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A SEMIOTIC PERSPECTIVE OF MATHEMATICAL ACTIVITY: THE CASE OF INTEGER

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ABSTRACT

Semiotics is defined as using signs to represent mathematical concepts in problem-solving. The mathematical semiotic process involves creating meaning from the triadic relationship between the representamen (R), object (O), and interpretant (I). Mathematical semiotics play an essential role in the cognitive processes of individuals as they formulate and communicate mathematical ideas. Therefore, this study aims to describe the stages of the semiotic process of junior high school students solving integers-related mathematical problems. In this qualitative analysis, the participant is a seventh-grade student categorized as pseudo-semiotic. The research instrument is a test on integers and interviews. The results demonstrate that the semiosis related to integers involves the representamen, object, and interpretant stages. For a subject with a pseudo-semiotic type, this meaning-making process requires the construction of a comprehensive understanding of the concept. Furthermore, the understanding is developed using various instruments, resulting in connection conflicts between different components of the semiotic system. Connection conflict occurs because of the mismatched relationship between the elements of semiosis: representamen, object, and interpretant. A pseudo-semiotic subject only has a superficial understanding of mathematical concepts, making it challenging to establish accurate connections between symbols and their underlying meanings. Consequently, this hinders the ability to understand mathematics profoundly and apply the concepts in real-life situations.

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1. INTRODUCTION

Semiotics is a scientific discipline investigating sign processes or something that represents something else, analogy, communication, and symbol (Palayukan et al., 2022; Semetsky, 2016). All signs can be perceived by the senses and conveyed to human activities by creating a systematic code. The discussion of semiotics centers around the comprehension and interpretation of meaning or significance by humans. Additionally, the concept of an integer examines the various constructions of meaning within signs and symbols. The semiotic perspective can be utilized specifically to examine diverse forms of communication in mathematics (Palayukan et al., 2022). To create meaning, mathematical communication employs multiple semiotic systems, particularly language, symbols, and visuals. Therefore, understanding semiotic systems is crucial for comprehending concepts and solving mathematical problems.

Semiosis is the process of creating meaning and interpreting signs. The process of meaning-making occurs in the relationship between "reality" and "what exists in human cognition" (Hoed, 2014). According to the thinking of Peirce's (Brier, 2018; Kralemann & Lattmann, 2013), semiosis, or the process of meaning-making from signs, is a triadic relationship between the representamen (R), object (O), and interpretant (I). The representamen (R) can be physically or mentally perceived and represented by the concept (O). At the same time, the interpretant (I) interprets the relationship between R and O. Therefore; semiosis is the cognitive process of sign formation initiated from the representamen, spontaneously associated with the object, and then interpreted by humans as an interpretant. This process is critical in constructing meaning from a sign and is relevant to mathematics education. An essential aspect of the connection between semiosis and mathematics involves the utilization of signs. Mathematical objects necessitate sign vehicles as representations for comprehension or interpretation. As a result, semiotics serves as a powerful theoretical framework with tremendous potential in exploring various study topics (Presmeg, 2006). Since semiosis is a cognitive process, it cannot be separated from the meaning of concepts in solving problems, which also involves cognitive processing. In this context, semiosis and conceptual meaning are interrelated and mutually influence the mathematical problem-solving process. As an illustration, when tackling a mathematical problem, it is crucial to comprehend the concepts before applying semiosis to construct and interpret the signs in the problem. This semiotic process ultimately contributes to the comprehension and resolution of the mathematical problem.

Semiosis is the cognitive process of creating signs from the representamen, which connects to the object in human cognition and is subsequently given a specific interpretation as interpretants. It is the mechanism of constructing meaning from signs and is intimately linked to mathematics education, specifically for using signs. Mathematical objects necessitate signs as representations for comprehension or interpretation. Therefore, semiotics has the potential as a strong theoretical lens for investigating various topics. As a cognitive process, it is also related to the signification of concepts in solving mathematical problems.

