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THE ROLE OF EQUILIBRATION IN THE FORMATION OF COGNITIVE STRUCTURES IN MATHEMATICS LEARNING

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ABSTRACT

The purpose of this study was to investigate the critical role of equilibration in organizing and building students' cognitive structure in the practice of learning mathematics. This qualitative research was designed using a phenomenological study approach. Participants in this study were 30 high school students of XI level. The main research instrument was the interview guide format and the document of cognitive test questions with the circle equation material. Research data obtained were tested for the validity of the data by triangulation. Method triangulation was done by comparing student work data and interview data. Theory triangulation is done by comparing the descriptions of the data obtained with the relevant theoretical sources. The results showed that in the context of knowledge construction (cognitive structure), there is a strong relationship between the process of assimilation and accommodation with equilibration. The role of balance in forming cognitive structures is to guide students to organize previous knowledge (schemas) and new knowledge and create thinking structures to become more complex by considering the socio-cultural dimension (environment) in mathematics learning. Further studies suggest that when selecting topics for equilibration-based activities, teachers should provide an effective learning environment to support the formation of cognitive structures to become more prosperous and complex.

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1. INTRODUCTION

Process skills and strategies in solving problems are very important for students to master in mathematics learning. Students are required to be able to find solutions in solving problems. This is because solving problems requires a new and different step or strategy compared to the steps or strategies in solving routine or usual problems. Students will carry out a thought process to find new solutions in problem-solving. In the process of thinking, there is processing between the incoming information and the schema (cognitive structure) that exists in the human brain (Aloqaili, 2012; Bormanaki & Khoshhal, 2017; Evtyugina et al., 2020). New experiences or information that come in will be processed with adaptation through the process of assimilation or accommodation (Dorko, 2019).

Learning mathematics is not just transferring knowledge from teachers to students. However, there are many things that must be understood by the teacher, including how students construct the knowledge that goes into their cognitive structure (Amineh & Asl, 2015). When the teacher has presented a very well-planned lesson, the Teacher wonders why some students "get it" and others don't. Then the Teacher tries to ask himself some tough questions: Are they (students) unmotivated? Are they unfocused, inattentive, lazy? Are they just "slow"? Meanwhile, there are students in the same class and there are those who excel. They thrive in well-prepared lessons, and they can learn from anyone, anytime by any method. They can do this because they know how to collect, process, and generate information. They have a well-developed cognitive structure (Garner, 2007).

Cognitive structures are the basic mental processes that a person (individual) uses to understand information (Garner, 2007). Cognitive structures are also called mental structures or thought patterns. Cognitive structures play a major role in students' information processing abilities as they serve as a frame of reference for understanding and working with several concepts. Teachers have an important role in developing students' mental representations through graphic representation content, diagram visualization, and symbolic thinking and abstraction. The greater the number of illustrations, the greater the mental representation, and the faster students' ability to process information develops (Navaneedhan & Kamalanabhan, 2017).

In problem-solving activities, students often experience uncertainty in determining whether the solution or reason stated/given is a right or wrong solution. Giving answers or reasons to a question is certainly related to the cognitive abilities of the individual. In situations of conflict that occur in connection with individual cognitive abilities, where the individual is unable to adapt his cognitive structure to the situation at hand in learning, it is said that there is a cognitive conflict within the individual (Maharani & Subanji, 2018). Cognitive conflict is a situation of cognitive imbalance caused by a person's awareness of information that is contrary to information stored in previous cognitive structures (Lee & Kwon, 2001). Cognitive conflict can also arise when there is a conflict of opinion or thought between an individual and other individuals in the individual's environment (Maharani & Subanji, 2018).

