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# Pre-service mathematics teacher conducting prospective analysis: A case study on practice didactical design research

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## Abstract

This study aims to understand how pre-service mathematics teachers conduct prospective analyses comprehensively. A qualitative research method with a case study design was used. The participant was Mrs N, a pre-service mathematics teacher who conducted research using DDR methodology in a grade 9 junior high school in Sindang, Majalengka, involving 48 students. Data were collected by analyzing empirical evidence from Mrs. N's prospective analysis process by uncovering the various stages of a prospective analysis. An iterative approach was used to analyze the data by refining the research questions through discussions and regular meetings with Mrs. N. The process ensured adaptability to new insights and understandings from the empirical data. Our findings reveal that the teacher draws upon fundamental philosophical principles from didactical design research, including hermeneutics, phenomenology, and ethnomethodology. By embracing these approaches, the teacher gains valuable insights into conducting research within an interpretive paradigm, allowing for a deeper exploration of the meaning of concepts, the purpose of learning, and the cultural influences that shape the educational process. Additionally, our study sheds light on the emergence of transpositional didactics theory as the prospective teacher delves into understanding the meaning of the geometrics transformation concept.

**Keywords**:
Didactical design research, Geometric transformation, Learning obstacles, Pre-service mathematics teachers, Prospective analysis

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## 1. INTRODUCTION

A growing body of literature emphasizes the crucial role of teaching, recognizing it as a multifaceted and intricate process (Hiebert et al., 2005; Maher et al., 2018). Professional and pedagogical competence is the foundation of teaching (Blömeke et al., 2020). The ability of teachers to create practical learning experiences that foster academic success for their students hinges on their possession of both professional and pedagogical competencies (Baumert & Kunter, 2013; Fennema & Franke, 1992; Kozikoglu, 2017; Tanang & Abu, 2014; Tichenor & Tichenor, 2005). Consequently, teachers' educational journey becomes closely intertwined with developing their professional and pedagogical competencies, shaped by their experience during higher education (Balbag et al., 2017; Reynolds et al., 2016). This experience offers valuable insights into crafting high-quality instructional design within the classroom (Allas et al., 2020; Keller-Schneider et al., 2020). Thus, maintaining and enhancing these competencies becomes a matter of ongoing professional development (Blömeke et al., 2020). Therefore, cultivating research capacity among pre-service teachers is indispensable, as it is vital to their professional growth and development.

Developing research capacity is a vital strategy for enhancing the competencies of pre-service teachers, making it a focal point in higher education worldwide (Bai et al., 2013). Teacher education has emphasized the importance of strengthening research capacity as a critical element of professional development for pre-service teachers (Oancea et al., 2021). Engaging in research activities enables a comprehensive understanding of the teaching profession, establishing a solid foundation for comprehending the educational process and advancing professionally (Christie & Menter, 2009; Maboya et al., 2022). Pre-service teachers' research objectives include enhancing their pedagogical competencies, evaluating existing teaching practices, adapting them to new situations and diverse learners, and incorporating fresh perspectives into the field of research. Consequently, building research capacity has garnered significant attention in professional development, necessitating research to shape pre-service teacher dispositions toward their profession (Pesti et al., 2018). Because students' performance is highly dependent on teacher performance, teachers' skills are heavily influenced by their prior skills or experiences (Suharta & Suarjana, 2018). The situation mentioned above greatly influences teachers' ability to critically assess the underlying beliefs, convictions, and ideologies within educational materials and teaching methods, enabling them to provide educational assessments based on a combination of academic research and practical competencies.

In the 2010s, a paradigm of mathematics education research emerged in Indonesia, commonly called didactic design research (DDR). DDR was developed based on the Theory of Didactic Situations (Brousseau, 2006) and underpinned by philosophies such as hermeneutics, phenomenology, and ethnomethodology (Suryadi, 2019). In simple terms, it aims to develop teaching materials ready to be taught to students (Bakker, 2018; Cobb et al., 2003) but has a higher level of complexity when compared to traditional pedagogical frameworks (Costică, 2013). In principle, DDR is a theoretical, conceptual, and methodological framework for developing teaching materials based on students' learning barriers (Suryadi, 2019). Since DDR refers to the concept of science didactics that prioritizes the concepts of cognitive psychology and learning theory, from a curriculum point of view (psychogenetic constructivism promoted by Piaget and sociocultural constructivism promoted by Vygotsky and Bruner), DDR can be considered as a science didactic (Costică, 2015). The concept of science didactics believes that teaching materials that are ready to be taught are the result of analyzing students' knowledge diffusion and acquisition (Chevallard & Bosch, 2020). It makes DDR very different from other theories, where others focus more on the diffusion process alone, be it learning methods or even the design research itself. As for the DDR theoretical framework to interpret the acquisition of knowledge, it is facilitated by the DDR interpretive research paradigm (Suryadi, 2019).

