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STUDENTS' COMMON ERRORS IN QUADRATIC EQUATIONS: TOWARDS IMPROVED MATHEMATICS PERFORMANCE

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ABSTRACT

With the aim to close the performance gap between the high performing and the low performing students, this descriptive qualitative study was conducted to analyze students' common and persisting errors in quadratic equations. Forty-six (N=46) Grade 9 students in a public high school in the Philippines participated in the study. Homework was given where students received instructions from video recordings and other online learning materials using blended learning. The common errors students committed were not following directions, mishandling signs, difficulty in recognizing a quadratic equation, inability to distinguish between solving a quadratic equation and simplifying an algebraic expression, failure to express quadratic equations in standard form, disregarding the negative roots, computational errors in basic algebraic conventions in simplifying radical and rational expressions, factoring, performing special products, and in completing the square. Students were interviewed to validate the error analysis. Immediate feedback through a whole class discussion was conducted the following day to discuss the errors and mistakes committed by students in order to address these and help them learn the necessary concepts and skills in quadratic equations. Instructional strategies and interventions for teaching quadratic equations are recommended for future studies.

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1. INTRODUCTION

With the advent of technology specifically the internet, instructional materials and resources in mathematics are within reach especially if enough infrastructure is in place. However, a wide gap between the low-performing and high-performing students still exists regardless of the accessibility of rich digital and technological resources because there are chances that what students understand from these instructional materials may not be necessarily correct or not conforming with the scientific conceptions or conceptions of the experts (Bell, 1993; as cited in Fitrianna & Rosjanuardi, 2021). According to the study by Smith and Harvey (2014), accessibility in the blended and online learning must have physical and sensory access to maximize student's learning. Moreover, the National Council of Teachers in Mathematics (NCTM, 2014; as cited in Buchheister et al., 2017) highlights effective teachers are expected to implement tasks and activities which give all students the opportunities to engage in higher order mathematical thinking and reasoning skills. Thus, there is really a need to assess, to evaluate and to complement learning tasks so that the online videos and related interventions become useful and effective to address students' learning needs.

Assessment is vital to ensure effective teaching (Rattadilok & Roadknight, 2018). Tomlinson (2005) stressed the purpose of formative or ongoing assessments is to help both the teacher and the student to see how learning is developing among learners and for the teacher to adjust instruction as necessary to make sure that learning stays on track. Assessment must be considered as a tool to underscore the strengths and weaknesses of the students so that proper remediation can be administered to address the obtained misconceptions. It is also important that teachers assess the existing knowledge and readiness of the students so that instruction can be planned strategically to meet the students' diverse needs. Through ongoing assessments, the data collected from the students are used to determine what to teach and whether further remediation or challenging tasks are needed for individual students in a heterogeneous classroom (Gregory & Chapman, 2013).

To ensure that students benefit from the learning activities and assessment inside the classroom, teachers are encouraged to provide timely feedback regarding student's performance. Quick feedback is defined as feedback that is immediately given to the students after they responded or completed a learning task (Sumarno et al., 2017). When teachers provide feedback regarding student's performance from the assessment, the students can easily identify what particular area of the lesson has to be mastered or remediated and consequently be reflected on their academic performance. The use of immediate feedback by the teacher reinforces student learning and motivation by rectifying students' persisting and common errors immediately after the in-class activities. Also, feedback enables students to adjust in their own learning, thus improving learning outcomes (Muis et al., 2015).

Teacher's feedback is considered as a deliberate practice in improving knowledge, skills, understanding, and acquisition of learning by the students such as problem solving (Sumarno et al., 2017). Al Wahbi (2014) contended that without teacher's feedback, students may not be able to rectify their errors and as a result, targeted competency levels might not be achieved. In addition, without teacher's feedback, students are not able to realize their mistakes and consequently, they have the tendency to repeat the same learning errors that they committed in the learning tasks provided by the teacher. Feedback increases student engagement since students can keep track of their own progress as they go through the learning process. For example, in the study by Mendoza and Lapinid (2022), the use of automated feedback significantly increased students' quiz scores and helped students understand mathematics concepts better.

There had been several studies conducted in analyzing students' errors in solving quadratic equations. Zakaria et al. (2010) used the Newman Error Hierarchical Model to classify and presented a frequency count for each of the types of errors students in Jambi, Indonesia, commit in solving quadratic equations using factorization, completing the square and quadratic formula. They found students are frequently committing computational errors and algorithmic procedures with very few students committing carelessness errors. Students failed to understand what is required of them in the problem and these were due to the lack of teachers' emphasis on the meaning of mathematical terms. Students fail to master basic computational skills such as handling positive and negative signs, and performing operations in simplifying algebraic expressions, and factoring that are prerequisite to solving quadratic equations. Nonetheless, the article did not present illustrative examples.

In a similar study using the Newman Error Model of analysis, Thomas and Mahmud (2021) found students in a secondary school in Malaysia commit transformation and comprehension errors with few students committing encoding errors. The errors were attributed to a lack of understanding of the basic concepts and skills. There were no reading errors or errors arising from reading important words in the test instruction. The test items are in word problems where students have to transform the problem into a quadratic equation. Findings indicate the lack of focus in the process of transforming word problems into quadratic equations in the teaching and learning process.