In fact, cognitive conflicts are formed and are related to the cognitive structure of the individual and their environment (Chang et al., 2009). There are several opinions of several experts who reveal how cognitive conflict is built: (1) cognitive imbalance, namely; an imbalance between a person's cognitive structure and the information that comes from their environment, in other words, there is an imbalance between internal structures and external inputs (Piaget, 1964); (2) cognitive imbalance or metacognitive conflict, namely: a conflict between schemata where there is a conflict between the old cognitive structure and the new cognitive structure (which is being studied or faced) (Chang et al., 2009; Kibalchenko & Eksakusto, 2020) and (3) Cognitive conflict, namely the conflict between new cognitive structures (involving newly studied material) and an environment that can be explained but the explanation refers to the initial cognitive structures possessed by individuals (Lee & Kwon, 2001). Some of the opinions put forward by experts can be concluded that cognitive conflict is a condition in which the new information it receives does not match the existing cognitive structure. Piaget's term cognitive conflict is called disequilibrium (Aloqaili, 2012; Bormanaki & Khoshhal, 2017; Piaget, 1964).

Piaget said that a cognitive structure is a well-organized knowledge structure in the brain that always integrates with the environment through a process of adaptation, namely assimilation and accommodation (Piaget, 1964). If assimilation and accommodation occur in a conflict-free manner or there is compatibility with the environment, the cognitive structure is said to be in equilibrium with its environment.

However, if this does not occur, it means that in the process there is a cognitive conflict so that a person is in a state of imbalance (disequilibrium). When a person experiences a disequilibrium, he will respond to this situation and seek a new balance (equilibrium) with his environment (Bormanaki & Khoshhal, 2017; Piaget, 1964; Rutherford, 2011; Simatwa, 2010). For more details, how the performance of equilibration in the formation of thinking structures through adaptation is described in Figure 1.

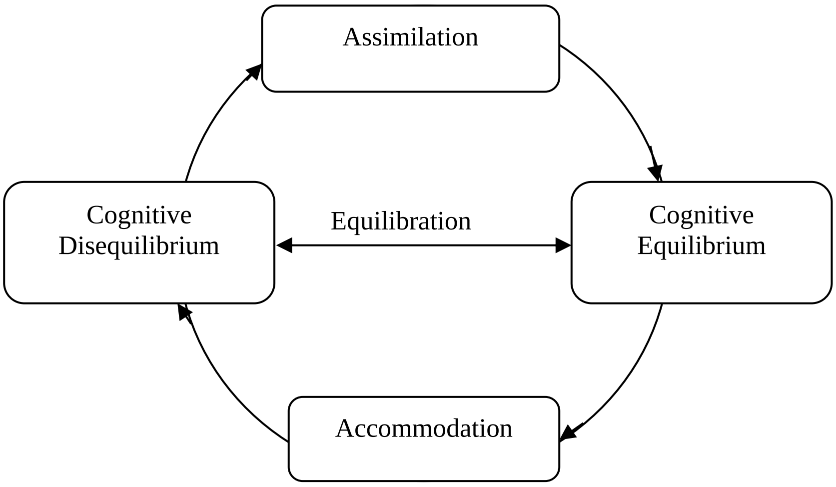

Figure 1. Cycles of adaptation and equilibrium (Bormanaki & Khoshhal, 2017)

Figure 1 shows that the cognitive development process is a cycle of adaptation and equilibrium according to Piaget (Bormanaki & Khoshhal, 2017). Figure 1 illustrates a cycle diagram of the adaptation and equilibration process, to illustrate the process of cognitive development. When a person learns or receives a new stimulus, a disequilibration will occur which leads to a process of adaptation (assimilation and accommodation). Through this process, the scheme will develop through the process of combining, changing or forming a new scheme until equilibrium occurs. In the adaptation and equilibration cycle, new experiences are assimilated for the first time into the existing schema. If it is not in accordance with the existing scheme, the result will be a cognitive imbalance (cognitive disequilibrium). Then, accommodating (adjusting) the scheme brings into a state of cognitive equilibrium until a new scheme challenges again (assimilation). This process will continue when a person learns or receives a new stimulus so that one's thinking process will be increasingly complex (mature) (Bormanaki & Khoshhal, 2017).