Prospective analysis within the DDR framework is part of the Interpretive Paradigm (Suryadi, 2019). Prospective analysis is vital in preparing for a learning design (Cobb et al., 2003), including analyzing learning obstacles (Fuadiah et al., 2019). The analysis becomes an essential reference for designing the ideal didactic situation (Carvalho et al., 2004) so students can develop a deep and relevant understanding (Aydın & Ubuz, 2010; Ramli et al., 2013). Such analysis also helps teachers anticipate the adverse effects of inappropriate didactic situations (Shriki & Lavy, 2012). The analysis process is also considered at the core of teacher practice (Mellone et al., 2020). Teachers' ability to analyze students' difficulties is crucial regarding conceptual knowledge, skills, attitudes, and procedures (Fennema & Franke, 1992; Tanang & Abu, 2014). By analyzing students' errors, teachers can encourage higher-quality thinking, provide more targeted assessments, and engage students in metacognitive processes. The acquisition of such knowledge profoundly influences their future pedagogical approach, instilling in them a sense of assurance and competence in effectively instructing mathematics (Lo, 2021; Rrustemi & Kurteshi, 2023).

Providing pre-service mathematics teachers with practical training in conducting prospective analyses will equip them with the necessary expertise to ensure the efficacy of learning materials developed through DDR in addressing learning obstacles and enhancing students' educational experiences. Prospective analysis is a notable advantage in design research (Steffe & Thompson, 2012). It has been shown to improve the effectiveness of learning design (Kanselaar, 1993). According to Bakker (2018), asking why a design is designed is the first step in a prospective analysis. In the domain of DDR, prospective analysis involves examining three categories of learning obstacles: epistemological, didactical, and ontological, as outlined by Brousseau (2006). The complex and open-ended process of prospective analysis relies on the researcher's creativity (Edelson, 2002). It needs theoretical knowledge and practical experience (Gravemeijer, 1994). Thus, prospective analysis emerges as a valuable and pragmatic approach to understanding and enhancing learning design. Of course, it can develop the knowledge and practice needed to be an effective teacher (Lloyd et al., 2020; Raiula et al., 2023; Warshauer et al., 2021), i.e., improving the quality of the learning design (Baumert & Kunter, 2013). Overall, it is essential to provide pre-service mathematics teachers with the experience of undertaking prospective analysis to identify learning obstacles, enhance their skills, and improve learning design for effective teaching.

Further comprehensive investigation is warranted in the field of transformation geometry. The above issue arises from the difficulties students encounter in comprehending and proficiently grasping the principles of transformation geometry. Through prospective analysis, pre-service mathematics teachers will acquire valuable insights into students' challenges when understanding transformation geometry. Based on research. Students have difficulty understanding the concept of geometric transformation in general (Albab et al.,

2014) and have problems understanding the concept of rotation (Ada & Kurtuluş, 2010; Edwards, 1997). Then, there are difficulties in identifying the center and the angle (Yanik & Flores, 2009). Students also have problems with proofs of geometric transformations (Portnoy et al., 2006). Students lack conceptual knowledge of coordinate systems (Malatjie & Machaba, 2019). The results above emphasize the significance of conducting additional research and formulating specific interventions to augment students' comprehension of transformation geometry. By identifying and resolving these obstacles, pre-service mathematics can cultivate a more profound understanding of transformation geometry.

The assimilation of novel mathematical concepts necessitates a comprehensive examination, wherein students must embrace mathematical principles by their preexisting beliefs, values, and experiences (Rösken & Rolka, 2007; Sullivan, 2011). Based on a study conducted by several researchers, it has been observed that students possess a restricted comprehension and familiarity regarding the characteristics of geometric Transformation (Jiang, 2008). Consequently, students' fundamental understanding is weak (Bardini et al., 2014). According to Yanik (2011), this occurrence arises due to the abstract nature of the learning design. It requires a learning design that links new concepts with students' prior understanding and experience and provides a relevant context to help students strengthen their knowledge of geometry transformation. Therefore, exposing pre-service mathematics teachers to prospective analysis is anticipated to yield more efficacious instructional methodologies, enhance students' comprehension of geometric transformations, and refine learning designs that cater to students' requirements. The objective of this study is to provide a comprehensive depiction of pre-service mathematics teachers engaged in the process of prospective analysis specifically focused on transformation geometry material.

## 2. METHOD

This study follows a qualitative approach, employing a case study design. Case studies are widely utilized in various social science disciplines, such as sociology, business, education, politics, and economics (Ellinger et al., 2005). In education, case studies are valuable for investigating specific issues. A case can refer to an individual, a group, a program, an institution, or a policy (Merriam & Tisdell, 2015; Yin, 2009). This study will focus on a pre-service mathematics teacher (N) conducting research for her senior thesis using the DDR methodology. DDR research typically encompasses three stages: prospective analysis, metapedadidactics analysis, and retrospective analysis. The case study approach is well-suited for investigating these three phenomena and exploring the experiences of the pre-service mathematics teacher throughout the DDR process.

### 2.1. Participant

The research was conducted in a grade 9 junior high school in Sindang, a sub-district of Majalengka district. The class consisted of 48 students, most of whom (92%) were of Sundanese ancestry. Additionally, 68% of the students were classified as socioeconomically disadvantaged. Mrs N, a pre-service mathematics teacher, chose to explore the DDR methodology for her final undergraduate project. It was Mrs N's first experience conducting

DDR-related research in the classroom. Mrs N employed a purposeful sampling method to ensure a representative sample, which allowed her to gather information and understand the study's key topics. As part of this method, six students were selected as the research focus and participated in interviews.