The role of semiotics in the learning and teaching process, as well as in problem-solving. Semiotics is viewed as a tool for explaining mathematical activities in learning and teaching various concepts, such as the analysis of the systems involved in basic arithmetic concepts (Santi, 2011), the semiotic structure related to geometry (Dimmel & Herbst, 2015), and the analysis of cognitive activities in proving absolute value functions (Pino-Fan et al., 2017). Miller (2015) conducted a semiotic study employing Peirce's principles and demonstrated that students actively participate in semiotic activities such as manipulation, discussion, and problem-solving situations while learning geometry concepts. In this regard, semiotics has been used to examine the different meanings associated with geometric diagrams (Mudaly, 2014). The study by Alshwaikh (2010) found that the understanding of geometry is determined by the semiotic structure contained in diagrams, which involves a complex process. These studies have proven that learning and teaching mathematics related to cognitive activity is semiotic.

Mathematical activities involve interpreting signs and transforming the concepts to develop knowledge (Hoffmann, 2006; Hundeland et al., 2014). Using signs, students can think about the relationships between mathematics and the objects in their environment (Suryaningrum et al., 2020). Peirce classified signs into three aspects: sign, object, and interpretant (Brier, 2018; Kralemann & Lattmann, 2013). The sign is a physical form perceived by human senses, while the object is the external information in the environment referenced by the sign. The interpretant is the conceptual thinking of an individual who uses the sign and derives a certain meaning from the object (Arzarello & Sabena, 2011; Suryaningrum & Ningtyas, 2019). Semiotics in mathematics use symbols to represent mathematical concepts in problem-solving (Ostler, 2011; Presmeg et al., 2016).

Integer is one of the basic concepts considered an essential prerequisite for learning mathematics (Cetin, 2019). This concept is typically introduced in elementary school, enabling students to acquire sufficient comprehension before advancing to higher-level material in subsequent grades. Understanding mathematical concepts is a dynamic and structured process (Sa'dijah et al., 2021). Consequently, students must grasp integers well before undertaking more complex topics.

Mathematical concepts heavily rely on how students represent and interpret signs or symbols (Mudaly, 2014). A deeper understanding of the concepts can be achieved when students accurately interpret signs. However, difficulties are frequently encountered in comprehending signs in mathematical concepts, such as integers (Bishop et al., 2014; Murtafiah et al., 2023; Whitacre et al., 2012). This includes negative or positive signs (Bofferding & Wessman-Enzinger, 2017; Vlassis, 2008) in the order relation of integers (Bishop et al., 2014; Schindler et al., 2017), student difficulties in solving problems related to negative integers arithmetic operations (Fuadiah et al., 2017a, 2017b) as well as signs in representation (Widjaja et al., 2011). The difficulty in understanding these signs affects the inability of students to solve mathematical problems correctly (Khalid & Embong, 2020; Kusmaryono et al., 2020). These studies provide an overview of the relationship between signs or symbols and the ability to solve integer problems. Difficulties in understanding these signs have an impact on the integer concepts (Akhtar, 2018), which ultimately affects their performance in problem-solving and causes errors (Khalid & Embong, 2020; Ostler, 2011; Presmeg et al., 2016).

This study highlights the cognitive processes related to interpreting signs in solving integer problems from a semiotic perspective. It is important because semiosis is a cognitive process that cannot be separated from the meaning of concepts in solving mathematical problems. In this context, semiosis and concept meaning are interrelated and mutually influence the mathematical problem-solving process. In solving a mathematical problem, it is crucial to grasp the concept initially and subsequently employ semiosis to establish and interpret signs within the problem. This semiotic process significantly contributes to overall comprehension and problem-solving abilities. Therefore, comprehending mathematical concepts and using semiotic systems in solving problems are fundamental to students' success in learning mathematics.