Makonye and Nhlanhla (2014) used constructivist perspective of learning to explain learners' errors in solving quadratic equations through factorization in Gauteng province, South Africa. The errors were categorized as systematic, random, conceptual, or procedural following Cox (1975; as cited in Makonye & Nhlanhla, 2014). While systematic errors are recurring, random errors are those that may occur due to slips and carelessness. Findings indicate most errors were categorized as conceptual and application errors. Specifically, students did not follow correct order of operations, disregarding the middle term in factoring a trinomial, unfamiliar with different forms of quadratic equations. The researchers posit learners use simple equation schema to solve the quadratic equation not minding the intricacies of applicability of previously learned concepts.

Tendere and Mutambara (2020) classified errors in solving quadratic equations as conceptual, procedural, and technical. The conceptual errors found were students' misconception on like terms, incorrectly applying factorization in a non-standard quadratic equation, and mishandling signs and radicals. The procedural errors involved memorizing and following the algorithms in solving without completely understanding the principles behind these. Some students used an incorrect quadratic formula. The others albeit using the correct quadratic formula, failed to understand the concept of coefficients in the standard form of a quadratic equation, thus, incorrectly substituted the values into the formula. The technical errors in their study refer to carelessness, slips, or silly mistakes.

The reviewed research studies above all agree that error analysis is part and parcel of the feedback process since students' difficulties in learning mathematics may be found in the errors they commit (Fitrianna & Rosjanuardi, 2021). This is all the more necessary in the post-pandemic were most of the mathematics classes are being conducted online or in the blended learning modes. Addressing students' errors depend largely on the types of errors students commit. For example, if a mistake is due to carelessness, pointing this out helps students to be more careful and to review their solutions when there is spare time left in the assessment; whereas a misconception would require the teacher to process this with the students towards the correct concept. Given the deteriorating mathematical skills of students in the Philippines, as reflected in international assessments, this study fills a significant gap in the existing research, where limited prior studies have focused on persisting errors of students in quadratic equations especially in the Philippines. Additionally, the Philippine mathematics education is faced with challenges such as the lack of face-to-face contact hours and overcrowded curriculum. Thus, the flipped classroom was conducted to maximize the face-to-face interaction in addressing students' misconceptions through consolidated discussions. Consequently, when errors are discussed in class with the aim to correct them, teachers can close the learning gap between those struggling students and the exceptional ones. It is in this premise that this study was conducted. It aimed to identify and classify the errors committed by students in quadratic equations in tiered worksheets after they have gone through online materials in a differentiated flipped classroom so that appropriate interventions may be introduced to address and minimize these. Specifically, the study underscores and reiterate all the more the need to do error analysis post-pandemic due to the shift of most face-to-face classes into the online or blended learning modes of instruction and to address diversity in terms of ability levels in the classroom.

2. METHOD

This study is a descriptive qualitative research. The participants of this study comprised forty-nine Grade 9 students enrolled in a regular public high school in the Philippines. Permission to conduct the study was granted by the school principal and informed consent was obtained from both students and their guardians. All forty-six (46) students in a heterogeneous classroom agreed to participate in the study. They were instructed by the teacher-researcher to do the online tasks as homework in the learning package by watching online instructional videos and self-reading of instructional materials in mathematics learning websites on the following lessons: Definition of Quadratic Equations and its Examples; and Solving Quadratic Equations by Extracting the Square Roots, Factoring, Completing the Square, and Quadratic Formula. Students were provided questions to serve as their guide as they go through the online instructional materials. The tiered worksheets served as the primary research instrument in this study for the aforementioned lessons. In each of the five lessons, the researchers created three worksheets corresponding to students' mastery levels: beginning mastery, approaching mastery, and high level of mastery. A total of 15 worksheets were analyzed throughout the duration of the study. Students were then given worksheets to answer as a formative assessment on the following day in the face-to-face class. Errors were identified and classified using content analysis by systematically analyzing the students' solutions and answers in their respective worksheets, and another face-to-face class instruction was conducted to discuss the committed errors in each lesson to underscore the important concepts, further enhance student's learning, and to help students correct their errors and misconceptions. Individual students were interviewed to further validate the interpretation for committing the mistakes.

3. RESULT AND DISCUSSION

3.1. Results

3.1.1. Definition of Quadratic Equations and its Examples

Difficulty in Recognizing a Quadratic Equation

Some of the students showed difficulty in discerning whether a given equation is a quadratic equation based on its degree. This is in relation to the question which prompts the students to determine whether a particular mathematical equation is an example of a quadratic equation or not. For example, in item 3 from Worksheet 2 (See Figure 1), seven (15%) students immediately said that the given equation is a quadratic equation since they saw the exponent 2. They did not realize that they still have to distribute 4x to all the terms contained inside the parenthesis which further result to a cubic equation. Clearly, this is not a quadratic equation contrary to what they answered. One student said,

> *"Sir, nakita ko po kasi na may exponent na 2 kaya akala ko quadratic na siya. Naoverlook ko po." (Sir, I immediately thought that it is a quadratic equation since I saw that there is an exponent that is 2. I just overlooked it.)*