## 2.2. Context

The research was conducted through continuous learning activities, specifically from February 2022 to June 2022. At the time of this report, Mrs N's research reached the prospective analysis stage. In the initial phase, Mrs N conducted preliminary observations to identify the most relevant study materials. Among the topics investigated, transformation geometry emerged as the most intriguing subject. However, the Geometry Transformation Competency Test result on November 1, 2021, revealed that only 60.42% of students met the Minimum Completion Requirements (MCR). This low student achievement highlights the urgent need for intervention and improvement. Mrs N's research objectives, as outlined in the research flow, are to investigate the meaning of the following aspects related to Geometric Transformation among junior high school students. This low student achievement highlights the urgent need for intervention and improvement. The research process followed the following sequence (see Figure 1).

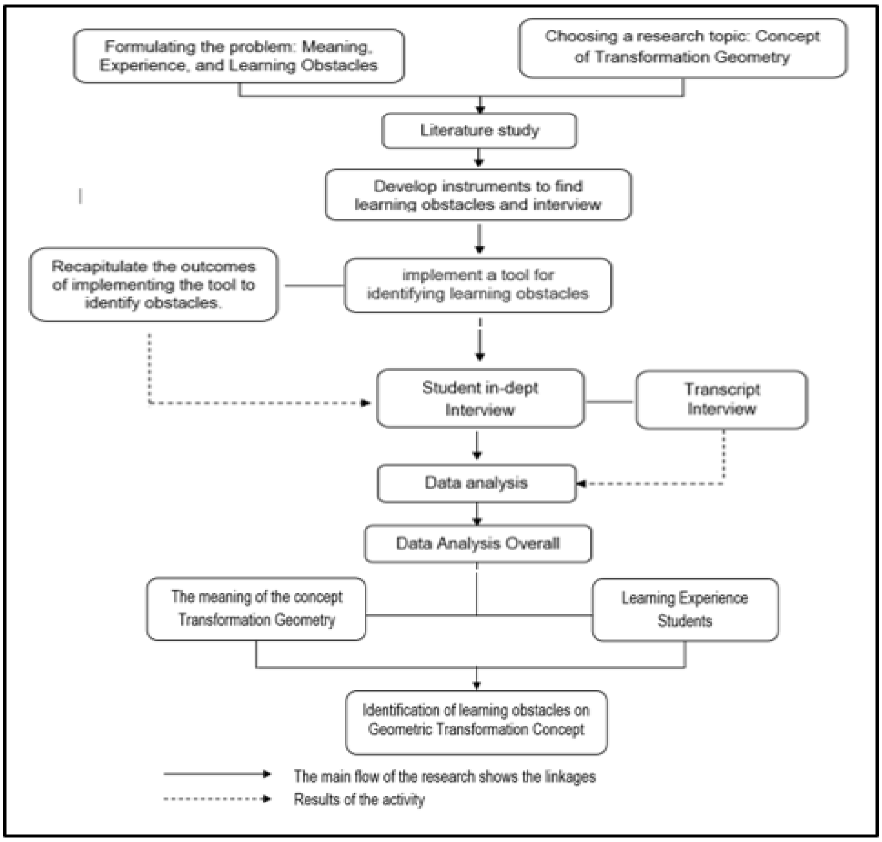



**Figure 1**. Prospective analysis conducted by Mrs. N's

The purpose of Mrs. N's research, as outlined in the research flow, was to investigate the meaning of various aspects of the concept of transformation geometry among junior high school students: (1) The student's perception of the interpretation of the concept of transformation geometry; and (2) To identify learning obstacles that students may face when learning geometric transformations.

Various research instruments were utilized to achieve these objectives. The written examination used in this study is the Respondent Ability Test, administered to 48 students and lasted 90 minutes. This test aims to assess students' understanding of Geometric Transformation, particularly in the context of problem-solving. It also seeks to identify potential learning barriers by examining the solution process and analyzing student responses. In addition to the written examination, in-depth interviews were conducted with a selected group of six students. These interviews aimed to delve deeper into the students' understanding of Geometric Transformation, explore their problem-solving approaches during the Respondent Ability Test, and analyze their experiences grasping the concept. All data collected from the Respondent's Ability Test, in-depth interviews, and supporting documents were meticulously researched to identify potential learning obstacles junior high school students may encounter when studying Geometric Transformation.

### 2.3. Data collection and analysis

The case study conducted in this research focuses on individual or group actors and aims to comprehend their perspective on specific phenomena (Hitchcock & Hughes, 1995). It seeks to capture how individuals define and interpret these phenomena within a particular context and at a specific time (Yin, 2009). The data collection process involved analyzing empirical data collected by Mrs N during her research. Initially, the data was collected primarily to evaluate the research process, but its potential for further analysis became apparent later. An iterative approach was adopted to analyze the empirical data (Corbin & Strauss, 1990). The observed data from all prospective analysis processes were analyzed iteratively to refine the research questions. Regular meetings were held with Mrs N to discuss and refine the observations, findings, and data analysis efforts. This iterative process allowed for continuous refinement of the research, ensuring that the analysis remained responsive to emerging insights and new understandings derived from the data.

