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Case study in a grounded theory perspective: Students' reasoning abilities in Lithner's framework across self-regulated

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Abstract

Students' performance in understanding and developing algorithms from numerical methods is crucial because these concepts require them to engage in reasoning. DL-CA facilitates learning for fifth-semester bachelor students, specifically those in mathematics education, in comprehending numerical methods, algorithm concepts, and program development. Computer-assisted learning is appropriate for internalizing discovery learning and supporting autonomous study. The condition that differentiates individual learners is the level of self-regulated learning (SRL), which significantly enhances reasoning abilities by enabling goal setting, progress monitoring, and strategy adaptation, resulting in improved critical analysis and problem-solving skills. This grounded theory research investigates the reasoning outcomes of students in learning algorithm concepts for non-linear equation problems using computer-assisted discovery learning (DL-CA) through a web platform. It examines levels of self-regulated learning (SRL) and explores students' reasoning perspectives to describe differences in each level of SRL. Data analysis involved open coding through to categorization as essential steps in grounded theory, supported by method triangulation to enhance the validity and reliability of the findings. It was conducted using HyperRESEARCH output, a tool that aids in following comprehensive qualitative research procedures. The output suggests the following conjecture: The reasoning abilities of students in the high self-regulated learning (SRL) group encompass four categories: memorization, algorithm, plausibility, and mathematical foundation. In contrast, students in the medium and low SRL groups only demonstrate imitative reasoning abilities. Novel reasoning abilities are not sufficiently explained by these students, potentially due to limitations in the instruments or misunderstandings of the problems.

Keywords:
Computer assisted, Discovery learning, Grounded theory, Mathematical reasoning, Self-regulated learning

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1. INTRODUCTION

Mathematical reasoning is a core aspect of both mathematics and computer science education, especially in solving complex problems (Angraini et al., 2023; Desti et al., 2020;

Palinussa et al., 2021; Rohaeti et al., 2019; Sari & Hidayat, 2019). Strong reasoning abilities enable students not only to solve problems but also to understand the underlying theories that support their solutions (Hidayat et al., 2022; Maulida et al., 2024b; Schoenfeld, 2014). In this study, mathematical reasoning is essential for evaluating students' ability to develop algorithms to solve non-linear equations (Lithner, 2008). Mathematics students, in particular, are expected to have a deep understanding of mathematical theories and concepts, which contributes to their reasoning abilities (Aisyah et al., 2023; Hidayat et al., 2018; Nuraziza et al., 2022).

Students in mathematics education often encounter significant challenges when applying theoretical concepts in real-world program development, especially for solving non-linear problems. Many students struggle to transition from theory to practical application, This highlights a gap between their academic knowledge and the skills needed to address complex, real-world situations (Hattie & Timperley, 2007). Mathematics education students may find it difficult to solve non-routine, story-based problems that reflect everyday scenarios, particularly when these problems cannot be solved using purely analytical methods. Although they might understand theoretical solutions, many students lack familiarity with numerical methods and are often unsure of how to use computational tools, such as programming or algorithm development, to approach these problems numerically. Consequently, enhancing students' reasoning skills and equipping them to integrate numerical methods with computer-assisted tools are essential steps. This requires adopting effective, student-centered learning strategies to support their understanding and practical application of mathematics problem (Hattie & Timperley, 2007).

To enhance students' reasoning abilities, an appropriate learning approach is essential, as its structure supports the reasoning process by guiding students to explore, question, and make connections autonomously (Anazifa & Djukri, 2017). Students follow steps like problem identification, exploration, hypothesis formation, testing, and conclusion drawing, which effectively facilitate the development of their reasoning skills. Discovery Learning is a pedagogical approach that emphasizes active student engagement in this process, encouraging learners to discover concepts independently. However, one limitation of this approach is that students may experience confusion or difficulty when encountering complex concepts on their own (Hwang & Oh, 2021). This drawback can be minimized with computer assistance, allowing students to access immediate feedback, simulations, and guided resources to support their exploration. This method, known as Computer-Assisted Discovery Learning (DL-CA), combines the strengths of discovery-based learning with computational support, enhancing engagement and comprehension (Corazza et al., 2021). By integrating technology into the learning process, students are better equipped to navigate challenges and develop a more profound understanding of the material (Lee et al., 2016).

However, Computer-Assisted Discovery Learning (DL-CA) requires students to have a strong awareness of the importance of learning and the ability to study independently. Students must manage their own learning pace, make decisions, and persevere through challenges without constant instructor guidance (Azmi & Arfianti, 2021). This self-regulation skill is crucial, as it enables students to fully engage in the discovery process and take ownership of their learning journey, fostering deeper understanding and improved reasoning abilities (Sudirman et al., 2017). Since self-regulated learning (SRL) varies among individuals, it becomes a key factor that differentiates learning outcomes and reasoning skills (Yandari et al., 2018). Students with higher SRL are better equipped to set personal goals, monitor their progress, and adapt strategies, allowing them to overcome challenges effectively (Delima et al., 2024; Pertiwi et al., 2021; Van Gog et al., 2011). This adaptability not only helps them grasp concepts but also enables them to apply these concepts in various contexts, thereby enhancing their problem-solving abilities and promoting a capacity for independent, lifelong learning (Xiao et al., 2019).

In Computer-Assisted Discovery Learning (DL-CA), students require a high degree of independence to revisit and reinforce their understanding of complex material with limited guidance, ultimately fostering both competence and confidence in mathematical reasoning. This independence is essential for the development of students' reasoning abilities, as learners who take initiative in managing their own study processes are more effective in grasping algorithmic and programming concepts (Pape et al., 2002; Zimmerman & Schunk, 2013). Students who demonstrate strong self-management skills not only engage more deeply with the content but also develop critical self-regulated learning strategies that empower them to navigate their educational journeys. Those who show greater effectiveness in understanding and applying the concepts taught highlight the necessity of fostering independent learning strategies to enhance mathematical reasoning (Talib et al., 2019).

The interplay between reasoning and self-regulated learning requires approaches that develop reasoning abilities while encouraging students to take charge of their own learning. Discovery learning, particularly when supported by computer assistance and designed to facilitate SRL, is an effective method for enhancing students' reasoning abilities. By providing illustrations, images, and interactive media, Computer-Assisted Learning (CAL) helps students better understand and develop algorithms (Gibson, 2001). This approach not only supports self-regulated learning but also offers guidance through clear, student-centered instruction (Clark & Mayer, 2016). Building on these foundational concepts, it is important to explore the integration of self-regulated learning (SRL) strategies within Computer-Assisted Discovery Learning (DL-CA) to enhance mathematical reasoning among students. This study will investigate how DL-CA, supported by SRL, can create a more effective learning environment to improve mathematical reasoning. By developing a structured framework that incorporates tailored SRL strategies into the DL-CA context, students will have the opportunity to personalize their learning experiences while benefiting from the interactive features of computer-assisted tools. The study examine the impact of this integrated approach on various facets of mathematical reasoning, including the ability to memorize essential concepts, apply algorithmic thinking to solve problems, evaluate the plausibility of solutions, generate novel approaches to challenges and reinforcing mathematical foundations accros SRL, to describe enhancement of students' reasoning abilities.

2. METHOD

This study employs a qualitative research methodology to investigate students' reasoning outcomes in learning algorithm concepts and developing programs for non-linear equation problems. A qualitative approach was chosen as it allows for an in-depth and detailed examination of individual or small group experiences within a specific context, utilizing grounded theory methodology. The context of this study involves mathematics education students enrolled in the Numerical Algorithms and Programming course. A purposive selection of 33 participants was made, with 5 participants chosen from each level of: high, medium, and low. The research process began with the selection of research samples, implementation of the research treatment, and the identification of grounded study objects. Subsequently, data analysis was conducted in three stages: open coding, axial coding, and selective coding. The overall research steps are depicted in the flowchart illustrated in Figure 1 (Charmaz, 2014; Creswell, 2014; Glaser & Strauss, 2017).