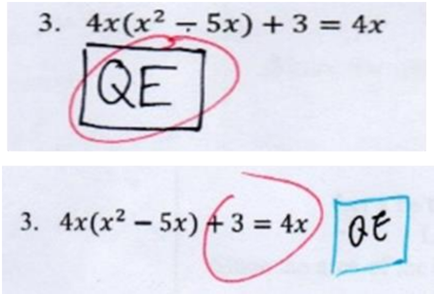

Figure 1. Difficulty in recognizing equation based on its degree

Inability to Express the Quadratic Equation in its Standard Form

There were also students (3 or 7% of them) who committed minor errors in terms of manipulating equations specifically the use of Addition Property of Equality. Please see Figure 2 where in the student was not able to provide the correct numerical coefficient for the constant term. During the face-to-face discussion, one student claimed that it is convenient to just add -7 to both sides of the equation and then rewrite the quadratic equation in its standard form. One student even mentioned that,

> *"Sir, nilipat ko po kasi si 3x tsaka -2x^2 sa kabila kaya nag-iba na 'yung sign. Kaso, ginawa ko ring negative si 7 na dapat positive pala, kaya ayun mali." (Sir, I put 3x and -2x^2 to the right side of the equation that's why, their signs were reversed. However, I also made the sign of 7 negative which is incorrect. I realized that it must be positive.)*

Moreover, one student (2%) was not able to rearrange the terms such that the given quadratic equation can be expressed in its standard form, thus, providing incorrect numerical coefficients. Yet another student has no clear conception of what makes terms similar or like terms and when they can do operations between and among the terms of the equations (see Figure 2).

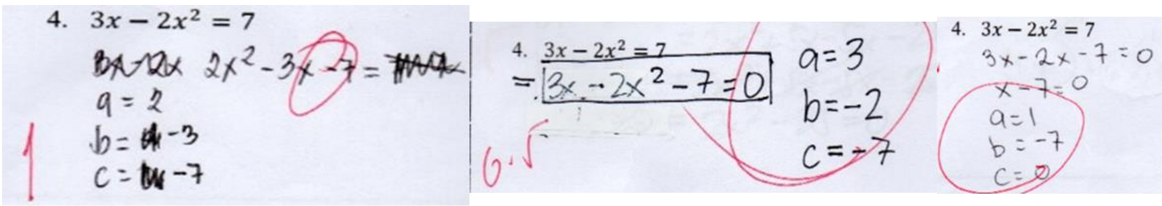

Figure 2. Inability to express the quadratic equation in its standard form

Incorrect Simplified Expressions

Eleven (24%) students did not know how to simplify a quadratic equation involving rational expressions and had errors in combining like terms. Some of them admitted that they were having difficulties when they see fractions in the given items while some said they were just careless in terms of adding terms.

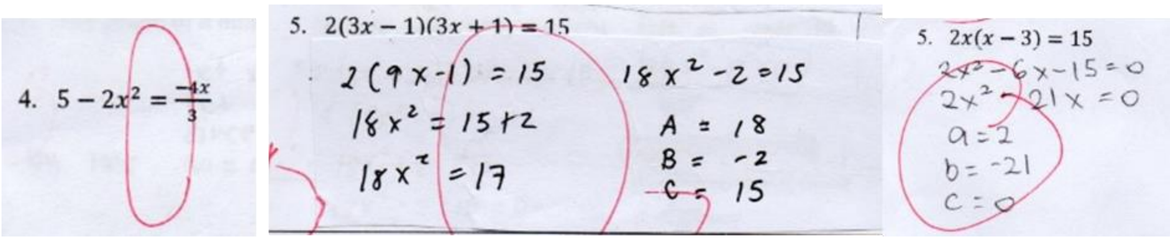

Figure 3. Mistakes in simplifying expressions and combining like terms

As reflected in Figure 3, a student neither provided a solution nor an answer to a quadratic equation that contains a rational expression. Another student missed out the exponent 2 in multiplying the binomials. The third solution in the figure illustrates a student combining unlike terms when these unlike terms are not supposed to be combined.

3.1.2. Solving Quadratic Equations by Extracting the Square Roots

Disregarding the Negative Roots

With regards solving equations by extracting the square roots, four (9%) students just wrote the principal square root of a given quadratic equation, i.e., disregarding the negative root. The teacher-researcher made it clear in the consolidated discussion that since we are treating quadratic equations which has its highest degree is 2, therefore, the number of solutions must also be two, whether those are real roots or imaginary roots.

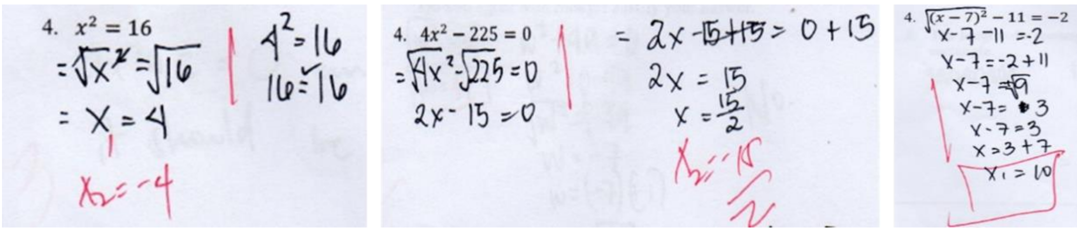

Figure 4. Disregarding negative root

The second and third solutions in Figure 4 illustrates the student incorrectly applied extraction of roots. This made the teacher-researchers realize the need to clarify with the students the required form of the quadratic equation where extraction of roots is applicable – i.e., the isolation of the constant term on one side of the equation and the other side containing the variable or unknown value is a perfect square expression.