## 3. RESULTS AND DISCUSSION

### 3.1. Results

This section will explore the empirical evidence of Mrs N's research, specifically the prospective analysis, which serves as this study's primary data source. The case study aims to evaluate three key findings made by Mrs. N, and the analysis outcomes are provided below:

#### *3.1.1. The student's perception of the interpretation of the concept of transformation geometry*

Mrs N examined students' understanding of geometric transformation and categorized their first interpretation into different groups. To assess the significance of this topic, Mrs N posed the following question to the students in the Respondent Ability Test:

"How do you define Geometric Transformation?". The students' responses are summarised in the Table 1.

**Table 1**. Student answers to the significance of geometric transformation concepts

| Categories of Meaning | Transformation Geometry: Explanation of the Definition | Number of Students |
|---|---|---|
| 1 | Position change or displacement | 9 |
| 2 | Displacement or changes in shape and size | 8 |
| 3 | The process of associating points on a plane with a set of points on the same plane is called mapping. | 13 |
| 4 | Combination of several formulas for flat shapes: reflection, Translation, rotation, and dilation | 8 |
| 5 | Miscellaneous | 8 |
| 6 | No answer | 2 |
| | Total | 48 |

Let's examine Table 1, which presents various responses related to the definition of geometric transformation. These responses provide insights into the students' understanding of the concept. Based on these findings, we can assess Mrs N's knowledge of the purpose and significance of her results within the framework of a prospective analysis.

*Researcher* : *Give a detailed explanation of the findings presented in Table 1.*

*Mrs N* : *According to the results of the analysis, the actual definition of the idea of Geometric Transformation written by certain students continues to contain an inconsistent and imprecise interpretation of the concept. To study the true meaning of the notion as understood by students through in-depth interviews, I selected six students as participants based on a purposive sampling procedure.*

*According to two assumptions to determine the meaning of the concept of Geometric Transformation: (1) each student's response contains a particular unit of meaning, and (2) the true meaning of the concept of Geometric Transformation can be observed by the consistency of the meaning used when solving the Respondent Ability Test questions and by what is revealed in the in-depth interview.*

*Based on the research assumptions, the distribution of responses in Table 1, the meaning units from the six students selected for interviews, and three conclusions regarding interpreting the concept of Geometric Transformation as students perceive it. These three findings are as follows:*

1. *Geometric transformation refers to mapping points on a plane to a distinct set of points on the same plane.*
2. *There exists ambiguity regarding the significance of the concept of geometric transformation.*
3. *Additional meaning units related to the concept of geometric transformation have been identified.*

### *3.1.2. Learning obstacles to the concept of geometry transformation*

According to Mrs N, the findings related to students' interpretations and experiences in understanding Geometric Transformations do not guarantee a comprehensive understanding of the concept. Thus, it is vital to investigate the presence of potential learning obstacles students face in Geometric Transformation. Based on the knowledge of the concept's meaning and the student's experiences in acquiring it, it has been identified that learning obstacles are present in the context of Geometric Transformation. Specifically, ontogenetic, epistemological, and didactical obstacles are commonly encountered.

To gain further insight, Mrs N was interviewed in-depth to explain how she believes students encounter ontogenetic, epistemological, and didactical obstacles during the learning process.

*Researcher* : *How can students with Ontogenic Obstacles be identified for the Concept of Geometric Transformation?*

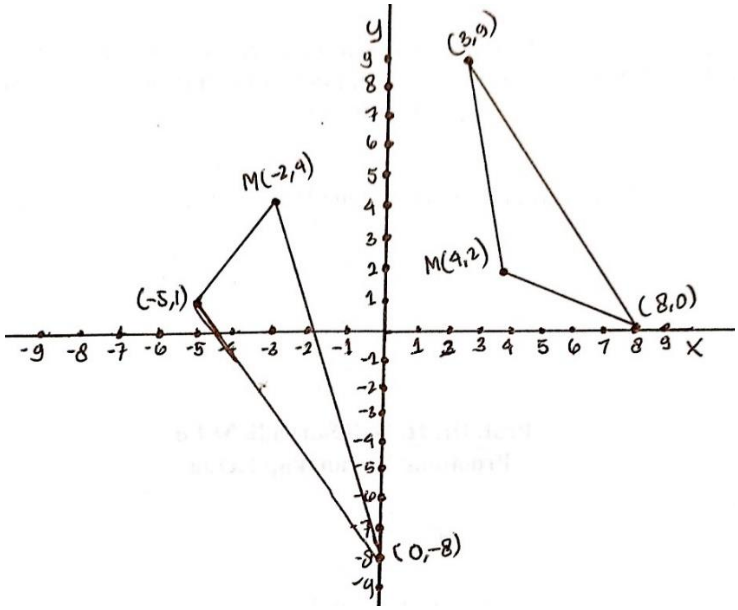
*Mrs. N* : *The researcher suspects that the idea of Geometric Transformation has learning difficulties that are ontogenetic or ontogenetic. Let's examine a few indications of ontogenetic impediments inside the Geometric Transformation notion.*

- *Students are not consistent in interpreting the concept of Geometric Transformation.*
- *Students are unfamiliar with the precise definition of the idea of Geometric Transformation.*
- *Students do not fully comprehend the meaning.*
- *The researcher observed a mismatch in the difficulty level of the questions given to students.*
- *Researchers discovered student solutions that did not correspond with the correct answer in the respondents' test ability results and student responses to interview questions.*
- *Students sometimes struggle with simple problems involving the direction of rotation, the center of rotation, and dilatation, among others.*