Mathematical semiotics makes a substantial contribution to mathematics learning. By emphasizing understanding representamen, mathematical objects, and interpretants, mathematical semiotics pave the way for students to achieve a deeper understanding of concepts. Semiotic analysis not only helps students overcome difficulties understanding mathematical notation or special symbols but also allows them to communicate mathematical ideas more clearly. Through the introduction of mathematical semiotic concepts, students are not only invited to understand but also stimulate their critical thinking regarding the use of symbols in mathematics. In addition, mathematical semiotics triggers students' creativity in solving mathematical problems by considering variations in representation. By developing metacognitive skills, students can become more aware of their thought processes, improve their ability to solve problems and become more independent in their mathematics learning. Overall, mathematical semiotics not only deepen understanding of concepts but also increase students' motivation and interest in mathematics subjects through a more interactive and relevant learning approach.

2. METHOD

2.1. Research Method

The method of this study was based on a qualitative approach and an exploratory, descriptive type used to describe, analyze, and interpret the meanings of the signs used by students. The qualitative approach was considered an ideal method for investigating students' interpretation and cognitive construction in their utilization of the semiotic process for interpreting signs while solving problems related to the concept of integers (Eco, 1978).

2.2. Participants

The study included one junior high school student who fulfilled the following criteria: 1) male and female students between the age 12 to 14 years studying in the seventh grade of junior high school, participants in this research who have learned the concept of integers and who have clear communication skills, 2) student participated in learning integer topics by solving items about integer operations. Students were determining integer problem solving based on their experience and knowledge of integer operations, 3) can communicate well (verbally or written), and 4) the student actively engaged in semiotic processes while solving integer math problems.

2.3. Research Procedure

Research activities initiated by determining the study sample. After the sample was set, each student was given a problem math test. The data was collected from the answers provided to the integer math problems and recordings of interviews. The problems explored the semiotic process in representamen, object, and interpretant. The recording device used in this study is a voice or audio recording device. This tool records the subject's responses in the interview process, both short interviews during the test completion process and interviews during in-depth investigations. Additionally, the interviews serve the purpose of verifying and clarifying any ambiguous information provided by the student during their responses.

2.4. Instruments and Data Analysis

This study used two instruments: interview guidelines and tests. His study used two instruments: interview guidelines and tests. The test used in this research consisted of one item. I was used to confirm and explore the semiotic processes related to the concept of negative integers. Researchers prepared and developed the instruments used through direction and guidance from supervisors I and II. The instrument was validated by two experts, a mathematics education expert and a mathematics expert. Validation of the instrument is directed at conformity between the questions and the research objectives, construction and language in the questions. The data collected in this study was analyzed descriptively. Data analysis was carried out after data collection, involving data from tests of number patterns for mathematical semiotic analysis, and interviews of student answers.

The data analysis was done gradually according to the procedure which involves data reduction, data presentation, and conclusion. A dominant subject in one of the three categorized types, namely the student with the incorrect representamen but the correct answer (type B, called pseudo-semiotics, was selected for data triangulation (Creswell, 2014). Meanwhile, the collected data was analyzed to describe the semiotic process of the student in solving integer math problems. The analysis was performed through three steps: 1) selecting appropriate data and eliminating unused data; 2) presenting the data by grouping based on Pierce's triadic semiotic analysis of representamen, object, and interpretant (Eco, 1978; Suryaningrum et al., 2020); and 3) drawing conclusions based on the study findings.

3. RESULT AND DISCUSSION

According to Peirce's semiotic perspective and the respective triadic components used, the process of semiotic construction occurs in three stages: the Representamen, object, and interpretant stages (Brier, 2018; Kralemann & Lattmann, 2013). In each stage, there are connections between the components, and the process of semiotic construction can be seen in Table 1.

Table 1. Semiotic indicators

Process/Stage	Indicator
Representamen	- Identify initial signs and connect/think with an Object or related concept - Create a Representamen based on the initial Object thought
Object	- Interpret the meaning through the confirmation of the correctness of the relationship of the Representamen produced with the initial Representamen - Creating a mathematical model based on the relationship between number lines, integers greater than or less than, and integer operations
Interpretant	- Infer meaning based on the corresponding Object - Inferring the meaning of the result from integer operations

The subject in type B is a student who answered the test question with a wrong representamen but a correct final answer. In this study, type B is referred to as the subject of pseudo-semiotic. The subject tends to obtain the correct solution but needs to construct an accurate meaning in connecting the semiotic components, resulting in a conflict of connections. Even though some of the used representamens differ, the pseudo-semiotic subject's semiotic process is as follows.