In the math forum at NCTM 2018 it is emphasized that in mathematics learning, students build their own understanding of mathematical concepts. Teachers do not lecture, or explain much, or try to "transfer" mathematical knowledge, but to create situations that will help them build the necessary understanding. Thus, students must be actively able to construct mathematical knowledge, build mathematical connection processes and develop habits of thinking about solving mathematical problems (Steffe, 2002; Wang, 2011). Meanwhile, the role of teachers in learning is as resource persons and facilitators who provide assistance as needed (scaffolding) to facilitate the knowledge construction process developed by students (Chang et al., 2009; Wang, 2011).

The purpose of this study is to investigate the importance of the role of equilibration in organizing and building the structure of students' thinking in mathematics learning practice. This study focuses on students' thought processes when building a thinking structure by empowering a balance in the accommodation process (assimilation and accommodation).

Through the results of this study, it is hoped that educators can provide an effective learning environment to support the formation of students' cognitive structures to become richer and more complex an investigation was conducted to answer the researcher's curiosity about the question: a) What is the role of equilibrium in shaping students' thinking structures when solving math problems? b) How is the role of the teacher so that students can build their thinking structure in a more complex direction?

2. METHOD

2.1. Research Design

This study uses a qualitative method with a phenomenological approach to explore data and find the meaning of facts and experiences experienced by objects that are the focus of research (Khan, 2014). Researchers conducted many interviews to build a sufficient dataset to find symptoms that emerged from the object under study (Creswell & Creswell, 2017). The phenomenon under study focuses on the cognitive structure of the object (students) when solving the circle equation problem.

2.2. Participants

Participants in this study were 30 students at level XI who took mathematics classes. After they have completed the test, a minimum of three students will be selected as representatives to be interviewed. The research was conducted at a high school, Semarang City, Indonesia.

2.3. Instruments

The main instruments in this study were the interview instruction format and the cognitive test question document with the circular equation material. The interview instrument contains a list of unstructured interview questions. Interview questions can be developed according to need. Cognitive test instrument consists of 1 item description of the equation of the circle which is presented below.

Problem : *A circle centered on the intersection of two lines g1 and g2. The equation for the line g1 is $x + 2y + 6 = 0$ and the equation for the line g2 is $x - 5y + 13 = 0$.*
The circle is through a line equation $2x - 3y + 6 = 0$.

Question : *Determine the general form of the circular equation.*

2.4. Procedure

Mathematics learning is carried out for 4 learning meetings. At the end of the meeting, students completed a written test with one of the things being tested was the equation of a circle problem. The results of student work are analyzed and grouped into three categories of answer quality, namely: correct and perfect answers, correct and imperfect answers, and wrong answers. Then each group is sampled to be interviewed to confirm its work.

Determination of sample data sources for interviews was carried out purposively and snowballing, namely, the data sources chosen were the people who knew best about what was being asked, and if they did not have the desired information, the number of data sources could be increased or increased. overtime (Miles & Huberman, 2016).

The results of the interviews were analyzed as a basis for describing the cognitive structure of students in solving circular equations. The research data obtained were tested for validity by triangulation. Method triangulation is done by comparing student work data and interview data. Theory triangulation is done by comparing the description of the data obtained with the relevant theoretical sources. to find out whether the findings are consistent or not with the findings of the previous theory.

2.5. Data Analysis, and Validation

Data analysis is done by reviewing texts or documents and in-depth interviews. The results of the interviews were analyzed as a basis for describing the cognitive structures of students when solving circular equation problems. Research data obtained were tested for the validity of the data by triangulation. Method triangulation was done by comparing student work data and interview data. Theory triangulation is done by comparing the description of the data obtained with the relevant theoretical sources. to find out whether the findings are consistent or not with the findings of the previous theory.

3. RESULT AND DISCUSSION

3.1. Result

Test result data from 30 respondents that were collected were grouped according to the quality of the respondent's answers, namely correct (perfect) answers, correct but imperfect answers, and wrong answers. The following shows the results of grouping the quality of respondents' answers in Table 1.

