesson were intermittent and weak internet connection, HyFlex learning classroom management, and getting students to express their mathematical ideas. Better infrastructures and some contingency measures, establishing classroom protocols in HyFlex learning, and anticipating student difficulty in classroom interaction based on their prior knowledge in planning the research lesson, are suggested for the next cycle of the research lesson implementation.

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