Difficulty in Simplifying Radical Expressions

Some solutions of quadratic equations are irrational. Because of this, simplifying radicals were also reviewed and included in the assessment. The majority of students (30 or 65%) did not know how to simplify radical expressions by trying to find two factors of the radicand (if the radicand is not a perfect square) in which one of the factors is a perfect square. Interestingly, there were also students who committed mistakes in trying to add, subtract, or combine an integer and a radical (see Figure 5).

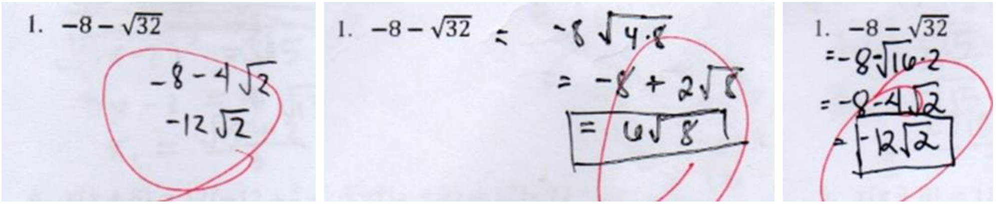

Figure 5. Difficulty in simplifying radical expressions

Twelve (26%) students failed to simplify their answers while the rest were careless in their solutions. One student (2%) realized that he/she forgot to write $\sqrt{3}$ in his/her final answer, while another student (2%) said that he/she mistakenly wrote ±9i instead of ±3i in item 6 of Worksheet 1, and some students committed mistake by combining -7 and $\frac{3\sqrt{5}}{7}$ directly without finding first the least common denominator (see Figure 6).

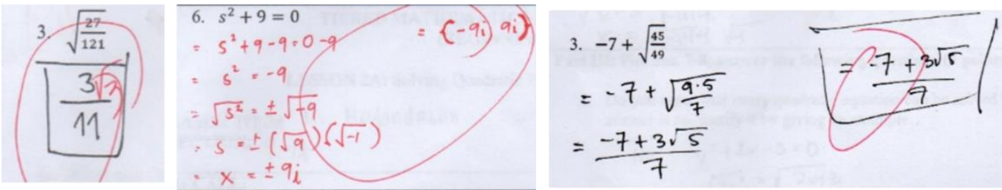

Figure 6. Carelessness of the students in simplifying radicals

Further, there were fourteen (30%) students who did not know how to get the simplified form of a square root of a decimal while some were able to express the decimal into a rational number to easily simplify the expression (see Figure 7).

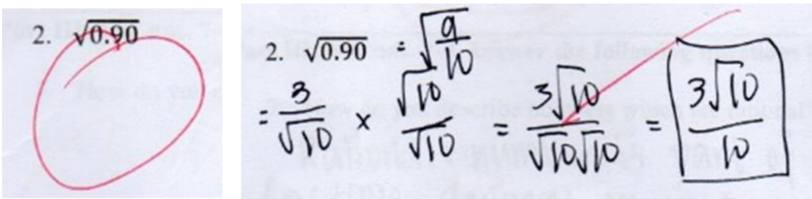

Figure 7. Students' answers in simplifying a radical involving a decimal radicand

Not Following Directions

There were seven (15%) students who did not follow directions in their worksheets. The direction indicated to just simplify the expressions involving radicals but some of them were solving for the solution set (see Figure 8). This error is closely related to students' inability to distinguish between an algebraic expression and an equation, and that solving for an unknown value is only applicable to equations.

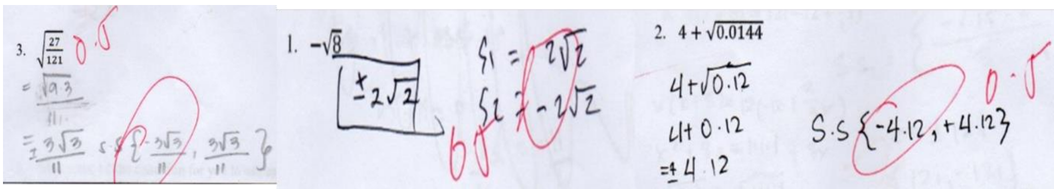

Figure 8. Simplified radical expressions indicated as solutions

3.1.3. Solving Quadratic Equations by Factoring

Not Following Directions

Likewise, students were so focused at solving quadratic equations even if the given direction is to factor the given algebraic expression. There was also a confusion in solving for the values of x and the procedure of factoring per se.

The teacher-researcher finds that this error can be classified as either not following the given direction or failure to distinguish between what constitutes a solution versus that of simply factoring a given quadratic expression. Thus, he clarified this further with the class that finding solutions is only applicable for quadratic equations, not for quadratic expressions.

Insufficient Knowledge in Factoring Quadratic Expressions

In addition, twelve (26%) of the students were still having difficulties in factoring quadratic expressions like common factoring and factoring the difference of two squares (see Figure 9).