Table 1. Quality of respondents' answers

Number of respondents	Quality of Respondents' Answers		
	True and perfect answers	True and imperfect answers	Wrong answers
N = 30	12	10	8
Percentage	40%	33%	27%

Based on the results of students' responses to the problems posed by the teacher (see Table 1), three students were selected as representatives to be interviewed. The three students are students (S-R1) with correct (perfect) answers, students (S-M1) with correct but imperfect answers, and students (S-F1) with wrong answers. The purpose of the interview was to find out the students' thought processes in solving problems through the process of assimilation and accommodation.

To find out the students' thought processes in solving problems, the researcher analyzed the text (documents) of students' responses and conducted a more in-depth interview with the subject (S-R1; S-M1; S-F1). So that later the subject's thought process can be explained when solving math problem. The following is the result of the subject's answer (S-F1) to the circle equation problem proposed in the assignment.

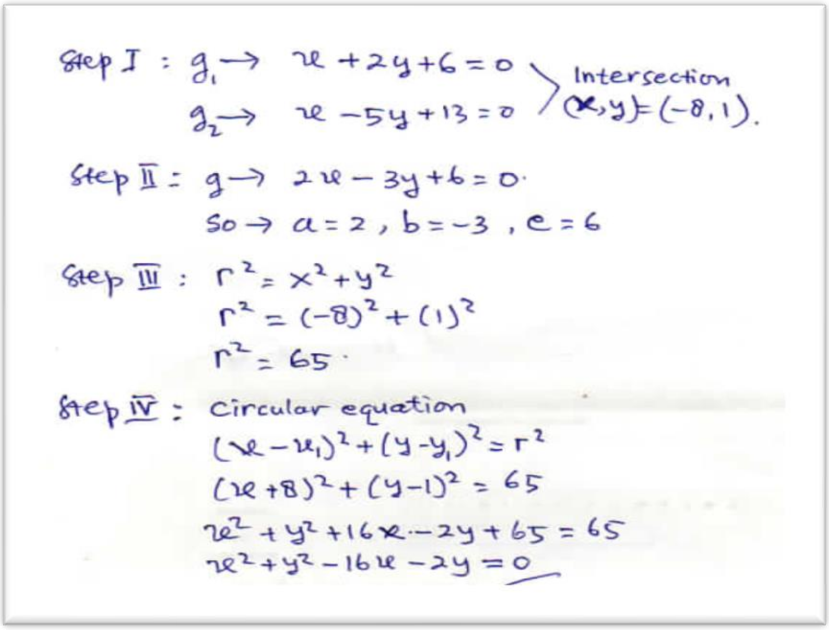

Figure 2. The answer from the subject (S-F1)

The answer response given by the subject (S-F1) in Figure 2 shows that solving the circle equation problem is the wrong answer. For this reason, the researcher (R) confirms through interviews with the subject (S-F1) as a basis for carrying out a theoretical discussion.

R	:	*What are the ideas for solving this problem?*
S-F1	:	*At first, I thought of finding the center of the circle by finding the intersection of the two lines g1 and g2.*
R	:	*Are you sure of the benefits of step two?*
S-F1	:	*I realize that this second step does not support the solution*
R	:	*Are you sure of the action in step three?*
S-F1	:	*I am confused thinking about it, maybe I took the wrong step to determine the radius of the circle*
R	:	*Is the solution to the circular equation already?*
S-F1	:	*I have to double-check.*
R	:	*Did you understand that step three only applies to the circle centered on the point (0,0)?*
S-F1	:	*I understand there have been errors in several places and I will fix these errors.*

Based on the results of the interview, the subject (S-F1) was given the opportunity to improve his work. Not long after, the subject (S-F1) succeeded in making improvements as presented in the following interview.