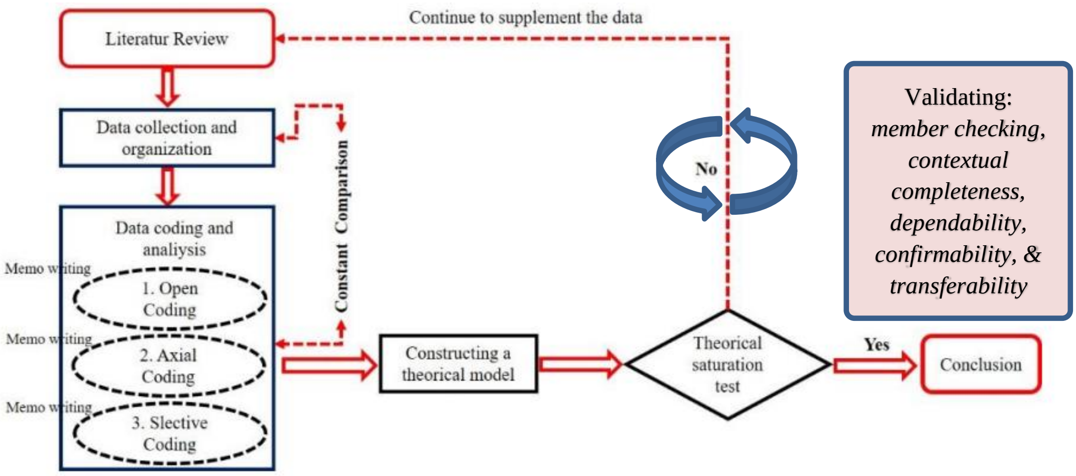


Figure 1. Grounded theory research procedure

In Figure 1, the grounded theory research process begins with a literature review to understand relevant theories, followed by data collection and organization. The data is then analyzed through three stages of coding: open coding breaks the data into initial concepts, axial coding connects these concepts into categories and subcategories, and selective coding identifies the core category that summarizes the emerging theory (Creswell, 2014; Glaser & Strauss, 2017). Once the theory is constructed, the researcher develops a conjunction of theory, which synthesizes the main concepts (Lukman et al., 2022; Sudirman et al., 2024). Grounded theory, originally developed by Glaser and further advanced by Charmaz, focuses not only on discovering "objective facts" but also on researchers' interpretations formed through their interactions with the data. Grounded theory, originally developed by Glaser and further advanced by Charmaz, focuses not only on discovering "objective facts" but also on researchers' interpretations formed through their interactions with the data. This approach makes the final stage essential for constructing a theoretical model and validating findings through methods such as member checking, contextual completeness, dependability, confirmability, and transferability, utilizing appropriate instruments, data collection, and analysis techniques (Charmaz, 2014).

2.1. Research Instruments

In the study, the primary instrument was developed by the researcher, supported by two types of written instruments: a Mathematical Reasoning Ability Test and a Self-

Regulated Learning (SRL) Questionnaire, to gather data.: a Mathematical Reasoning Ability Test and a Self-Regulated Learning (SRL) Questionnaire. The Mathematical Reasoning Ability Test aimed to assess students' reasoning skills in creating algorithms and solving non-linear equations, including three questions across five reasoning categories: memorization, algorithm, plausibility, novelty, and mathematical foundation. To ensure validity and reliability, the test was based on established mathematical reasoning frameworks and was reviewed for alignment with these categories. The SRL Questionnaire, adapted from a validated and reliable SRL scale, measured students' levels of self-regulated learning, with items carefully aligned to SRL constructs to confirm its appropriateness for the study. Both instruments served as the basis for interviews conducted to provide in-depth qualitative data, complementing the tests and questionnaire. These interviews were designed as open-ended questions to explore students' thought processes, allowing for detailed and focused responses. Emphasizing the validity and reliability of these instruments strengthens the study's findings, ensuring that the tools effectively capture both reasoning abilities and self-regulated learning skills.

2.2. Data Collection Techniques

The data were collected from three main sources: reasoning ability test scores, an SRL questionnaire, and interviews. The reasoning ability test was administered to 33 students, with scores collected both before and after the learning process to assess changes in their reasoning skills. The SRL questionnaire was administered to all students, and 15 students were selected based on their responses for further analysis to evaluate their self-regulated learning strategies. Based on the SRL scores, students were categorized into three groups: low SRL (53-105), medium SRL (106 -156), and high SRL (157-208). Five students from each SRL group were selected for interviews to gain deeper insights into their experiences and explore how their self-regulated learning strategies influenced their reasoning abilities. Additionally, interviews were conducted with students from each SRL group to provide a more comprehensive understanding of the relationship between self-regulated learning and reasoning skills.

2.3. Data Analysis Techniques

Qualitative analysis from a grounded theory perspective begins with the collection of written data in the form of reasoning ability tests and self-regulated learning (SRL) questionnaires, as well as interviews to explore the relationship between mathematical reasoning abilities and SRL. The collected data is then analyzed through three coding stages: open coding, axial coding, and selective coding (Creswell, 2014). In the open coding stage, the data is broken into smaller parts and labeled to identify initial patterns. The qualitative analysis software, HyperResearch, is used in the open coding stage to organize and analyze the information gathered from interviews and questionnaires, facilitating the researcher in systematically identifying and categorizing important concepts. Next, in the axial coding stage, relevant codes are grouped into larger categories, illustrating the relationships between various concepts (Charmaz, 2014; Creswell, 2014). The final stage, selective coding, identifies the core category that links these findings into a broader theory concerning the relationship between SRL and reasoning abilities (Glaser & Strauss, 2017). Once the theory is constructed, the theory construction stage is carried out to explain the relationship between SRL and reasoning abilities in the context of learning. The resulting theoretical conjecture is tested through several validation techniques: member checking to verify the alignment of findings with participants' perspectives, contextual completeness to ensure the theory encompasses all relevant contexts, dependability to check the stability of findings, confirmability to confirm the results originate from valid data, and transferability to ensure the findings can be applied to similar contexts (Creswell, 2014). After going through this validation process, the theory that is formed is accepted as a conjecture describing the relationship between mathematical reasoning abilities and self-regulated learning (SRL).

3. RESULTS AND DISCUSSION

3.1. Results

The reasoning test data for 33 students are grouped based on SRL classification as follows: students with high SRL (scores 157-208), medium SRL (scores 106-156), and low SRL (scores 55-105). Each group has a specific number of students and an average mathematical reasoning ability score describe in the following Table 1.

Table 1. Reasoning ability test scores based on SRL level

Learning	SRL Level	Frequency(N)	Mean	Std. Dev
DL-CA	High	16	77.38	6.29
	Medium	12	58.48	7.18
	Low	5	33.09	17.50
Total		33	63.80	15.66

The distribution, mean, and standard deviation of students at each level of self-regulated learning (SRL) have been presented in Table 1. In the high SRL group, there are 16 students with an average mathematical reasoning score of 77.38 and a standard deviation of 6.29, indicating that students with high SRL tend to have strong reasoning abilities, with scores that are relatively consistent within the group. In the medium SRL group, consisting of 12 students, the average reasoning score is 58.48 with a standard deviation of 7.18. This suggests that students with medium SRL have moderate reasoning abilities, slightly lower than the high SRL group, with a slightly greater variation in scores. Meanwhile, the low SRL group, consisting of 5 students, has the lowest average reasoning score at 33.09, with a standard deviation of 17.50, indicating the lowest reasoning ability and significant score variation within the group. Overall, with an average mathematical reasoning score of 63.80 and a standard deviation of 15.66 across all groups, these results indicate a difference in reasoning abilities based on SRL levels, with higher SRL associated with higher average reasoning scores. The analysis results show variations in students' mathematical reasoning abilities based on their level of self-regulated learning (SRL). The analysis results indicate that there is a direct relationship between reasoning ability and SRL level, suggesting a progressive increase in reasoning skills as SRL levels rise. This finding serves as an initial step toward a deeper examination of the relationship between SRL and reasoning abilities, allowing researchers to make conjectures on how SRL may impact mathematical reasoning.

Conjectures on Mathematical Reasoning Abilities at Each SRL Level Obtained Through Grounded Theory Analysis:

3.1.1. Open Coding

Open coding was performed on the written responses from the mathematical reasoning test and interviews by dividing the data into small segments and assigning codes to represent the meaning of each segment. Each question and sub-question reflects mathematical reasoning abilities based on framework of Lithner's (2008), which includes memorization reasoning, algorithmic reasoning, plausible reasoning, novelty reasoning, and mathematics foundation reasoning. Identification using HyperRESEARCH provided an overview of six mathematical reasoning abilities for each case, categorized by students' Self-Regulated Learning (SRL) scores (high, medium, and low). The identification resulted in codes, sub-categories, and categories at each SRL level as follows.