3.1. Representamen Stage

The direct stage of pseudo-semiotic begins with obtaining External Representamen, a problem related to the concept of order. Furthermore, the problem is understood and converted into an initial Representamen, namely the order of integers. The subject identifies this Representamen and connects with the concept of the meaning of numbers between -3 and 3. Subsequently, the subject creates a Representamen as a line and mentions all the numbers between -3 and 3, which are -3, -2, -1, 0, 1, 2, 3. The conflict in this connection arises because, according to the subject, all numbers between -3 and 3 start from -3 and end at 3 (Interpretant). This direct stage of the subject corresponds to the interview with the subject as follows:

Researcher : *Do you understand this question?*
Subject : *Yes.*
Researcher : *What did you think first when you read this question?*
Subject : *About addition and subtraction of integers.*
Researcher : *Oh, integer operations. How do you do integer operations?*
Subject : *Using a number line.*
Researcher : *What is a number line?*
Subject : *Like what I did here (pointing to the answer to the question).*

Researcher : *oh Ok. Please explain the meaning.*
Subject : *Yes ma'am. The numbers that are greater than -2 move to the right of the number line and the number to the left of -2 is smaller than -2.*
Researcher : *Do you understand the numbers between -3 and 3?*
Subject : *understand ma'am.*
Researcher : *Please explain to me!*
Subject : *The integer between -3 and 3 is this ma'am (points to the answer)*

Researcher : *Is the number 3 included between -3 and 3?*
Subject : *yes ma'am.*
Researcher : *Why? Try to explain!*
Subject : *because it's between -3 and 3. Therefore 3 is included, ma'am.*
Researcher : *oh 3 included yeah. Number -3 is also included huh?*
Subject : *Yes ma'am. I have not written the answer.*

The subject's direct stage in the sequence concept can be seen in the following scheme (see Figure 1).

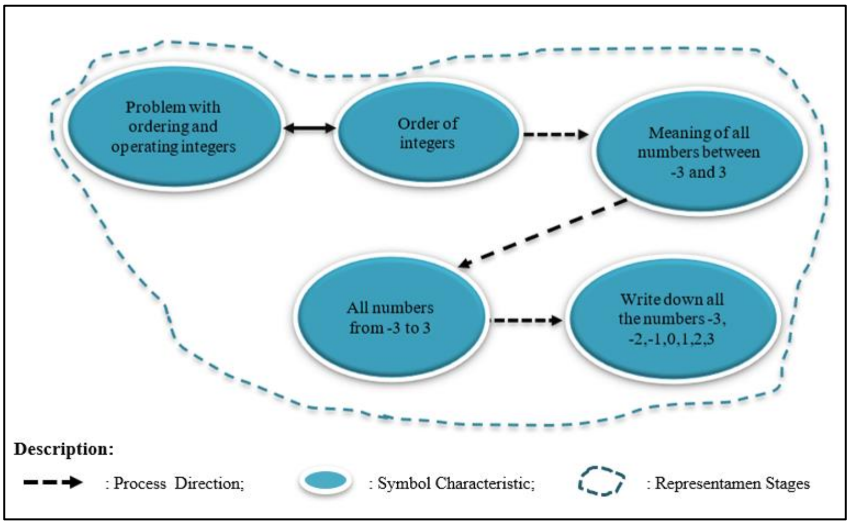

Figure 1. The representamen stage

3.2. Object Stage

In the object stage, the Subject pseudo-semiotic identifies the number sign (Initial Representamen) and connects the concept by thinking of the Number Line (Object). Furthermore, the subject draws (Representamen) the "number line" on the answer sheet and answers questions about the order of integers. In the pseudo-semiotic, the student draws numbers with zero as the limit (Interpretant). At this stage, the subject can interpret the representation of the number line correctly. Therefore, the pseudo-semiotic concludes that the number line is divided into three parts: positive, negative, and zero. As signs, the three parts of the number line, namely "Positive," "Negative," and "Zero," are identified by subjects as placements or positions on a number line. The pseudo-semiotic subject sorts the numbers correctly, and the object stage that occurs with the subject is under this description.