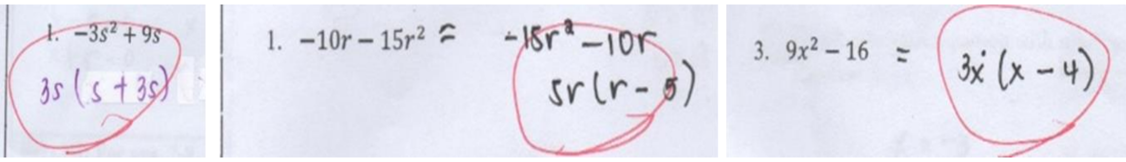

Figure 9. Errors in factoring quadratic expressions

The first and second solutions indicate difficulty in factoring when the leading coefficient is negative. The solutions indicate the negative sign was disregarded by the student. Both solutions seemingly indicate students' difficulty in determining the second factor when factoring the common factor of the terms in a given algebraic expression. The student in the third solution used common factoring when there is no common factor between the given terms in the expression. This illustrates students' need to recall and distinguish the different types of factoring.

Mishandling Signs

Thirteen (28%) students were careless in handling signs. Students' sampled works below illustrate students' difficulty in handling signs in applying the zero-product property wherein they were to set each factor equal to zero and use the Addition Property of Equality to find the solutions of the quadratic equation or to write the quadratic equation given its roots. It may be deduced that due to carelessness, students got incorrect solutions because they often incur errors in terms of signs along the process (see Figure 10).

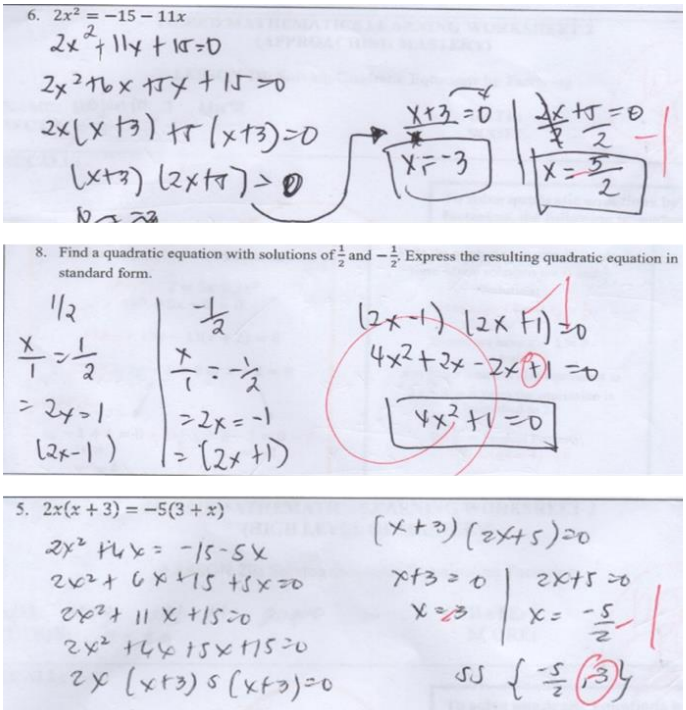

Figure 10. Students' carelessness in handling signs

Mathematical Writing Error (Missing "=0" Symbol)

Minor errors were also observed. One of these is the omission of the "= 0" symbol to denote factors of a quadratic expression in the quadratic equation. This implies that students (7 or 15%) tend to follow procedures demonstrated in the videos they watched but failed to assimilate the reason why factoring works in solving a quadratic equation – i.e., to apply the Zero Product Property wherein the factors should have a product equal to zero if and only if one of the factors or both factors is/are equal to zero (see Figure 11).

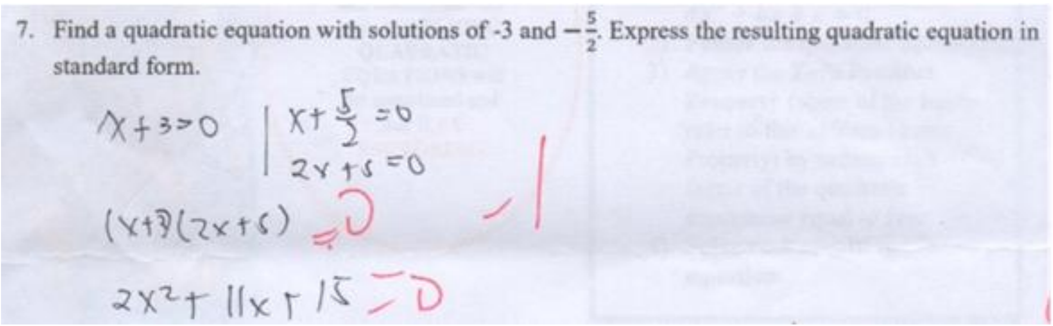

Figure 11. Carelessness of the students in setting the factored form of the quadratic equation to zero

Computational Error

Students (9 or 20%) were careless in their mathematical computations such as in multiplying and simplifying algebraic expressions (see Figure 12).

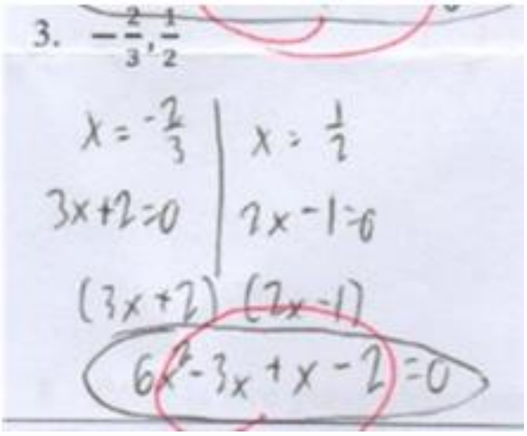

Figure 12. Computational error in multiplying algebraic expressions

3.1.4. Solving Quadratic Equations by Completing the Square

There seems to be more chances of errors for an algorithm that entails multiple and complex steps such as the completing the square method of solving a quadratic equation. Following is a discussion in detail how this becomes a challenging task for students.