*Researcher* : *How can students with Epistemological Obstacles to the Concept of Geometric Transformation be identified?*

*Mrs. N* :

- *Students' responses on the Respondent Ability Test demonstrate inconsistent knowledge of Geometric Transformation.*
- *Researchers discovered that students struggled to solve contextual problems involving the application of the idea of Geometric Transformation, meaning understanding and applying knowledge (factual, conceptual, and procedural). It suggests that students had little context. The prevailing circumstances tend to emphasize procedural solutions.*
- *Another finding is in the example of student response completion:*




*The representation of Geometric Transformation in Cartesian diagrams so far obtained by students tends to be oriented to the overall shape of the building and does not map point to point.*

- *This entire explanation boils down to a single issue: students' limited understanding and comprehension of geometric transformation. The implication is that students have difficulty and make mistakes when confronted with challenges or problems involving Geometric Transformation principles outside the context and form they have typically acquired.*

*Researcher* : *How can students with Didactical Obstacles to the Concept of Geometric Transformation be identified?*

*Mrs. N* : *Based on the findings of interviews, students acknowledged openly and implicitly that the benefits of the idea of Geometric Transformation in everyday life are not integrated into the learning process. In addition, the researchers discovered that the material was not presented in the correct order since the translation material was not presented first when the teacher explained the material. Rotation, Translation, Reflection, and Dilation are the topics provided in the order in the Class XI Senior High School student book (Ministry of Education and Culture, 2017). The researcher observed that translational material is better if presented at the beginning before other material. Therefore, this indicates that there is an inappropriate order of material. The researcher found that the idea of Geometric Transformation lacked a precise definition. Providing a formal definition is essential. Researchers also note that this is another type of didactical obstacle.*

### *3.1.3. The acquisition of the definition of the concept of geometry transformation by students*

Mrs N proceeds to explore the students' learning experiences with geometry transformation. She emphasized the significance of tracing these experiences to understand the learning process and gather valuable insights for improvement. Based on previous research, the following key points were identified.

a. The concept of Geometric Transformation encompasses mapping an individual point located on a given plane to a collection of points that also reside on the same plane;
b. Ambiguity exists regarding the importance of the concept of Geometric Transformation; and
c. Additional meanings associated with the concept of Geometric Transformation were discovered.

The finding indicated that students' understanding of geometric transformation is not solely derived from inherent knowledge but is influenced by their experiences and significance. It is crucial, therefore, to explore the various factors that contribute to the meaning attributed to Geometric Transformation, particularly the students' experiences in grasping its purpose. Mrs. N conducted in-depth interviews and discovered that student's comprehension of the concept was primarily shaped by the teachings of their instructors and the information they obtained from textbooks. The six selected students openly and implicitly expressed that their understanding of Geometric Transformation was primarily derived from the resources provided by their teachers and the content covered in their textbooks. Thus, it can be observed that students' understanding of Geometric Transformation is heavily influenced by the guidance of their instructors and the information presented in their course materials.

When exploring the meaning of Geometric Transformation as a displacement of position or a change in shape and size, it was revealed that some students acquired and constructed this understanding by introducing related concepts. Before learning about Geometric Transformation, students were introduced to the idea of change in appearance, encompassing aspects such as form, nature, and function (as defined in the Big Indonesian Dictionary). Illustrations played a significant role in this process, including examples like a child looking in a mirror, a student moving a table, a propeller rotating, and the magnification of a natural landscape photograph. These visual aids effectively created a mental image in students, helping them grasp the concept's meaning. The following excerpt is from Mrs N's group interview with third- and fourth-grade students, shedding light on their understanding of Geometric Transformation (see Figure 2).

| | | |
|---|---|---|
| Mrs N | : | Why ....? |
| S4 | : | Yes because of that, I remember that picture. When taught geometry transformation. There is a child shifting the table, then the mother said that the train was running and the passengers moved and everything in it he.he.. |
| Mrs N | : | Why geometry transformation is defined by S3 like this? |
| S3 | : | at that time I've seen in the back of the book or on google yes, uh in the book mom, when mom gave group assignments. Yes, there is at the back of the book (glossary) |

**Figure 2**. Interview the student

From the excerpts from the interview (see Figure 2), Mrs. N. arrived at a similar conclusion as S4 and S3 regarding the importance of geometric transformation in the mapping and alteration of points or objects on the same plane. The interviews with S4 and S3 revealed that the student's understanding of Geometric Transformation primarily revolved around example problems and procedural knowledge. They were more inclined towards comprehending problems with predetermined solutions rather than engaging with conceptual issues.

### 3.2. Discussion

When examining Mrs N's research findings, we can draw four conclusions based on the significance of the geometric transformation concept as reported by the students. First, geometric Transformation maps points on a plane to a set of points on the same plane. This definition is a combination of various meanings generated by students. It aligns with formal definitions provided by experts and can be found in college-level textbooks. For instance, in the college-level textbook, geometric transformation is defined as a transformation on the plane is a one-to-one correspondence from the set of points in the plane onto itself (Martin, 2012), a transformation of the plane is a function $\rightarrow: R^2 \rightarrow R^2$ with domain $R^2$ (Umble & Han, 2015), a transformation of the plane is a one-to-one function from the plane onto the plane (Eccles, 1971), and a transformation of the plane, or more simply, a transformation, $\alpha$ is a one-to-one map or function of the points of the plane onto themselves (Dodge, 2012). However, in some junior high school textbooks, the concept of transformation geometry is not clearly explained, but specific types of transformations, such as reflection, Translation, rotation, and dilation, are directly addressed (Subchan et al., 2018). Other textbooks explicitly discuss translations, reflections, rotations, and dilations (Dudeja & Madhavi, 2018). It is worth noting that although Mrs N's description of the concept aligns with the formal definition used by mathematicians, the two junior high school books she referred to lack a formal definition of geometry transformation.