Researcher	:	*What is your stance on placing negative and positive numbers?*
Subject	:	*The positive numbers are on the right, while the negative numbers are on the left.*
Researcher	:	*Please show this number line, where are the positions of the positive and negative numbers*
Subject	:	*This is ma'am. (student shows the number line on the answer sheet). Therefore, to the left of zero is negative, and to the right of zero is positive.*
Researcher	:	*What does it mean less than or more than stated in the question?*
Subject	:	*It means ma'am; if less than that, it moves to the left, while more than it moves to the right.*
Researcher	:	*What do you mean by moving? Try to explain with pictures.*
Subject	:	*I use this, ma'am. See the line jumps on the number line below, and when less than -3, jump to the left, and more than -3, jump to the right.*

Researcher	:	*How is the jump conducted?*
Subject	:	*Jump according to the required number, ma'am. For example, the four numbers less than -4 are -5,-6,-7,-8. Jump one-on-one four times to the left.*
Researcher	:	*If the number is "between," in which direction does it jump?*
Subject	:	*Jump left or right (students think for a moment).*

The Subject Interpretan Stage in the sequence concept can be seen in the following scheme (see Figure 2).

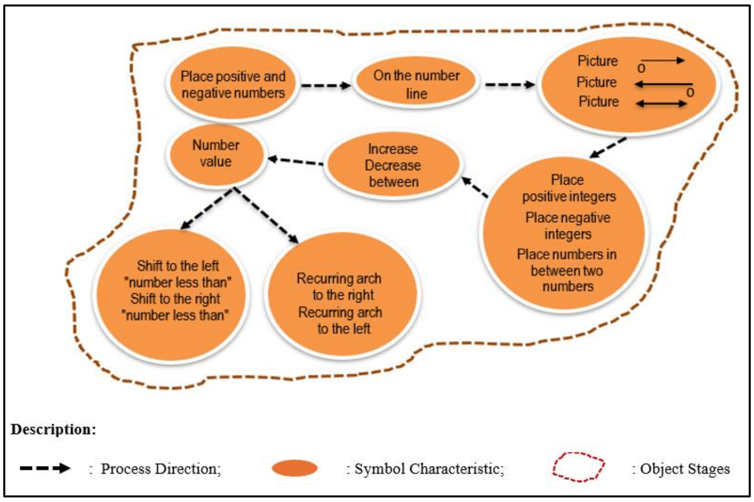

Figure 2. Object stages in symbolic-semiotic subjects

3.3. Interpretant Stage

Based on the previous initial representamen, the pseudo-semiotic subject identifies the repeated shift as addition and subtraction operations to create a representamen. According to students' perspective, for part a, determining four integers greater than -3 involves a gradual increase in numbers from -3 based on the integer sequences requested, which is six jumps. Similarly, in part b, determining five integers less than -2 involves gradually decreasing numbers from -2, according to the number of sequences requested. The final stage of the pseudo-semiotic is illustrated in the following scheme (see Figure 3).