Difficulty in Completing the Square and Expressing Perfect Square Trinomial into Square of a Binomial

Given a general quadratic equation written in its standard form, $ax^2+bx+c=0$, the initial strategy is usually to isolate the constant term c to the right side of the equation. After this, it was observed that fourteen (30%) students had difficulty in determining the constant term to be added to both sides of the equation to make the left-hand side of the equation a perfect square trinomial (see Figure 13).

Despite the examples provided in the online class instruction, some of the students still had problems using the completing the square method especially when the numerical coefficient of the linear term is not an integer. Moreover, students who had insufficient skill in factoring had difficulty expressing the perfect square trinomial into a square of a binomial. Solutions at the right of Figure 13 illustrates students not paying attention to the significance of the sign of the middle term in a perfect square trinomial in terms of its effect to the factored form.

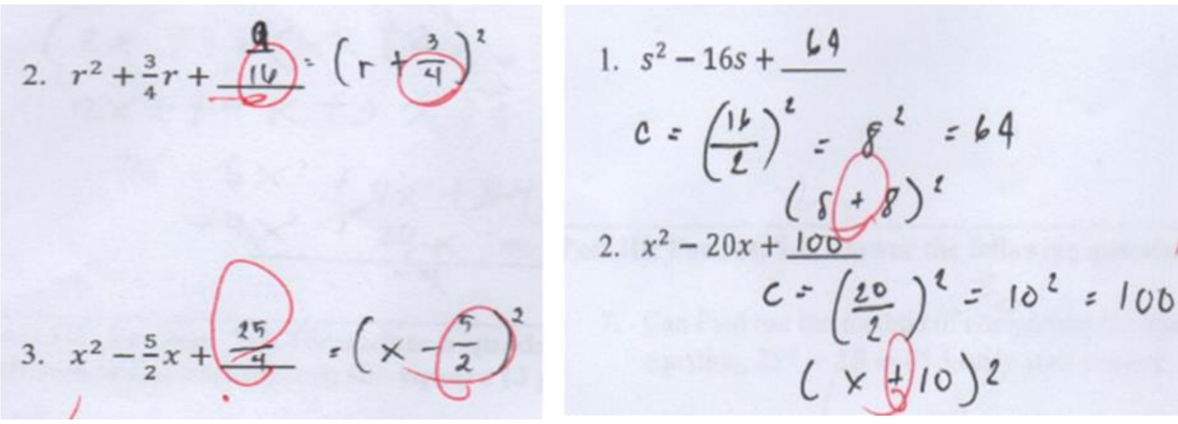

Figure 13. Students' errors in expressing perfect square trinomial into square of a binomial

Incomplete Solutions

Because of the many steps to be considered, eleven (24%) of the students lost track of their goal to solve the equation for the values of x – i.e., they still had to go back to the process of solving quadratic equations by extracting the square roots. See Figure 14 where the student stopped at the given quadratic equation $(x + 2)^2 = 16$. This was also emphasized in the class discussion so that students could realize the connection between the the methods completing the square and extracting the square roots.

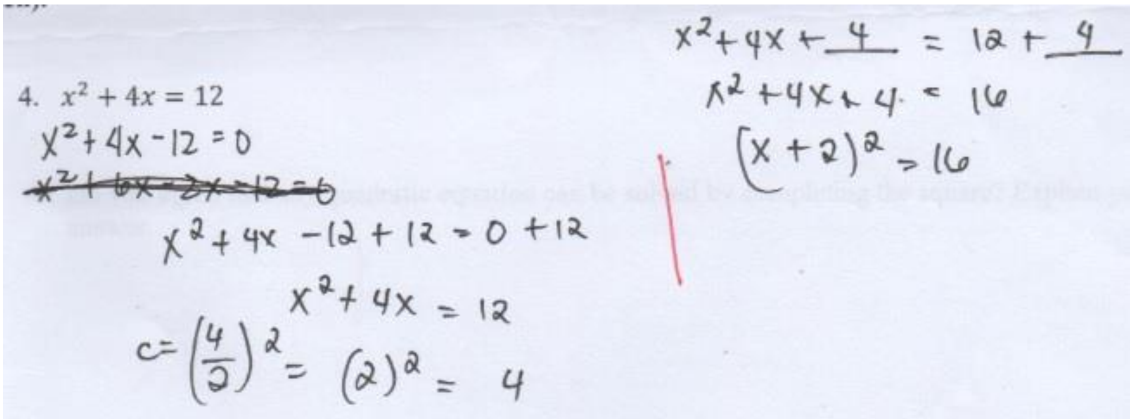

Figure 14. Incomplete solution in solving quadratic equations by completing the square

Not Following Directions

In addition, three (7%) students did not follow the instructions given to him/her because he/she used the method of factoring instead of the instruction to use completing the square in solving for the word problems provided in the worksheet (see Figure 15).