Open Coding for High-group of SRL

In the high SRL group, convergence from interviews with 5 respondents resulted in 153 codes, 36 sub-categories, and 15 categories reflecting mathematical reasoning abilities (see Table 2).

Table 2. Sub-categories and categories to define reasoning abilities in high SRL

Sub-Category	Category
Formula of quadratic solution, Formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$,	Remembering the Quadratic Function Root Formula
Function Factorization, Factorization Method, Guessing values $x_1 + x_2 = -3$ and $x_1x_2 = -5$, Perfect Square, Variable Squaring Method	Remembering Function Factorization
Table Method (Trial and Error), Table Method (Trying Different x Values)	Misconception of Analytical Calculation
Bisection Algorithm, Formulating Bisection Steps, Bisection Algorithm Iteration Process, Pseudo Code for Bisection Algorithm, Writing Pseudo Code in a Programming Language, Bisection Algorithm Flowchart, Simple Flowchart for Bisection Steps	Bisection Algorithm
Determining New Bounds in the Bisection Algorithm, Choosing Bounds [a:b] that Contain a Solution if $f(a)\cdot f(b)<0f(a)$, Determining New Bounds and Replacing Old Bounds, Dividing Bounds, Halving the Range, Range Division, Midpoint, Range Iteration	Determining Bounds
Iteration Table, Documenting Each Iteration, Record Iterations	Iteration Table
Stopping After the 4th Iteration, Ending Iteration, Result Approaching Solution, 4th Iteration, Continuing Iterations, Further Iterations, Result Improvement, Result Validation.	Stopping Iterations
Determining Tolerance, Tolerance Value, Iteration Tolerance, Tolerance Bound	Determining Tolerance
Choosing New Bounds, Evaluating Function Values, Bound Selection, Checking Bounds, Bounds Contain Solution, Testing Bounds, Evaluating $f(a)\cdot f(b)<0$, Checking Function Sign, Referring to Analytical Solution, Bounds Contain Solution, Result Matches Analytical Solution	Choosing Bounds Containing Solutions

Sub-Category	Category
Not Matching Analytical Solution, Result Discrepancy, Iteration Result vs Analytical Result, Analytical Error.	Not Matching Analytical Calculation
Containing Solution, Effective Bounds, Smaller Range More Efficient.	Bounds Efficiency
Choosing Bounds Based on Function Evaluation, Bounds Containing Solution, Iteration Tolerance.	Argumentation
Iterations Approaching 0, Range Not Too Large, Effective Iteration, Validating Results with Graphs, Result Consistency with Analytical Method.	Iteration Effectiveness
Adjusting Range Based on Solution, Misunderstanding Bounds, Error in Bound Selection, Misconception in Solution Determination, Using Large Range for Solution, Finding Only One Solution, Failing to Identify Other Solutions, Misunderstanding Instructions, Error in Solution Identification, Misconception in Bound Range.	Misconception
Converting Units from Liters to cm^3, Ensuring Unit Consistency.	Unit Conversion
Formulating Equations Based on Volume and Dimensions, Checking Equation Consistency, Simplifying Equations or Reducing Them to 0,	Formulating Equations
Identifying Equations as Non-Linear Functions with Exponents Greater than 1.	Understanding Non-Linear Equations

The open coding process for high-SRL students, as shown in Table 2, categorizes data into sub-categories and broader categories related to solving non-linear equations and the bisection algorithm. Sub-categories like "Formula of quadratic solution" and "Function Factorization" are grouped under categories such as Remembering the Quadratic Function Root Formula and Remembering Function Factorization. On the other hand, errors and misconceptions, as seen in "Table Method" and "Not Matching Analytical Solution," are categorized under Misconception of Analytical Calculation. Other sub-categories related to the bisection iteration process, such as "Determining New Bounds" and "Stopping After the 4th Iteration," are classified into categories like Determining Bounds and Stopping Iterations. Thus, open coding organizes the data into meaningful categories based on emerging patterns, facilitating axial coding.

Open Coding for Medium group of SRL

In the group of medium-level SRL students, interview convergence was obtained from 5 respondents, resulting in 102 codes, 29 sub-categories, and 8 categories that describe six mathematical reasoning abilities (see Table 3).

Table 3. Sub-categories and categories to define reasoning abilities in medium SRL

Sub-Category	Category
Analytical method, Quadratic function, Quadratic formula in obtaining the roots of a quadratic function, Difficulty in determining the values x1 + x2 = -3 and x1 * x2 = -5, Quadratic formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$, Simplified value, True solution, Irrational number	Remembering the Quadratic Function Root Formula
a = Xa b = Xb Xc = (Xa + Xb) / 2, The interval contains the solution if f(a) * f(b) < 0, c = (a + b) / 2, If f(a) * f(c) < 0 then b = c otherwise a = c,	Determining Bounds

Sub-Category	Category
Determining the initial bounds, Determining the midpoint of the interval, Selecting a new bound, Iteration table, Adjusting to analytical calculations, If Abs(f(c)) < e then the solution = c	Applying the Algorithm
Obtaining the midpoint, Approximating the true solution, Testing the bounds, The function value approaches 0, Satisfies the tolerance, Function value < Tolerance	Determining the Solution
The bound refers to the answer, Determining the bound referring to the analytical solution, The interval contains the solution,	Consistency with Analytical Results
Contains the solution, The function value of the bound is negative or positive, Testing the bound, Argument for selecting the interval, Determining the effective bound	Arguing
Error in testing the bound, Evaluating the accuracy of the bound, Testing the bound with a sketch	Misconception
Misunderstanding the problem	Misunderstanding

The open coding process, shown in Table 3, identified several categories and sub-categories related to solving non-linear equations and the bisection algorithm from medium-SRL students. Under Remembering the Quadratic Function Root Formula, sub-categories include the analytical method, the quadratic formula, and common difficulties in determining specific values. The Determining Bounds category encompasses concepts such as establishing initial bounds and conditions for the interval to contain solutions. The Applying the Algorithm category details steps like selecting new bounds and creating iteration tables. Within Determining the Solution, key aspects include obtaining midpoints and ensuring function values approach zero within tolerance limits. The Consistency with Analytical Results category emphasizes the importance of verifying that bounds refer to analytical solutions. The Arguing category involves justifying the selection of intervals based on function values. Additionally, misconceptions and misunderstandings, categorized under Misconception and Misunderstanding, highlight common errors in testing bounds and interpreting problems.

Open Coding for Low group of SRL

Open coding in a low-level student group, interview convergence was obtained from 5 respondents, resulting in 83 codes, 33 sub-categories, and 6 categories that describe six mathematical reasoning abilities (see Table 4).

Table 4. Sub-categories and categories to define reasoning abilities in low SRL

Sub-Category	Category
Quadratic Function Root Formula, Remembering the Use of the ABC Formula, Difficulty in Factorization, Factorization Failure	Remembering the Quadratic Function Root Formula
Initial Range Selection, Range Division, Boundary Testing, determining a new range, Determining Bounds Containing Solutions, Function Value of Bounds, Negative or Positive, Midpoint = (a+b)/2	Iteration Steps
Solution Approaching Zero, Solution Argumentation, Adjustment Based on Reference Adjustment of Stopping Criteria	Stopping Iterations

Sub-Category	Category
Boundary Selection, Proof that Bounds Contain Solution, Choosing Optimal Bounds Rationalizing Initial Bounds, Determining Effective Bounds, Boundary Testing	Argumentation
Errors in Understanding the Question, using analytical answers as reference, failure to choose bounds containing other solutions	Misconception
Errors in Formulating Mathematical Equations, Rectangular Prism Volume Formula, Surface Area Without Lid	Misunderstanding

Table 4 shows the open coding process from low-SRL student, identified several categories and sub-categories related to solving quadratic equations and addressing misconceptions. Under Remembering the Quadratic Function Root Formula, key sub-categories include the Quadratic Function Root Formula, challenges in factorization, and the Difficulty in Factorization. The Iteration Steps category includes elements such as Initial Range Selection, Range Division, and Boundary Testing, as well as determining new ranges based on the function value of bounds. Within the Stopping Iterations category, concepts like Solution Approaching Zero and adjustments based on reference criteria are highlighted. The Boundary Selection sub-category emphasizes proof that bounds contain a solution while choosing optimal bounds for accuracy. The Argumentation category focuses on rationalizing initial bounds and ensuring their effectiveness through boundary testing. Finally, misconceptions and misunderstandings, categorized under Misconception and Misunderstanding, involve errors in interpreting questions and formulating mathematical equations, such as the Rectangular Prism Volume Formula.