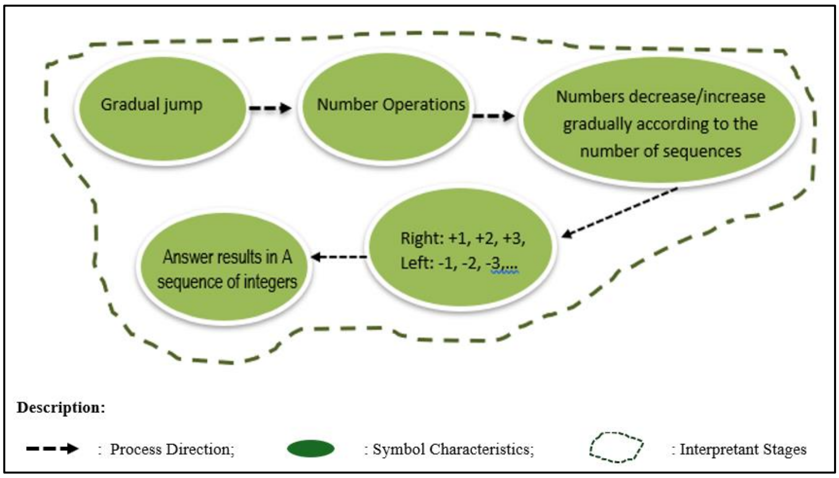

Figure 3. The interpretan stage

Subjects exhibiting pseudo-semiotics employ an instrumental understanding to construct the concept of integers. This entails forming meanings that do not adequately connect the components of semiosis, hindering the proper connection in the process. Even though the correct solution is obtained, the subjects need help to employ accurate concepts because their comprehension is confined to instrumental understanding. This understanding refers to the capability to precisely apply memorized rules to resolve tasks or problems without comprehending the rationale behind the rules (Skemp, 2012). It occurs when students do not have adequate relational understanding and cannot accommodate the concept accurately. In this case, students rely solely on tools or procedures that have been previously learned and need to understand the concept fully. In addition, students need to truly understand their meaning and context to memorize formulas or procedures. This can hinder the ability to understand the concept more deeply and apply the understanding in different contexts. Bofferding (2014) also identifies this type of subject as students with a mental model capable of understanding negative symbols correctly but needing a correct interpretation based on understanding whole numbers. Similarly, Yung and Paas (2015) also found that many students apply principles of operation less appropriate in their efforts to reach a final solution without relational and logical understanding. In pseudo-semiotic, subjects only memorize the "encounter" of signs without truly understanding the reasons behind the rule. In the case of the multiplication of signs, this rule transforms the operation into an additional form, making it easier for pseudo-semiotic subjects to apply the debt analogy. Furthermore, the subjects rely solely on quick learning techniques and methods focusing on short-term memory without truly understanding the underlying mathematical concepts. This sign multiplication aims to transform the operation into an additional form, making it easy for pseudo-semiotic subjects to apply the debt analogy.

Since the representamen of "encounter" is not an accurate understanding, students cannot make proper connections between symbols, objects, and interpretations (Palayukan et al., 2022). This is consistent with Bosse et al. (2016), where integer representations are manipulated. In line with previous studies, this subject provides an overview that students still have difficulties understanding and conceptualizing negative numbers (Cetin, 2019). The main problem students face is using the "+" and "-" symbols and the conflicts can lead to pseudo-constructions in the pseudo-semiotic stage of semiosis. Meanwhile, errors in representing mathematical symbols can lead to misinterpretations and misunderstandings in constructing an integer concept. Difficulties in comprehending and using these symbols also impede students' development of accurate comprehension. This can adversely affect the overall ability to comprehend mathematical concepts, potentially hampering the future academic achievements of students (Purwasih et al., 2023).

The signs and representations used to understand, teach, and learn mathematical concepts are often visual in mathematics. For example, pictures, diagrams, graphs, tables, or mathematical notation can be used as visual media that help visualize and present mathematical concepts. The use of visual media in mathematics learning has the advantage of providing clear images, assisting in understanding concepts, and facilitating more effective mathematical communication. Visual media allows students to see relationships between mathematical concepts, identify patterns, and make better generalizations. The use of semiotics as a theoretical framework allows mathematics education research to investigate the complex relationship between signs and understanding of mathematical concepts. The use of semiotics in the context of integers allows researchers to study the role of signs in creating a person's understanding of mathematics and to find effective learning strategies. Therefore, mathematics education studies that use semiotic integration can provide a deeper understanding of how the concept of Whole Numbers is understood, represented, and taught in mathematics education learning.