R	:	*At which step do you begin to reflect?*
S-F1	:	*I calculated the radius of the circle* $r = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert = \frac{-13}{\sqrt{13}}$
R	:	*How to solve the circular equation?*

S-F1	:	*I will show you the equation of the circle*
		$(x + x_1)^2 + (y - y_1)^2 = r^2 \Leftrightarrow (x + 8)^2 + (y - 1)^2 = 13$
R	:	*Is this the final solution?*
S-F1	:	*I still have one more step to complete.*
		$x^2 + y^2 + 16x - 2y + 65 = 13 \Leftrightarrow x^2 + y^2 + 16x - 2y + 52 = 0$

At the beginning of the problem-solving activity, the subject (S-F1) was still confused thinking about possible strategies or steps to be carried out, as shown in the interview excerpt. The mental condition of this confusion in the subject (S.F1) is called equilibration, which is the process of adjusting balance from disequilibrium to equilibrium for the purpose of increasing one's thinking and knowledge to a more complex or mature stage (Bormanaki & Khoshhal, 2017; Zhiqing, 2015). Because this problem is considered very complex, the subject (S.F1) is guided by the researcher to complete this task. Vygotsky's theory states that educators (teachers) should help students engage in complex or higher-level thinking through structured assistance (Liu & Matthews, 2005).

In mentoring activities with the help of sufficient scaffolding, the subject (S-F1) realizes that there are an error in determining the radius of the circle $r^2 = x^2 + y^2$ (see Figure 2). This error is because in the subject's thinking process imperfect assimilation occurs so that when it is continued into the accommodation process it also experiences a cognitive disequilibrium. Then after the subject reflects with the help of scaffolding, the subject can replace it with the appropriate formula. At the final completion stage of the subject (S-F1) succeeded in compiling the circle equation $x^2 + y^2 + 16x - 2y + 52 = 0$ as the correct solution. This means that the cognitive structure of the subject has reached cognitive equilibrium (Bormanaki & Khoshhal, 2017; Zhiqing, 2015). The results of the response of the subject's answer (S-M1) to the circle equation problems proposed in the assignment.

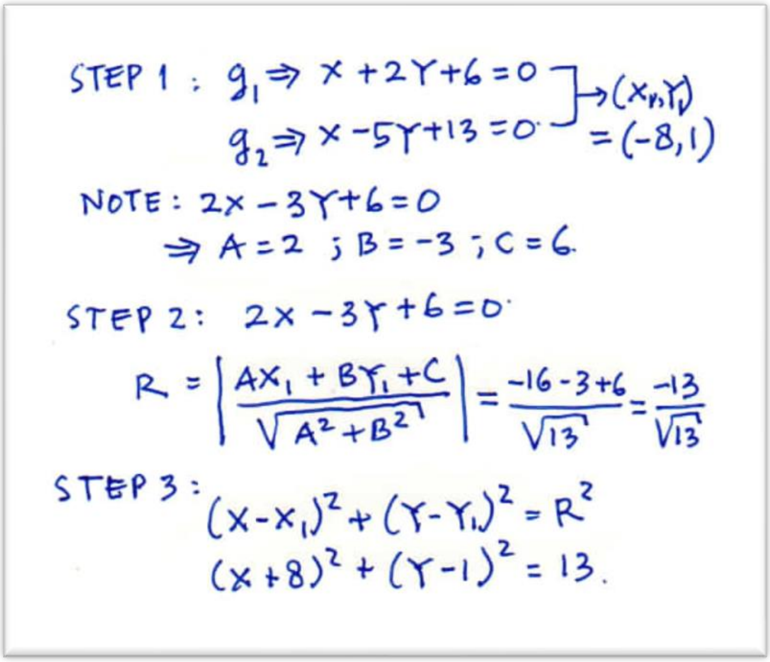

Figure 3. The answer from the subject (S-M1)

The following is an excerpt from the researcher (R) interview with the subject (S-M1) to confirm the process of solving the circle equation problem (Figure 3) as a basis for theoretical discussion.