3.1.2. Axial Coding

In the axial coding stage, each category derived from the open coding process can describe the central phenomenon. Categories, as causal conditions in the context of axial coding, refer to variables considered as causes or main factors affecting or explaining the phenomenon under investigation. This is illustrated in the following axial coding flowchart (see Figure 2).

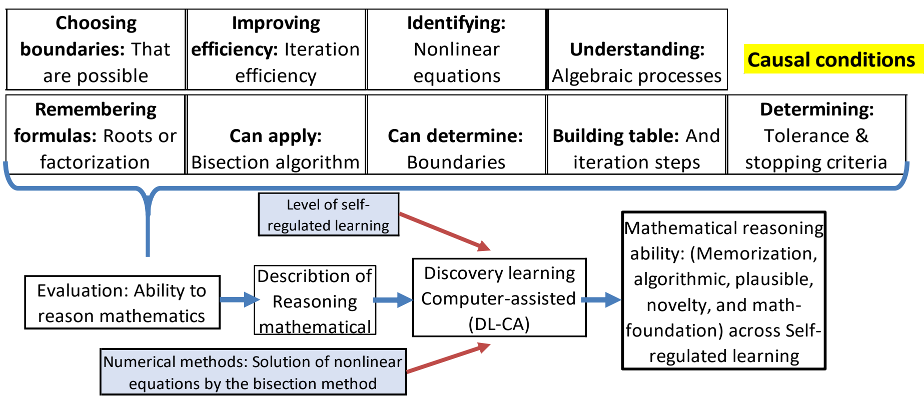


Figure 2. Grounded theory research procedure

In the diagram from Figure 2, the columns of categories as causal condition factors illustrate 1) mathematical reasoning abilities, through 2) computer-assisted discovery learning, supported by the internal factor 3) level of self-regulated learning, resulting in the 4) central phenomenon at each SRL level, which pertains to mastery of core reasoning abilities.

3.1.3. Selective Coding

A conceptual model illustrating key aspects of the mathematical reasoning process, particularly in solving non-linear equations, is shown in the following Figure 3.

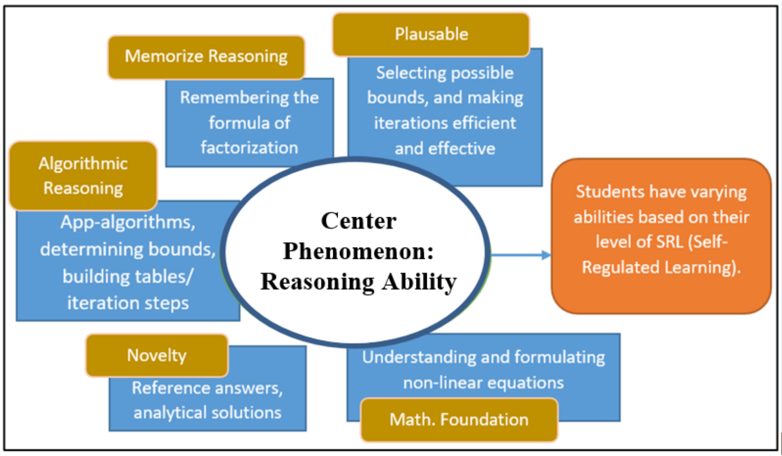


Figure 3. Selective coding

Figure 3 illustrates the central phenomenon derived from the categorization and grouping of mathematical reasoning abilities at each level of Self-Regulated Learning (SRL). This framework delineates how students' mathematical reasoning evolves as they progress through different stages of SRL. Each category reflects specific competencies, including recalling formulas, selecting appropriate methods, and effectively applying iterative processes. 1) Memorize Reasoning: This type of reasoning relates to the ability to recall formulas and procedures that have been learned, 2) Algorithmic Reasoning: This involves the ability to apply algorithms or structured steps to solve problems, 3) Plausible Reasoning: This refers to the ability to determine bounds that are likely to contain solutions, considering the effectiveness and efficiency of iterations, 4) Novelty Reasoning: This is the ability to provide alternative solutions or handle new or unfamiliar problems, 5) Math Foundation: A strong foundation in basic mathematics is crucial for understanding more complex concepts (Lithner, 2008).

3.1.4. Theorization

Theorization is the stage where the researcher formulates a theory based on the results from open coding, axial coding, and selective coding according to the hypothesis derived from the literature review, this study adopts the Lithner perspective. The hypothetical syllogism that builds the theory is as follows:

Hypothetical 1 : "If participants possess all five mathematical reasoning abilities, they are considered to have excellent mathematical reasoning skills."

Hypothetical 2 : "If participants possess four mathematical reasoning abilities, they are considered to have good mathematical reasoning skills."

Hypothetical 3 : "If participants possess three mathematical reasoning abilities, they are considered to have fairly good mathematical reasoning skills."

Hypothetical 4 : "If participants possess two mathematical reasoning abilities, they are considered to have poor mathematical reasoning skills."

Hypothetical 5 : "If participants possess only one mathematical reasoning ability, they are considered to have very poor mathematical reasoning skills."

Hypothetical 6 : "If participants do not possess any mathematical reasoning abilities, they are considered to have inadequate mathematical reasoning skills."

Hypothetical 7 : "If participants possess the abilities of memorize and algorithmic mathematical reasoning, they are considered to have imitative reasoning skills."

Hypothetical 8 : "If participants possess the abilities of plausible and novelty mathematical reasoning, they are considered to have creative reasoning skills."

Hypothetical 9 : "If participants possess the ability of mathematical foundation reasoning, they are considered to have reconstructive reasoning skills."

The conclusions drawn using HyperRESEARCH output, as a theory linking SRL levels and mathematical reasoning abilities, are as follows:

Testing your Theory on Case: High SRL
The following rules were found to apply to this case:

Rule 3 was applicable:
If Memorize Reasoning AND Algorithmic Reasoning AND Plausible Reasoning AND Mathematics Foundation Reasoning THEN GOAL REACHED Mathematical reasoning ability in the good category

Figure 4. Theory testing on students with a high SRL level

Figure 4 presents the output of the HyperRESEARCH analysis on reasoning abilities at the high-SRL: Students who received DL-CA (Discovery Learning with Computer Assistance) and have a high level of self-regulated learning exhibit good mathematical reasoning abilities because they have four indicators of mathematical reasoning: memorized reasoning, algorithmic reasoning, mathematical foundation, and plausible reasoning. Students with a high level of self-regulated learning who received DL-CA demonstrate imitative and constructive reasoning abilities, but not creative reasoning.

Testing your Theory on Case: Medium SRL
The following rules were found to apply to this case:

Rule 4 was applicable:
If Memorize Reasoning AND Algorithmic Reasoning AND Plausible Reasoning THEN GOAL REACHED Mathematical reasoning ability in the fairly good category

Figure 5. Theory testing on students with a medium SRL level

Figure 5 presents the output of the HyperRESEARCH analysis on reasoning abilities at the medium-SRL: Students who received DL-CA and have a medium level of self-regulated learning exhibit fairly good mathematical reasoning abilities because they have three indicators of mathematical reasoning: memorized reasoning, algorithmic reasoning, and plausible reasoning. Its level received DL-CA exhibit imitative reasoning abilities.