Mathematics is a scientific discipline that exhibits an abstract nature and characteristics. By using a special mathematical language, mathematical explanations and arguments can be built in a structured and systematic manner. Each mathematical concept forms an abstract framework that allows us to delve into a world of ideas that is not directly visible (Kadarisma et al., 2020; Kariadinata, 2021; Murtianto et al., 2019). In the process, the language of mathematics became a universal means of communication among mathematicians, making it possible to convey thoughts and findings clearly and without ambiguity. Thus, mathematics is not only about numbers and formulas but also produces a deep understanding of mathematical structures that are abstract and full of character, of course involving mathematical language in providing mathematical explanations and arguments (Ningsih et al., 2023; Setambah et al., 2021).

Students' ability to solve mathematical problems depends on understanding concepts and is greatly influenced by mathematical communication skills through test answers. In presenting answers on the answer sheet, students use mathematical representation as a representation. This representation includes students represent concepts, operations, variables, or ideas in the form of symbols, use notation, symbols, and mathematical language structures, and students create mathematical models of these concepts. In the context of mathematical semiotics, every mathematical representation is considered a sign or symbol that carries meaning. Therefore, semiotic analysis can reveal how students construct meaning through these signs in their answers. Appropriate notation, consistency, and creativity in representation can reflect students' understanding of mathematical concepts. Additionally, clarity and readability in delivering problem-solving steps are important in evaluating students' mathematical communication skills through student answer sheets (Ardina & Sa'dijah, 2016).

When students face difficulties expressing the results of problem-solving using their language, this may reflect their challenges in interpreting and deciphering the mathematical signs involved. The ability to convey mathematical thinking through one's language involves understanding the concepts in students' minds and their ability to translate these thoughts into a form that others can understand (Rachmawati et al., 2021). Students' ability to associate symbols with appropriate objects and interpret the meaning of the results of mathematical operations is related to their ethics in conveying mathematical thinking through their language; both reflect the process of inference and interpretation construction in mathematical semiotics.

4. CONCLUSION

In the process of semiosis or forming an understanding of integer concepts by students, three stages of semiosis should be considered: the representamen, the object, and the interpretant Stages. The representamen Stage is the initial level where students directly encounter mathematical representations, such as symbols and numbers. Students rely on instrumental understanding in comprehending mathematical concepts at this stage. Furthermore, in the object stage, students form connections and relationships between the mathematical representations encountered and their previous experiences and knowledge. Students assimilate new information and adjust the concept to their residual knowledge at this stage. The interpretant stage is where students have successfully built a proper understanding of integer concepts through semiosis. At this stage, connections and relationships between different mathematical concepts in real-life situations can be built. In each semiotic stage, students are subjected to a semiotic connection, a cognitive process to achieve understanding. Therefore, the process of forming an understanding of mathematical concepts does not only occur at the level of symbolic representation but also involves the overall understanding and experiences of the students.

In the case of pseudo-semiotic, building an understanding of the concept of integers involves using instrumental understanding. However, due to their mismatched meanings, this can lead to conflicts in the connections between the semiotic components, namely the Representamen, Object, and Interpretant. Subjects with pseudo-semiotics only have a shallow understanding of mathematical concepts. As a result, proper connections cannot be formed between mathematical symbols and their underlying meanings, which can hinder their ability to understand concepts in real life. Situations.

The findings of this research reveal that mathematical semiotics has a vital role in the process of solving integer problems. Semiotic analysis not only helps students overcome difficulties understanding mathematical notation or special symbols but also allows them to communicate mathematical ideas more clearly. Through the introduction of mathematical semiotic concepts, students are invited not only to understand but also to stimulate their critical thinking regarding the use of symbols in mathematics. These findings contribute to how understanding representations, mathematical objects, and interpretants influence students solving integer problems.

The limitation of this research is that the number of participants is relatively small, consisting of 3 selected participants. For further research, more participants need to be considered. Based on research limitations, the recommendation for further research is that the results of this research become a reference in developing modules or teaching materials that can develop students' learning processes so that students can solve algebra problems better.

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