So, according to Mrs. N's research, the understanding of transformation geometry among students varies. They tend to have a less specific knowledge of the formal definition, which may differ from the definition provided in university textbooks. Initially, students perceived transformation geometry as altering objects' position or displacement on a two-dimensional plane. Furthermore, specific individuals interpreted it as a modification in the form and dimensions of the entity. Moreover, students believe that the essence of transformation geometry lies in mapping points inside a specific plane, whereby each point is initially assigned a particular location within that plane. Ultimately, students integrate the notion of transformation with different flat geometry formulas, including reflection, Translation, rotation, and dilation. The various interpretations of transformation geometry among students reflect their lack of clarity in grasping the idea. This lack of clarity might cause significant difficulties in learning, including ontological, epistemological, and didactic obstacles.

Mrs. N's Prospective analysis focuses on the interpretive paradigm of the DDR framework. Its primary goal is to analyze students' knowledge acquisition, specifically the impact of the didactical design employed. The process for acquiring knowledge in the concept of didactic transposition is commonly referred to as "learnt knowledge," specifically referring to the knowledge obtained from students. The study conducted by Mrs. N in this research yields knowledge about the definition of transformation geometry and the learning obstacles students encounter. Based on the French didactic tradition, when engaged in didactic research. It has three main pillars: Vergnaud's theory of conceptual fields (Vergnaud, 2009), the theory of didactical situations (Brousseau, 2006), and the anthropological theory of the didactic (ATD) that emerged from Chevallard's theory of didactic transposition (Chevallard, 2019). Unconsciously, Mrs. N engages in didactic transposition during her prospective analysis. It becomes evident as she can justify the accuracy of definitions and identify learning obstacles.

According to Chevallard (2019), knowledge $\mathscr{k}$ is a subset of a body of knowledge $\mathcal{K}$, a subset of discipline $\mathcal{D}$: $\mathscr{k} \subseteq \mathcal{K} \subseteq \mathcal{D}$. When delving deeper, we find that Mrs N has undergone the process of external didactic transposition, which takes place outside the school setting. It involves analyzing the definition of transformation geometry from the perspective of mathematicians. Additionally, Mrs N has undergone internal didactic transposition within the school context, involving the analysis of textbooks and the meaning of transformation geometry as students understand (Bosch et al., 2021). The terms "external" and "internal" didactic transposition were introduced by Østergaard (2015). Without realizing it, Mrs N has gained valuable experience by navigating these three institutions, which undoubtedly contributed to her effectiveness as a teacher and her ability to gauge students' understanding of mathematical topics. Figure 3 illustrates the external didactic transposition, in which Mrs. N draws inferences about the definition of Transformation Geometry based on a comparison with the student's understanding of the topic, as presented in the college textbook.

*Based on the research assumptions, the distribution of responses in Table 1, and the meaning units from the six students selected for interviews, three conclusions regarding interpreting the concept of Geometric Transformation as students perceive it. These three findings are as follows:*

1. *Geometric Transformation refers to mapping points on a plane to a distinct set of points on the same plane.*
2. *There exists ambiguity regarding the significance of the concept of geometric Transformation.*
3. *Additional meaning units related to the concept of geometric Transformation have been identified.*

**Figure 3**. Interview excerpt on the didactic transposition process

The second conclusion about Mrs N's research reveals another significant finding: students experience "Ambiguity in the importance of the concept of Geometry Transformation." This ambiguity stems from students' diverse interpretations of the concept of transformation in geometry. The lack of clarity surrounding the established meaning of transformational geometry in the learning environment contributes to this ambiguity. It signifies a misalignment between students' concept image and formal concept definition, which should be minimized to ensure a solid concept understanding. According to Tsamir et al. (2015), it is crucial to establish alignment between students' concept image and concept definition as early as possible. Both concept images and definitions are essential components of students' cognitive structures (Vinner, 1983). Concept image refers to the mental framework of a mathematical concept constructed through students' experience (Dreyfus et al., 2014; Tall & Vinner, 1981), while concept definitions are explicit formal descriptions of mathematical concepts (Tall & Vinner, 1981). Therefore, when students are asked to explain the meaning of the concept of geometric transformation, the success of their explanations heavily relies on their previous learning experiences.