Testing your Theory on Case: Low SRL
The following rules were found to apply to this case:

Rule 4 was applicable:
If Memorize Reasoning AND Algorithmic Reasoning AND Plausible Reasoning THEN GOAL REACHED
Mathematical reasoning ability in the fairly good category

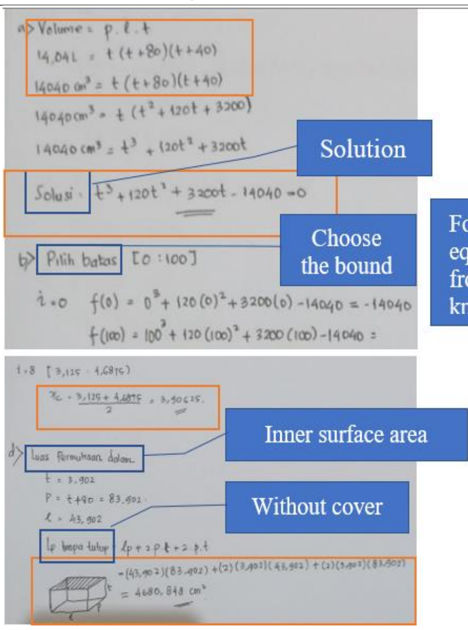
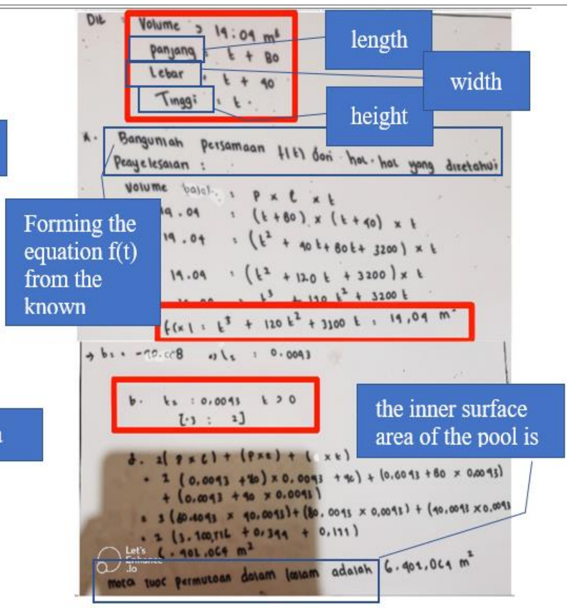
Figure 6. Theory testing on students with a low SRL level

Figure 6 presents the output of the HyperRESEARCH analysis on reasoning abilities at the low-SRL: Students who received DL-CA and have a low level of self-regulated learning exhibit fairly good mathematical reasoning abilities because they have three indicators of mathematical reasoning: memorized reasoning, algorithmic reasoning, and plausible reasoning. Its level received DL-CA exhibit imitative reasoning abilities.

3.1.5. Verification and Validation

The research findings are validated through assessments of credibility, confirmability, transferability, and dependability of the results (Creswell, 2014). A member check was conducted with 33 participants, which involves comparing the data obtained by the researcher (etic) with that provided by the data sources (emic) (Cohen et al., 2002; Krefting, 1991). After completing the data analysis, an overview of students' mathematical reasoning abilities influenced by learning and assessed from the perspective of self-regulated learning (SRL) levels was obtained. The researcher then returned to the participants to validate the findings. One example of the validation results is the difference in the depiction of mathematics foundation reasoning abilities across the three levels of SRL: 1) Research findings (Etic): Students with high SRL levels exhibit mathematics foundation reasoning abilities, while students with medium and low SRL levels are unable to demonstrate mathematics foundation reasoning abilities, 2) Data providers (Emic).

Table 5. Student answers across different levels of SRL

Aspect	High SRL Level	Low/Medium SRL Level
Writen Answer		

Aspect	High SRL Level	Low/Medium SRL Level
Interview Answer	P: Do you understand what needs to be done for question? T1: ***Formulate a nonlinear equation*** based on the given information in the problem. P: Explain how you arrived at that answer. T1: The ***swimming pool is rectangular***, with the volume formula being length×height× width. The volume is given as 14.04L, the length = t + 80 cm, width = t + 40 cm, and height = t cm. Thus, the equation becomes 14.04 = (t + 80)(t + 40)(t). P: Is the equation complete or correct? T1: Not yet, sir. You can see that the ***units are not the same***. When solving problems involving solid figures like this, the ***units must first be converted***. I converted the units to cm, so 14.04 L = 14.04 dm^3 = 14,040 cm^3, which changes the equation to 14,040 = (t + 80)(t + 40)(t). P: Is the equation now complete for use? T1: It still needs to be adjusted by expanding it and turning it into a zero-value equation: $(t)(t^2 + 120t + 3200) = 14,040\ t^3 + 120t^2 + 3200t = 14,040\ t^3 + 120t^2 + 3200t - 14,040 = 0$ P: Do you think it's necessary to change (t)(t + 80)(t + 40)? T1: It ***seems necessary***, sir, to ***simplify the calculation for each value of x***. P: Actually, it would be easier if you left it unchanged. Is this a nonlinear equation? T1: Yes, sir. It appears that when multiplied out, it forms a ***cubic function***.	P: Do you understand what needs to be done for question? S1: ***Formulate a nonlinear equation*** based on the basic formula for the volume of a rectangular prism, which is volume = length × width × height. P: Explain how you arrived at that answer. S1: We know that ***the swimming pool is shaped like a rectangular prism***, so the formula for the volume is volume = length × width × height. The volume is given as 14.04 liters, with length = t + 80 cm, width = t + 40 cm, and height = t cm. Thus, the equation is 14.04 = (t + 80)(t + 40)(t). P: Did you not consider the units? S1: ***No, I didn't, sir. I didn't pay attention*** to the different units and ***should have converted them first***. P: So, what do you mean by t1 and t2? S1: I calculated the values of the variable t using an analytical method, but since the volume value was not converted into cm^3, I didn't account for that in the calculation.
Analysis	T1 explains the process of formulating a nonlinear equation based on the problem data. They started by using the volume formula for a rectangular prism and converting the volume unit from liters to cubic centimeters to obtain the correct equation. Although the initial equation was 14.04 = (t + 80)(t + 40)(t), T1 recognized the need to convert the units and simplified the equation to 14,040 = (t + 80)(t + 40)(t). To facilitate calculations, T1 then transformed the equation into the polynomial form $t^3 + 120t^2 + 3200t – 14.040 = 0$. T1 ultimately confirmed that this equation is a cubic function, indicating that it is nonlinear.	Common errors include a lack of attention to units and insufficient understanding of basic mathematical concepts in the context of solid geometry. Students S1 and S2, who were used to identify reasoning abilities in mathematical foundations, showed similarities in formulating mathematical equations correctly but failed to consider the consistency of the units for the dimensions of the pool.

The analysis of responses (see Table 5), reveals clear differences in reasoning abilities between students with high and low/medium Self-Regulated Learning (SRL) levels. The interview responses highlighted as keywords in the open coding process and used to determine categories. Students at the high SRL level (T1) demonstrate a comprehensive understanding of problem-solving, accurately formulating a nonlinear equation and recognizing the critical importance of unit consistency. T1 effectively converts volume units and transforms equations into polynomial form, showcasing their strong grasp of mathematical concepts. In contrast, students at the low/medium SRL level (S1) exhibit a basic understanding of the volume formula but fail to pay attention to unit conversions, which leads to errors in their calculations. While S1 arrives at a similar equation, their lack of attention to unit consistency indicates gaps in their reasoning skills. Overall, high SRL students show a methodical approach to problem-solving and a deeper understanding of mathematical principles, whereas low to medium SRL students struggle with fundamental concepts and details. The results presented in Figures 3 and 4 further illustrate the disparities in mathematical reasoning abilities among students, aligning with the feedback gathered during the member check process for those at both high and medium SRL levels.

Contextual completeness refers to the researcher's use of literature studies to maintain contextual completeness through various references (books, journals, and scientific articles) to strengthen the validity of the information produced. The reference sources related to learning theories, discovery learning methods, and theories on self-regulated learning, align with several research findings as follows (see Table 6).

Table 6. Literatur to strengthen the validity of the information produced

No	Citation	Title	Conclusion
1	Rahayuningsih et al. (2021)	The effect of self-regulated learning on students' problem-solving abilities.	Students with high self-regulation and high problem-solving ability tend to demonstrate strong literacy skills, high metacognitive awareness, and proactive yet inflexible cognitive processes.
2	Öztürk and Sarikaya (2021)	The relationship between the mathematical reasoning skills and video game addiction of Turkish middle schools students: A serial mediator model.	Students with higher self-regulation skills are better able to manage the negative effects of video game addiction, which supports their reasoning abilities. Conversely, lower self-regulation skills can exacerbate the negative impact of video game addiction on reasoning.
3	Maulida et al. (2024a)	Differences in the influence of self-regulated learning levels on enhancing students' mathematical reasoning abilities.	The level of self-regulated learning significantly influences the enhancement of students' mathematical reasoning abilities. Variations in the level of self-regulated learning lead to differences in the improvement of these abilities.