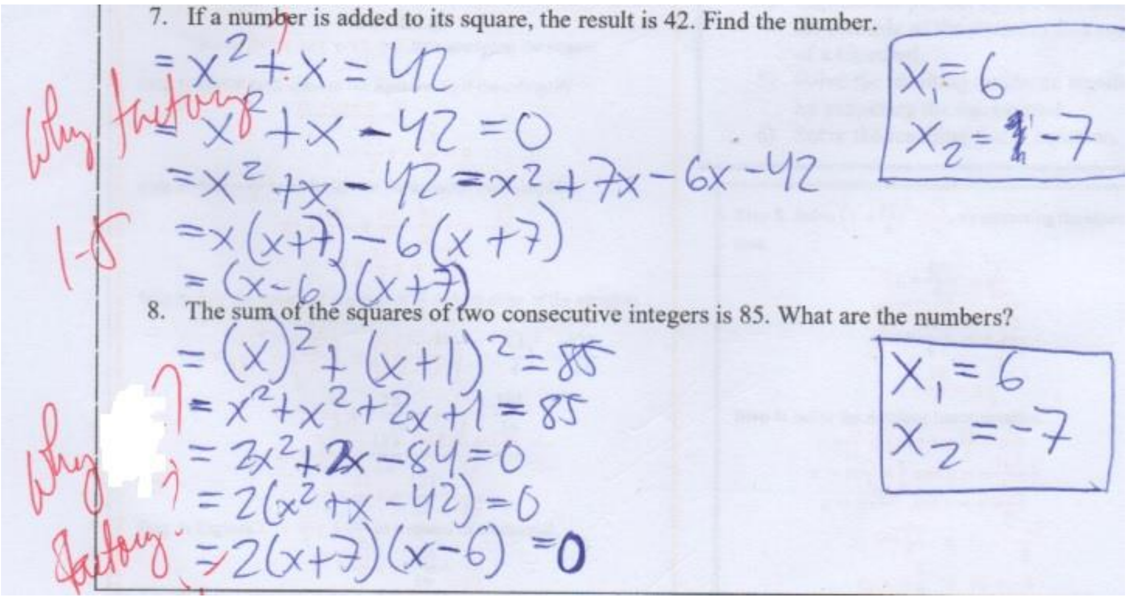

Figure 15. Sample works on not following instruction – factoring was used instead of completing the square

3.1.5. Solving Quadratic Equations by Quadratic Formula

Computational Errors in Simplifying Radical Expressions

Very few students (3 or 7%) committed mistakes in using the quadratic formula because they perceive this method is easier than the completing the square. At this point, students find it easier to identify the numerical coefficients of any quadratic equation expressed in standard form and plugging these values to the quadratic formula. Students' errors were mostly in simplifying radical expressions.

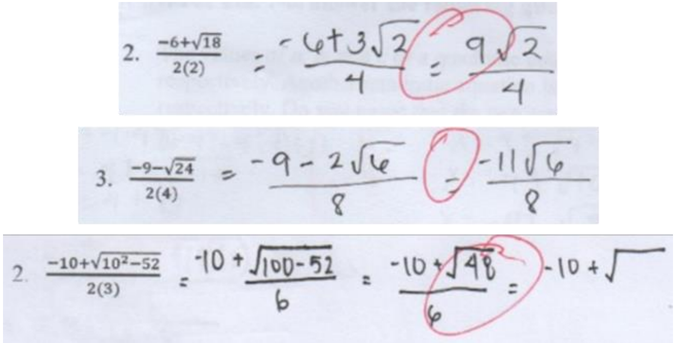

Figure 16. Sample works on computational errors in simplifying radical expressions

Consistent with the previous analysis, solutions in Figure 16 illustrate students did not know the instances that warrant combining like terms when radicals are involved – i.e., the integer and an irrational number are incorrectly combined by adding or subtracting the integer with the integral factor of the radical. The third solution shows the student had difficulty in proceeding with transforming the entire radical $\sqrt{48}$ into a mixed radical to further simplify the radical expression.

3.2. Discussion

Results of error analysis reveal what aspects of the lesson needed clarification and emphasis in order to help correct students' errors and improve their performance. Some of the errors require students improve their test-taking skills specifically on reading and understanding the given direction and to check their solutions and answers before turning their work in to avoid careless mistakes such as not following directions and mishandling signs and computational mistakes. This is consistent with the findings of the study of Peng et al. (2014) and Zakaria et al. (2010) that test taking strategies has an impact on students' mathematics performance within the self-regulated learning theory. Albeit some students admitted mishandling signs and computational mistakes were due to carelessness, the others were just unsure of the correct sign and had poor algebraic prerequisite skills.

O'Connor and Norton (2022) suggested purposeful attention to prerequisite knowledge as the lack thereof impede understanding of quadratic equation. The lesson on solving quadratic equations require ample prerequisite skills. In this study, these prerequisite skills were identified as notable persistent computational errors in simplifying algebraic expressions involving factoring, special products, radicals, fractions, and completing the square among the basic algebraic conventions which were supposedly skills that have been acquired prior to the quadratic equation. These results are consistent with previous studies conducted in analyzing errors in solving quadratic equations (Makonye & Nhlanhla, 2014; Tendere & Mutambara, 2020; Zakaria et al., 2010). We noticed that even at this point of their mathematics learning, students have poor understanding of fundamental concepts in algebra: what constitutes like terms involving variables, integers, and radicals; how to correctly identify the second factor in common factoring; distinguishing which among the types of factoring is applicable in a given algebraic expressions; and how to complete a square. The error analysis reveals that the errors are not solely arising from the expected knowledge and skills from learning quadratic equations per se, but is further complicated by students' poor prerequisite knowledge and skills. Students who had insufficient knowledge in factoring tend to avoid it and resort to other methods they are more comfortable with such as the extraction of roots method. Whereas other students who are more knowledgeable in factoring use this method over completing the square since they find the latter a much comparatively difficult method.