Mrs N's research highlights the presence of ambiguity in students' understanding of geometry transformation, leading to a misalignment between their concept images and concept definitions. Consequently, it becomes crucial for teachers to enhance students' perceptions and concept images about problem-solving (Nurwahyu & Tinungki, 2020). Concept images may not fully correspond to concept definitions as they evolve with exposure to new mathematical ideas (Engelke Infante et al., 2018). These findings suggest that students may lack sufficient knowledge of concept definitions, resulting in limited images and difficulties in understanding and applying the concept (Clements & Sarama, 2011). Mrs N's experience will contribute to her ability to design optimal learning experiences that engage students in exploring and understanding the meaning of concepts through active participation in the learning process. Vinner (2002) emphasized that teachers' expertise in lesson design is crucial in improving students' understanding and aligning their concept images with scientific conceptions. Failure on the part of teachers to effectively communicate the subject matter can lead to deviations in students' conceptualizations, causing them to diverge from the intended concept definitions. Therefore, teachers need to employ practical teaching approaches that foster accurate concept images and facilitate meaningful learning experiences for students.

The third conclusion about Mrs. N's research findings reveals three significant learning obstacles in geometry transformation: ontogenic, epistemological, and didactical. To reach this conclusion, Mrs N followed a series of steps. Initially, she administered a respondent ability test to assess students' understanding of transformation geometry. After analyzing the test results, Mrs N conducted in-depth interviews to uncover specific learning obstacles. This approach aligns with the methodology employed by previous researchers in identifying learning obstacles in subjects such as the System of Linear Equations Using the Gauss-Jordan Method (Siahaan et al., 2023), one-variable linear inequality (Rohimah, 2017), and the derivative concept (Musyrifah et al., 2022). Through this process, Mrs N discovered that some students lacked a fundamental understanding of Geometric Transformation, leading to difficulties tackling more advanced problems.

Mrs. N found numerous less precise definitions of Geometric Transformation. The "No answer" category (number 6) submitted by two respondents did not help define the notion. Second, the "Miscellaneous" group (number 5) of 8 students had unclear responses, suggesting a lack of concentration on Geometric Transformations' specific descriptions. Third, definition 4, which defines Transformation Geometry as a mixture of flat shape formulas like reflections, translations, rotations, and dilations, may be excessively technical and distract from the concept's mathematical essence. From an academic perspective, these ideas may be unsuitable because they don't correctly explain Transformation Geometry.

Based on these findings, Mrs. N confirmed the respondents who answered that they did not follow transformation geometry to identify student learning obstacles. The researcher confirmed how Mrs. N determined each learning obstacle, such as ontogenic obstacles. Mrs. N replied, "Based on the findings, I concluded "Students are not consistent in interpreting the concept of Geometric Transformation," which indicates that students' understanding is not solid, possibly due to limited relevance in their previous basic concepts.

Concerning the epistemology of obstacles, Mrs. N concluded that students encounter obstacles when answering contextual problems. "The findings indicate that students may have limitations in understanding the fundamental background." This topic emphasizes an approach that centers around adhering to a prescribed set of steps to arrive at a solution. According to the research conducted by Mrs. N, various cognitive obstacles that inhibit students' comprehension of the idea of Geometric Transformation were discovered. Firstly, students openly acknowledge that they fail to perceive the relevance or advantage of this concept in their daily lives.

Furthermore, there exists a disparity in the sequencing of the content among the utilized textbooks. The sequence of Rotation, Translation, Reflection, and Dilation is presented; however, in university textbooks, the order of Translation is presented first. It aims to enhance comprehension of geometric transformation, which is intricately linked to the concept of displacement. This arrangement disparity can serve as an obstacle to maintaining uniform and efficient instruction on this subject.

Furthermore, students exhibited limited contextual and spatial comprehension, resulting in a narrow concept image of geometric transformation. Additionally, Mrs N observed that Geometric Transformation's practical applications and benefits were not effectively integrated into everyday life. The organization of the material itself was deemed inadequate from a functional perspective, and the concept of Geometric Transformation lacked a specific and well-defined definition. Teachers should facilitate rule-finding investigations while learning transformation geometry to enable students to apply the rules confidently in the future (Bansilal & Naidoo, 2012). Teachers must follow the research recommendations to ensure an efficient and effective learning process in geometric transformation that aligns with the desired learning objectives. Mrs N's observations regarding students' experience in understanding the meaning of geometric transformations highlight the uncertainty and lack of clarity in the learning environment. This ambiguity surrounding the importance of transformation geometry impacts students' confidence in their thoughts and written work. Students' knowledge has not yet reached the internalization stage, as per the theory of knowledge (Uriarte Jr, 2008). The explicit knowledge obtained from various sources, such as teachers, books, media, and peers, has not been fully integrated into their tacit knowledge. According to the theoretical perspective, the didactic situation has not yet reached the stage of institutionalization (Brousseau, 2006). As a result of this situation created by the teacher, students have not fully comprehended and developed the meaning of the Geometric Transformation concept, leading to difficulties in solving various problems. Consequently, it can be concluded that students undergoing the learning process of transformation geometry have not acquired a solid foundation of knowledge in the subject.

The fourth conclusion pertains to the study findings of Mrs. N, which are directly linked to the research methodologies employed. The previous discussion provides an overview of Mrs N's research methodology. The analysis phase involves three steps: prospective, metapedadidactic, and retrospective analysis, which are part of the didactical design research methodology. However, due to limitations in the field, only the prospective analysis phase was conducted in this study. Even though only one stage was completed, the research philosophy of didactical design encompassing hermeneutics, phenomenology, and ethnomethodology. DDR focuses on understanding the reality associated with students' concept images in the given learning context. Mrs N's initial analysis focuses on students' interpretations of the definition of geometry transformation and the ambiguity surrounding its meaning. The disparity between concept image and concept definition contributes to the ambiguous description of the importance of transformation geometry. Another object of analysis pertains to identifying learning obstacles. These realities are subject to analysis using the philosophy of Hermeneutics (Suryadi, 2019). According to Mrs N's research, there are three learning obstacles in the context of geometry transformation.