Table 6 compares the findings from the current analysis with previous research, highlighting the positive impact of Self-Regulated Learning (SRL) on mathematical reasoning abilities. For instance, Rahayuningsih et al. (2021) established that students with high SRL skills tend to demonstrate strong literacy, high metacognitive awareness, and effective cognitive processes, which are related to better problem-solving abilities. Similarly, Öztürk and Sarikaya (2021) found that higher SRL skills help students manage the negative effects of video game addiction, supporting their reasoning abilities. Moreover, Maulida et al. (2024a) showed that the level of SRL significantly influences the enhancement of students' mathematical reasoning abilities, with variations in SRL levels leading to differences in these abilities. These studies collectively support the conclusion that developing SRL competencies can facilitate better mathematical understanding and reasoning, consistent with the disparities observed in reasoning abilities between high and low/medium SRL level students in the current research.

Dependability in this research is demonstrated through an external audit conducted by a validator who inspects all activities performed by the researcher, including themes, categories, sub-categories, and the hypothetical or substantive theories generated. Confirmability is shown by examining all findings related to themes, categories, sub-categories, and hypothetical conclusions obtained by the researcher by other experts beyond the validators and researchers. The validity of the findings and the results of the data analysis were verified by two independent mathematics education experts who reviewed the mathematical reasoning assessment instruments and self-regulated learning questionnaires, as well as other supporting documents.

Transferability refers to the effort to generalize the findings of this study and apply them to other situations and contexts. The researcher acknowledges that it cannot be definitively claimed that the findings are fully applicable to other contexts due to the research limitations related to the sample of mathematics education students from a state university in Langsa, using DL-CA learning for solving nonlinear equations with the bisection method. Nevertheless, by applying rigorous research procedures, the researcher hopes that the results of this study can provide valuable and relevant insights in different situations and contexts.

3.2. Discussion

Based on the research findings, a conjecture (substantive theory) was formulated linking the level of self-regulated learning (SRL) to mathematical reasoning abilities in DL-CA learning. This conjecture reveals the characteristics and classifications of mathematical reasoning abilities at each SRL level. Specifically, students in the DL-CA learning group with high levels of SRL demonstrate strong mathematical reasoning abilities, characterized by memorized reasoning, algorithmic reasoning, mathematical foundations, and plausible reasoning . Students in the DL-CA learning group with moderate and low levels of SRL exhibit reasonably good mathematical reasoning abilities, marked by memorized reasoning, algorithmic reasoning, and plausible reasoning. students exhibiting high SRL are better equipped to understand and apply mathematical concepts, as they actively engage in self-monitoring and reflection (Zimmerman & Schunk, 2013). Each aspect of reasoning at every SRL level differs in students' responses, reflecting their unique approaches to problem-solving based on their SRL abilities.

Students' responses, solving nonlinear equations using the bisection method, involve various aspects of mathematical reasoning, such as memorization, algorithmic reasoning, mathematical foundations, novelty, and plausible reasoning, which are interconnected and support the problem-solving process. Memorization is evident when students recall the main requirements of the bisection method, such as the function f(x) being continuous and having opposite signs at the endpoints of the interval [a,b]. This serves as the foundation for starting the process. Algorithmic reasoning is applied in the systematic steps of the bisection method, including dividing the interval into two parts, evaluating the function at the midpoint, and selecting a new subinterval based on the sign of the function until the desired tolerance is achieved. The structured approach inherent in the bisection method not only reinforces mathematical foundations but also encourages students to engage in deeper reasoning processes, thereby enhancing their overall problem-solving skills (Rahayuningsih et al., 2021).

Mathematical foundations strengthen this process by providing an understanding of why the bisection method works, such as through the properties of continuous functions and the intermediate value theorem. These foundational concepts are crucial as they allow students to grasp the underlying principles that govern the method's effectiveness (Navarro-López & Licéaga-Castro, 2010). In some student responses, novelty emerges when they attempt to modify the approach, for instance, by adjusting the tolerance level or iteration strategies to improve efficiency. Plausible reasoning complements the process by ensuring that the solution at each iteration remains within the valid interval and meets the error criteria. By integrating these aspects, students demonstrate a deep understanding of the bisection method and effectively solve nonlinear equations in a systematic and logical manner showcasing their ability to apply self-regulated learning strategies that enhance their academic performance (Dai et al., 2022).

Overall, at a group of students with a high level of SRL, the students' responses reveal a comprehensive structure in explaining mathematical techniques and algorithms through various in-depth and detailed categories. This includes not only basic formulas such as the quadratic root formula and factorization methods but also advanced techniques in bisection algorithms, including step-by-step procedures, iteration processes, and boundary evaluations (Jordan et al., 1999). Students with a high level of Self-Regulated Learning (SRL) demonstrate a comprehensive and methodical approach to mathematical problem-solving. They can explain basic concepts, such as the quadratic root formula and factorization methods, as well as more advanced techniques like the bisection algorithm. The bisection method, in particular, is noted for its robustness and simplicity, making it a preferred choice for solving nonlinear equations due to its guaranteed convergence properties (Dumas et al., 2015). Furthermore, students' ability to adapt and modify their approaches, such as adjusting tolerance levels in the bisection method, reflects a deeper understanding of mathematical principles and their applications.

Students with a low level of Self-Regulated Learning (SRL) tend to exhibit fragmented and less structured reasoning in mathematics. They often rely heavily on rote memorization without fully understanding the underlying concepts, which limits their ability to construct logical arguments or apply reasoning to new or complex situations. For instance, they may struggle to explain the rationale behind basic formulas, such as the quadratic root formula or factorization methods, and are unlikely to engage effectively with advanced reasoning processes like those required in the bisection algorithm. Their lack of strategic planning, reflection, and self-monitoring results in superficial reasoning that hampers their ability to develop deeper mathematical insights. The widespread attitude toward rote learning constitutes a significant obstacle, as many students expect to memorize and reproduce material passively, which stifles their engagement with the subject matter (Jordan et al., 1999). Furthermore, research indicates that students' dislike of mathematics often stems from ineffective instructional methods that fail to foster a deeper understanding of concepts, leading to a reliance on memorization rather than meaningful learning (Ukobizaba et al., 2021). This reliance on rote learning not only limits their mathematical reasoning but also inhibits their ability to adapt their knowledge to solve novel problems, as they lack the necessary conceptual framework to do so (Kollosche, 2021).

In contrast, students with a higher or lower level of SRL show significant limitations in their reasoning abilities. The difference is shown clearly in constructing mathematical models for real-world situations, such as calculating the surface area of a swimming pool. Students with high SRL approach this problem in a structured and comprehensive manner. They can identify key variables, such as length, width, depth, and additional features of the pool, and develop mathematical equations that accurately represent the real-world scenario, including combining formulas for flat and curved surfaces when necessary. Students with high self-regulated learning exhibit a strong understanding of the material, which enhances their ability to apply cognitive skills effectively in problem-solving situations (Miatun & Muntazhimah, 2018). Moreover, self-regulated learning has been shown to significantly impact learning outcomes, with studies revealing that students who engage in self-regulation strategies are more likely to achieve higher academic performance (Siregar et al., 2021). This ability to self-regulate not only fosters a deeper comprehension of mathematical concepts but also equips students with the skills necessary to adapt their knowledge to various contexts, thereby improving their overall problem-solving capabilities (Dai et al., 2022).

Some may be able to create basic equations, but these are often based on memorized reasoning—recalling formulas and steps from memory without fully understanding their application to the real-world context. Others frequently overlook crucial details, such as converting measurement units, leading to inaccuracies in their models. This reliance on memorized reasoning results in incomplete or incorrect models because they are not rooted in a deeper understanding of the mathematical principles. Additionally, their lack of reflective practices means that errors go unnoticed until the final stages. This contrast highlights that students with high SRL not only excel in constructing appropriate mathematical models by combining memorization with a foundational understanding but also ensure these models thoroughly reflect real-world conditions. In comparison, students with low SRL are more prone to errors due to their reliance on memorization without a solid conceptual foundation and the lack of strategy or evaluation in their reasoning process. the ability to engage in metacognitive reflection is crucial for identifying errors and refining reasoning processes, which ultimately enhances students' mathematical reasoning skills (Pape et al., 2002; Zimmerman, 2002; Zimmerman & Schunk, 2013).