Moreover, owing to the fact that students in the class where we conducted the study are diverse, we deemed the worksheets be based on students' mastery levels for differentiated learning. Thus, the instrument used in this study was mainly the tiered worksheets. In order to differentiate the higher tiered from the lower tiered worksheet, we incorporated more complex algebraic expressions such as those that include radicals. Nonetheless, we deemed radicals and rational expressions are inevitable since solutions of a quadratic equation may be irrational numbers or imaginary numbers.

Almost similar to the finding of Tendere and Mutambara (2020) where a student blindly followed the procedure and lacks understanding that a quadratic equation always yields two solutions including possibly imaginary numbers, the student gave 3 solutions to a quadratic equation. However, in this study, since we included solving quadratic equations using extraction of roots, students tend to disregard the negative root of the quadratic equation and just gave the positive root as a solution.

Some of the mistakes may appear as if students were not following the given direction to simply factor or to simplify a radical expression. They solved it as if the given is a quadratic equation. This implies students' poor understanding of what solving an equation means, and their inability to distinguish between an equation and an expression. Additionally, students' error in disregarding the negative root implies poor understanding of the concepts behind extraction of roots method and solutions of an equation which pertains to an exhaustive list of possible applicable values that make the equation true.

Consistent with the study of O'Connor and Norton (2022), student errors in quadratic equations were associated with misconceptions of the null factor law (or the zero-product property). The error on the zero-product property application was detected in students' mathematical writing error. The incorrect use or omission of a mathematical symbol was referred to as a mathematical writing error (Liew et al., 2022). It seems like students are blindly following the procedure of solving the given quadratic equation by factoring and just wrote the answers without explicitly indicating that each factor is equal to zero. This error was also identified in the study of Tendere and Mutambara (2020). This raises doubt if students really understood the reasoning behind the method of factoring in quadratic equation which requires the use of the zero-product property. According to Liew et al. (2022), students' mathematical writing error do not usually affect the final answer and are seemingly irrelevant to students' mathematics knowledge.

4. CONCLUSION

It was essential for the teacher to let the students have advance readings at home since the contact hours in the face-to-face instruction were not enough to cover the topics provided the jampacked nature of the curriculum. Considering the new generation of students, employing the responsible use of technology can help students achieve optimal learning. Nonetheless, since many schools especially public schools cannot afford their own infrastructures, giving assignments and additional readings in their handheld devices can benefit the students to realize the goals of blended learning. However, by looking at the data presented in this study, students must do their assigned tasks and come to class prepared if they want to master the contents of the lesson.

The mere presence of online instructional videos and materials with accompanying guide questions does not guarantee student learning as exhibited in the errors they committed because some students did not come to class prepared, or students may have lacked focus and understanding of the materials, among others. As what Rattadilok and Roadknight (2018) argued, teachers who enable timely assessment and immediate feedback improve student engagement and performance. Since formative assessment is one of the most effective ways in improving student achievement through quick feedback (Rattadilok & Roadknight, 2018), the worksheets as formative assessment served its original intent to see if students learned from the videos or if they have watched the videos. Consequently, its use in this study aimed to maximize student achievement and learning engagement.

Additionally, through the formative assessment, teachers can be made aware of what aspects of the lesson needed reteaching, or lacking through analyzing students' solutions and answers to identify and correct their errors. This involved a tedious analysis of students' common errors which is necessary in leading to an enhanced understanding of mathematical concepts by identifying the root causes of their incorrect solutions and answers. Thus, in this study, there was a whole class discussion conducted which also played a key factor in processing students answers and correcting their errors after these errors have been identified. The iterative process of providing regular assessments aligned with students' ability levels will enable quick identification of persistent errors and timely intervention. With proper synthesis and deepening of the previously learned concepts from the online class instruction and worksheets, students realize their committed errors and mistakes so as to lessen its occurrence in their future assessments. Teachers owe their students the appropriate, quick and timely feedback on their performance that can help them be more reflective in the learning process. The consolidated discussion provided them the opportunity to recognize their mistakes and errors committed so they could be aware that whenever they encounter similar problems in the future, they will be able to recall the right procedures to answer the items. The results of students' summative test were not analyzed as we deemed it to be beyond the scope of this study. Future studies may be conducted to determine the effects of a consolidated class discussion in processing or rectifying students' errors and/or other similar interventions on minimizing students' errors.

Results of the error analysis show students can avoid unnecessary mistakes brought by carelessness, mishandling of signs or not following directions by double checking their answers when time can afford. And lastly, students' answers in this study have shown poor prerequisite skills and concepts which greatly affect students' performance in mathematics. In solving quadratic equations, the prerequisite skills that have affected students' performance were distinguishing a quadratic equation from other types of equations and expressions, understanding what it means to solve an equation, writing a quadratic equation in standard form, mathematical writing error particularly omitting "equals zero" which implies the application of the zero product property, and simplifying algebraic expressions that involve radicals, rational expressions, factoring, and special product. In the light of the error analysis findings, future studies may look into appropriate instructional interventions and strategies in these kinds of errors when teaching quadratic equations.

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