Phenomenology is the subsequent philosophy that underlies the research process within the framework of DDR. It involves studying an individual's perception and interpretation of the world they inhabit (Marton, 1981). In the framework of DDR, phenomenology is employed to explore students' post-learning situations. Mrs N's research has shed light on how students experience the learning process and its impact on learning outcomes. This aspect is one of the focal points that can be investigated using the phenomenological philosophy (Suryadi, 2019). Ethnomethodology, conversely, pertains to understanding how a community achieves its goals more effectively and meaningfully based on its interests (Garfinkel, 2005). Within the education framework, a community of educators can collaborate to develop learning techniques, educational resources, and didactic designs that facilitate students' knowledge acquisition more effectively, relevant, and practically. Mrs N's research findings provide valuable insights that can be used to enhance the overall learning process.

The prospective analysis conducted by Mrs. N has yielded three significant findings: 1) The understanding of the concept of geometry transformation among students; 2) Obstacles encountered by students in learning the concept of geometry transformation; 3) Students' experience in grasping the definition of the concept of geometry transformation. These three discoveries provide invaluable insight into investigating each student's unique circumstances. DDR offers an alternative approach to increase the research capabilities of pre-service teachers. Mrs N's research experience has led her to engage with Chevallard's concept of didactic transposition, which stands as one of the most influential theories in her research. Several research studies related to the theory of didactic transposition are worth mentioning. For instance, Lombard and Weiss (2018) explore the transformation of the cognitive and environmental ecosystem into the educational ecosystem, emphasizing the requirement for specific knowledge in each ecosystem and highlighting the necessity of knowledge transposition. Hausberger (2017) examines the possibilities and challenges in connecting the formalism of homomorphism to the cognitive process of comparison and identification. Additionally, Bosch et al. (2021) employ didactic transposition in an introductory pre-service teacher education course, using a dialogical process incorporating students' prior knowledge, the history of optics, and desired outcomes in geometric optics.

A fresh perspective of mathematics education research in Indonesia emerges when examining the outcomes gained through the lens of didactical transposition theory. The prevailing research trends in Indonesia primarily focus on general learning issues without delving into specific difficulties experienced by individual students. Noteworthy studies in the field of mathematics education research in Indonesia, as revealed through bibliometric analysis, include "Ethnomathematics Research in Mathematics Learning in Indonesia (2017-2022) by Muhammad et al. (2023), "Google Classroom Learning Media Research Trends (2017-2020) by Ajinegara and Soebagyo (2022), and "Trends in the Use of ICT in Mathematics Learning (2019-2021) by Supinah and Soebagyo (2022). These studies collectively reflect a tendency in Indonesian mathematics education research to extensively analyze and apply the learning process, with an average sample size of 200 articles. Conducting prospective analysis provides Mrs N with a challenging research experience and a valuable tool for self-reflection on teaching practices. The findings obtained through prospective analysis offer comprehensive information to enhance the quality of classroom learning, aligning with teachers' moral and professional responsibility to administer fair and appropriate assessments, akin to how physicians evaluate patients and conduct relevant tests (Scriven, 1991). Understanding each student's unique circumstances is crucial to meeting these requirements, which can be achieved through didactic design research employing prospective analysis. Each stage of prospective analysis contributes to understanding student knowledge and facilitating the analysis process (Huberman, 1983). The prospective analysis is the initial phase in the teacher professional development process, characterized by its systematic and ongoing nature. This phase is crucial in ensuring the effective implementation of educational policies (Yaakob et al., 2020).

## 4. CONCLUSION

The approach to DDR involves three steps: prospective analysis, metapedadidactic, and retrospective analysis. Hermeneutics, phenomenology, and ethnomethodology are the underlying philosophical perspectives guiding this research. This study focuses on analyzing the prospective study conducted by Mrs N., which involves analyzing challenges and making decisions to reorganize specific learning plans. The key findings from Mrs N's research include 1) The students' understanding of the concept of geometry transformation, 2) Learning obstacles related to the concept of geometry transformation, and 3) Students' experiences in grasping the definition of the concept of geometry transformation. These findings are from the core of the researcher's investigation. Findings No. 1) and No. 2) are derived from a research procedure rooted in hermeneutic philosophy, while No. 3) is based on phenomenology. Each step in Mrs N's research journey provides valuable insights into addressing specific student problems, enhancing her research capacity and professional development as a future teacher. Moreover, the researcher's perspective reveals that Mrs. N unknowingly underwent a process of didactic transposition when analyzing the meaning of geometry transformation and examining the learning obstacles students face. Investigating knowledge transposition (didactics transposition) is crucial in DDR as it provides a deeper understanding of transformation geometry and facilitates comprehension of students' learning experiences. However, this study only analyzed the prospective analysis stage conducted by Mrs N. Therefore, future research should encompass all the necessary steps to identify additional measures to enhance prospective mathematics teachers' research capacity.

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| | |
|---|---|
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