The difference in reasoning ability in the aspects of algorithmic reasoning and plausible reasoning is also clearly evident between students with high and low levels of Self-Regulated Learning (SRL), especially in constructing iteration steps. Students with high SRL tend to build iteration steps systematically and structured, whether by using tables or careful per-iteration calculations. They pay close attention to each step, starting with determining boundaries using appropriate function value indicators for testing, and then proceeding to the next iteration while considering accurate results. In this process, they are also able to consider plausible outcomes based on their deep understanding of the concepts being applied. Students who engage in algorithmic reasoning are more adept at applying systematic approaches to problem-solving, which enhances their ability to construct accurate mathematical models (Zimmerman, 2002). Furthermore, the development of plausible reasoning skills allows students to evaluate potential outcomes critically, leading to more effective decision-making in mathematical contexts (Palengka et al., 2022).

Students' responses also highlight misconceptions and common errors that may arise during calculations and iterations, along with strategies for addressing these issues (Ancheta, 2022). In contrast, students with low SRL often determine boundaries arbitrarily or by guessing, without testing or verifying the values properly, leading to calculation errors at the end of the process. Although both groups may apply the same method, students with low SRL tend to fail to pay attention to crucial details that can affect the final result, causing errors even though the steps taken theoretically might be correct. Efficiency and effectiveness of iterations are also considered, demonstrating how results can be validated with graphs and consistency of analytical methods (Ladecký et al., 2019). Through subcategories such as unit conversions and equation formulation, a deep understanding of how to apply basic mathematical concepts in a broader context is demonstrated (Carnevale & Ahlfeld, 2019).

The theory found from this research regarding the relationship between mathematical reasoning and SRL has limitations in several aspects, such as the limited sample size, which only involved students from one university, and the focus on face-to-face learning, which may differ from online or hybrid learning. These limitations imply that future research should expand the sample by involving students from different universities or cities to obtain a more representative picture. Furthermore, future studies should consider the implementation of a hybrid learning model to examine its impact on mathematical reasoning abilities. Variations in learning settings, both online and face-to-face, should also be explored to assess their influence on students' self-regulated learning levels. The scope of the learning materials to test the effectiveness of this learning model on other mathematical topics. A more comprehensive measurement of other cognitive aspects, such as problem-solving abilities or creativity, would provide a more complete understanding. Affective factors, such as motivation and self-efficacy, should also be investigated to understand their impact on students' mathematical reasoning abilities. This study was also limited in its ability to explore the novelty reasoning aspect due to constraints in the instruments used to measure students' creative or innovative thinking. Additionally, the influence of affective factors on self-regulated learning should be further explored.

4. CONCLUSION

The research findings indicate that the relationship between mathematical reasoning and students' levels of Self-Regulated Learning (SRL) is closely intertwined, as described in Lithner's perspective. Students with high SRL levels demonstrate stronger and more structured mathematical reasoning abilities. They can effectively connect various aspects of mathematical reasoning, such as memorization, algorithmic reasoning, mathematical foundations, plausible reasoning, and novelty reasoning, in the problem-solving process. This aligns with Lithner's theory, which emphasizes the importance of reflection and self-regulation in the learning process to develop a deep understanding and accurate application of concepts. With high SRL, students are able to build a better conceptual understanding and avoid errors caused by reliance on memorization or algorithmic procedures that are not fully understood.

In contrast, students with low SRL levels tend to exhibit fragmented mathematical reasoning and rely heavily on memorization without a deep understanding of the underlying concepts. They often struggle to link algorithmic reasoning with more complex mathematical theory. This limits their ability to solve problems effectively, especially in tasks that require innovation or novel reasoning. From Lithner's perspective, the lack of self-regulation leads to an inability to reflect on and critically assess solutions, which in turn hinders the development of their mathematical reasoning abilities. Therefore, it is crucial for mathematics education to focus on developing better SRL skills in students to enable them to fully utilize their potential in mathematical reasoning.

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REFERENCES

Aisyah, N., Susanti, E., Meryansumayeka, M., Siswono, T. Y. E., & Maat, S. M. (2023). Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning. *Infinity Journal*, *12*(2), 377-392. https://doi.org/10.22460/infinity.v12i2.p377-392

Anazifa, R. D., & Djukri, D. (2017). Project- based learning and problem- based learning: Are they effective to improve student's thinking skills? *Jurnal Pendidikan IPA Indonesia*, *6*(2), 346-355. https://doi.org/10.15294/jpii.v6i2.11100

Ancheta, C. M. D. (2022). An error analysis of students' misconceptions and skill deficits in pre-calculus subjects. *Journal for Educators, Teachers and Trainers*, *13*(5), 283-295. https://doi.org/10.47750/jett.2022.13.05.026

Angraini, L. M., Larsari, V. N., Muhammad, I., & Kania, N. (2023). Generalizations and analogical reasoning of junior high school viewed from Bruner's learning theory. *Infinity Journal*, *12*(2), 291-306. https://doi.org/10.22460/infinity.v12i2.p291-306

Azmi, K. R., & Arfianti, K. (2021). Self regulated learning (SRL): Skills in improving learning motivation. *KONSELI : Jurnal Bimbingan dan Konseling*, *8*(2), 199-206. https://doi.org/10.24042/kons.v8i2.9958

Carnevale, M., & Ahlfeld, R. (2019). Mathematical formulation. In F. Montomoli (Ed.), *Uncertainty quantification in computational fluid dynamics and aircraft engines* (pp. 67-155). Springer International Publishing. https://doi.org/10.1007/978-3-319-92943-9_3

Charmaz, K. (2014). Grounded theory in global perspective: Reviews by international researchers. *Qualitative Inquiry*, *20*(9), 1074-1084. https://doi.org/10.1177/1077800414545235

Clark, R. C., & Mayer, R. E. (2016). *E-learning and the science of instruction: Proven guidelines for consumers and designers of multimedia learning*. John Wiley & Sons. https://doi.org/10.1002/9781119239086

Cohen, L., Manion, L., & Morrison, K. (2002). *Research methods in education*. Routledge. https://doi.org/10.4324/9780203224342

Corazza, G. R., Lenti, M. V., & Howdle, P. D. (2021). Diagnostic reasoning in internal medicine: a practical reappraisal. *Internal and emergency medicine*, *16*(2), 273-279. https://doi.org/10.1007/s11739-020-02580-0

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Sage.

Dai, W., Li, Z., & Jia, N. (2022). Self-regulated learning, online mathematics learning engagement, and perceived academic control among Chinese junior high school students during the COVID-19 pandemic: A latent profile analysis and mediation analysis. *Frontiers in psychology*, *13*. https://doi.org/10.3389/fpsyg.2022.1042843

Delima, N., Kusuma, D. A., & Paulus, E. (2024). The students' mathematics self-regulated learning and mathematics anxiety based on the use of chat GPT, music, study program, and academic achievement. *Infinity Journal*, *13*(2), 349-362. https://doi.org/10.22460/infinity.v13i2.p349-362

Desti, R. M., Pertiwi, C. M., Sumarmo, U., & Hidayat, W. (2020). Improving student's mathematical creative thinking and habits of mind using a problem-solving approach based on cognitive thinking stage. *Journal of Physics: Conference Series*, *1657*(1), 012042. https://doi.org/10.1088/1742-6596/1657/1/012042

Dumas, A., Dantan, J.-Y., Gayton, N., Bles, T., & Loebl, R. (2015). An iterative statistical tolerance analysis procedure to deal with linearized behavior models. *Journal of*

Zhejiang University-SCIENCE A, *16*(5), 353-360. https://doi.org/10.1631/jzus.A1400221

Gibson, I. W. (2001). At the intersection of technology and pedagogy: considering styles of learning and teaching. *Journal of Information Technology for Teacher Education*, *10*(1-2), 37-61. https://doi.org/10.1080/14759390100200102

Glaser, B., & Strauss, A. (2017). *The discovery of grounded theory: Strategies for qualitative research*. Routledge. https://doi.org/10.4324/9780203793206

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research*, *77*(1), 81-112. https://doi.org/10.3102/003465430298487

Hidayat, W., Rohaeti, E. E., Ginanjar, A., & Putri, R. I. I. (2022). An ePub learning module and students' mathematical reasoning ability: A development study. *Journal on Mathematics Education*, *13*(1), 103-118. https://doi.org/10.22342/jme.v13i1.pp103-118

Hidayat, W., Wahyudin, W., & Prabawanto, S. (2018). Improving students' creative mathematical reasoning ability students through adversity quotient and argument driven inquiry learning. *Journal of Physics: Conference Series*, *948*(1), 012005. https://doi.org/10.1088/1742-6596/948/1/012005

Hwang, Y., & Oh, J. (2021). The relationship between self-directed learning and problem-solving ability: The mediating role of academic self-efficacy and self-regulated learning among nursing students. *International Journal of Environmental Research and Public Health*, *18*(4), 1738. https://doi.org/10.3390/ijerph18041738

Jordan, D. W., Smith, P., & Spector, D. (1999). Mathematical techniques: An introduction for the engineering, physical, and mathematical sciences, 2nd ed. *American Journal of Physics*, *67*(2), 165-169. https://doi.org/10.1119/1.19219

Kollosche, D. (2021). Styles of reasoning for mathematics education. *Educational Studies in Mathematics*, *107*(3), 471-486. https://doi.org/10.1007/s10649-021-10046-z

Krefting, L. (1991). Rigor in qualitative research: The assessment of trustworthiness. *The American journal of occupational therapy*, *45*(3), 214-222. https://doi.org/10.5014/ajot.45.3.214

Ladecký, M., Pultarová, I., Zeman, J., & Vondřejc, J. (2019). Reference material preconditioning for FFT-based solvers. *PAMM*, *19*(1), e201900283. https://doi.org/10.1002/pamm.201900283

Lee, J., Lee, Y., Gong, S., Bae, J., & Choi, M. (2016). A meta-analysis of the effects of non-traditional teaching methods on the critical thinking abilities of nursing students. *BMC Medical Education*, *16*(1), 240. https://doi.org/10.1186/s12909-016-0761-7

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255-276. https://doi.org/10.1007/s10649-007-9104-2

Lukman, L., Wahyudin, W., Suryadi, D., Dasari, D., & Prabawanto, S. (2022). Studying student statistical literacy in statistics lectures on higher education using grounded theory approach. *Infinity Journal*, *11*(1), 163-176. https://doi.org/10.22460/infinity.v11i1.p163-176

Maulida, A. S., Wahyudin, W., Turmudi, T., & Nurlaelah, E. (2024a). Differences in the influence of self-regulated learning levels on enhancing students' mathematical reasoning abilities. *DWIJA CENDEKIA: Jurnal Riset Pedagogik*, *8*(2), 221-231. https://doi.org/10.20961/jdc.v8i2.89291

Maulida, A. S., Wahyudin, W., Turmudi, T., & Nurlaelah, E. (2024b). The effect of experiential learning and directed instructions assisted by augmented reality on students' self-regulated learning. *Infinity Journal*, *13*(2), 553-568. https://doi.org/10.22460/infinity.v13i2.p553-568

Miatun, A., & Muntazhimah, M. (2018). The effect of discovery learning and problem-based learning on middle school students' self-regulated learning. *Journal of Physics: Conference Series*, *948*(1), 012021. https://doi.org/10.1088/1742-6596/948/1/012021

Navarro-López, E. M., & Licéaga-Castro, E. (2010). Combining passivity and classical frequency-domain methods: An insight into decentralised control. *Applied Mathematics and Computation*, *215*(12), 4426-4438. https://doi.org/10.1016/j.amc.2010.01.012

Nuraziza, N. E., Susanto, S., Suwito, A., Trapsilasiwi, D., & Ambarwati, R. (2022). Analysis of student's mathematical reasoning in terms of learning independence during distance learning. *Journal of Education and Learning Mathematics Research (JELMaR)*, *3*(1), 22-32.

Öztürk, M., & Sarikaya, İ. (2021). The relationship between the mathematical reasoning skills and video game addiction of Turkish middle schools students: A serial mediator model. *Thinking Skills and Creativity*, *40*, 100843. https://doi.org/10.1016/j.tsc.2021.100843

Palengka, I., Juniati, D., & Abadi, A. (2022). Mathematical reasoning of prospective mathematics teachers in solving problems based on working memory capacity differences. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(12), em2193. https://doi.org/10.29333/ejmste/12670

Palinussa, A. L., Molle, J. S., & Gaspersz, M. (2021). Realistic mathematics education: Mathematical reasoning and communication skills in rural contexts. *International Journal of Evaluation and Research in Education*, *10*(2), 522-534. https://doi.org/10.11591/ijere.v10i2.20640

Pape, S. J., Zimmerman, B. J., & Pajares, F. (2002). This Issue. *Theory Into Practice*, *41*(2), 62-63. https://doi.org/10.1207/s15430421tip4102_1

Pertiwi, C. M., Rohaeti, E. E., & Hidayat, W. (2021). The students' mathematical problem-solving abilities, self-regulated learning, and VBA Microsoft Word in new normal: A development of teaching materials. *Infinity Journal*, *10*(1), 17-30. https://doi.org/10.22460/infinity.v10i1.p17-30

Rahayuningsih, S., Hasbi, M., Mulyati, M., & Nurhusain, M. (2021). The effect of self-regulated learning on students' problem-solving abilities. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(2), 927-939. https://doi.org/10.24127/ajpm.v10i2.3538

Rohaeti, E. E., Nurjaman, A., Sari, I. P., Bernard, M., & Hidayat, W. (2019). Developing didactic design in triangle and rectangular toward students mathematical creative thinking through Visual Basic for PowerPoint. *Journal of Physics: Conference Series*, *1157*(4), 042068. https://doi.org/10.1088/1742-6596/1157/4/042068

Sari, V. T. A., & Hidayat, W. (2019). The students' mathematical critical and creative thinking ability in double-loop problem solving learning. *Journal of Physics: Conference Series*, *1315*(1), 012024. https://doi.org/10.1088/1742-6596/1315/1/012024

Schoenfeld, A. H. (2014). *Mathematical problem solving*. Academic Press.

Siregar, H. M., Solfitri, T., & Siregar, S. N. (2021). The relationship between perceptions of online learning and self-regulation of mathematics education students. *Jurnal Didaktik Matematika*, *8*(2), 208-221. https://doi.org/10.24815/jdm.v8i2.21882

Sudirman, M., Fatimah, S., & Jupri, A. (2017). Improving problem solving skill and self regulated learning of senior high school students through scientific approach using quantum learning strategy. *International Journal of Science and Applied Science: Conference Series*, *2*(1), 249-255.

Sudirman, S., García-García, J., Rodríguez-Nieto, C. A., & Son, A. L. (2024). Exploring junior high school students' geometry self-efficacy in solving 3D geometry problems through 5E instructional model intervention: A grounded theory study. *Infinity Journal*, *13*(1), 215-232. https://doi.org/10.22460/infinity.v13i1.p215-232

Talib, C. A., Aliyu, F., Malik, A. M. b. A., & Siang, K. H. (2019). Enhancing students' reasoning skills in engineering and technology through game-based learning. *International Journal of Emerging Technologies in Learning*, *14*(24), 69-80. https://doi.org/10.3991/ijet.v14i24.12117

Ukobizaba, F., Ndihokubwayo, K., Mukuka, A., & Uwamahoro, J. (2021). From what makes students dislike mathematics towards its effective teaching practices. *Bolema - Mathematics Education Bulletin*, *35*(70), 1200-1216. https://doi.org/10.1590/1980-4415v35n70a30

Van Gog, T., Kester, L., & Paas, F. (2011). Effects of concurrent monitoring on cognitive load and performance as a function of task complexity. *Applied Cognitive Psychology*, *25*(4), 584-587. https://doi.org/10.1002/acp.1726

Xiao, S., Yao, K., & Wang, T. (2019). The relationships of self-regulated learning and academic achievement in university students. In *Forum on Psychological Health Education and Counselling for School Students (PHECSS2018)*, (Vol. 60, pp. 01003). https://doi.org/10.1051/shsconf/20196001003

Yandari, I. A. V., Nindiasari, H., Khaerunnisa, E., Pamungkas, A. S., Karso, K., & Nurjanah, N. (2018). Self-regulated learning in designing explorative learning tools among mathematics pre-service teachers through explorative module. In *Global Conference on Teaching, Assessment, and Learning in Education (GC-TALE 2017)*, (Vol. 42, pp. 00106). https://doi.org/10.1051/shsconf/20184200106

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice*, *41*(2), 64-70. https://doi.org/10.1207/s15430421tip4102_2

Zimmerman, B. J., & Schunk, D. H. (2013). *Self-regulated learning and academic achievement: Theoretical perspectives*. Routledge. https://doi.org/10.4324/9781410601032