R	:	*What knowledge did you use to solve this problem?*
S-M1	:	*I started by determining the center of the circle through the intersection of two lines; determine the radius of the circle, and construct the equations of the circle.*
R	:	$(x + 8)^2 + (y - 1)^2 = 13$ *Is this a solution to the circular equation?*
S-M1	:	*It seems I am not sure and interested in finding a better solution to the circular equation.*
R	:	*Please reflect so that you can find the right answer.*
S-M1	:	*Here is the solution to the circular equation in question.* $x^2 + y^2 + 16x - 2y + 65 = 13 \Leftrightarrow x^2 + y^2 + 16x - 2y + 52 = 0$
R	:	*Where did you get this solution idea?*
S-M1	:	*I feel that I have enough knowledge to better construct the equations of the circle.*

In the confirmation of the interview with the subject (S-M1), it turned out that the subject was not sure of the answers given. When an individual (subject S-M1) thinks about solving a problem, the adaptation process (assimilation and accommodation) will take place during the equilibration process until a condition of conformity (cognitive equilibrium) occurs. When someone has obtained a solution but is not satisfied, unsure, or unsure of the solution, there is still a cognitive disequilibrium (Bormanaki & Khoshhal, 2017; Dorko, 2019). Conversely, when someone is satisfied with the answer, the person's thought process has reached a state of equilibrium (Bormanaki & Khoshhal, 2017). This condition encourages the subject (S-M1) to reflect (check again) on the answers they get. When the reflection takes place the subject interacts with the environment through scaffolding given by peers. Finally, the subject succeeded in finding a solution to the circle equation. This means that after the subject interacts with the environment (peers), equilibration has guided the subject's cognitive processes to improve their cognitive structure.

In terms of cognitive development, when they (S-F1 and S-M1 subjects) can find themselves the intersection of the two lines as the center of the circle, thus they can be said to have been at the level of CPD (construction of proximal development) (Kusmaryono et al., 2021), and when they receive assistance in the form of scaffolding in a social environment (from teachers or peers) the subject can find the radius of the circle and the solution of the circular equation, then the subject has reached the level of potential development (Vygotsky, 1987). However, if it is understood more deeply, it is possible that even though in the end the subjects (S-F1 and S-M1) arrive at the cognitive equilibrium stage, but before getting scaffolding they cannot accommodate the scheme. Theoretically, this can lead to disjunctive generalizations (Dorko, 2019).

The results of the response of the subject's answer (S-R1) to the circle equation problems proposed in the assignment.

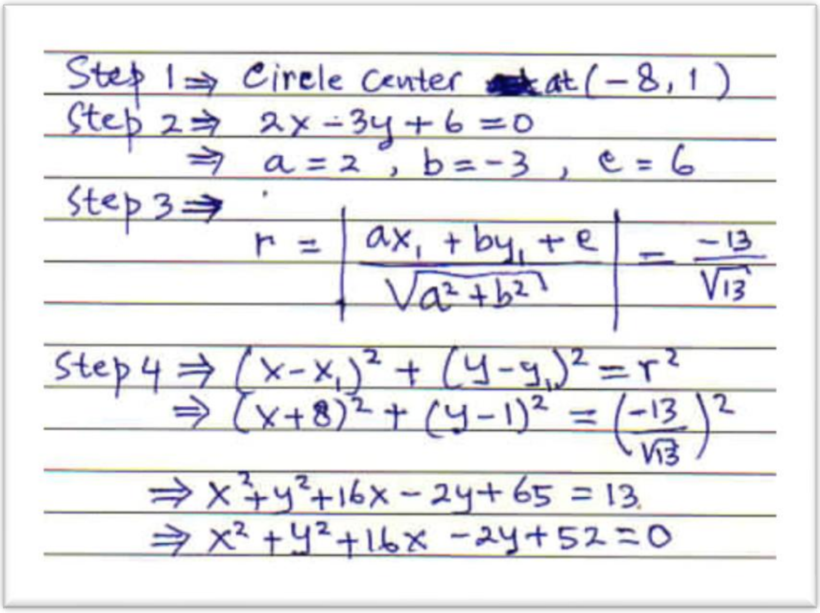

Figure 4. The answer from the subject (S-R1)

Snippets of the researcher (R) interview to the subject (S-R1) to confirm the process of solving the circle equation problem (see Figure 4) as a basis for theoretical discussion are presented as follows.

R	:	*Are you sure this answer is correct?*
S-R1	:	*I really believe it is*
R	:	*Have you reflected?*
S-R1	:	*I have done my reflection by checking the completion steps*
R	:	*Try to describe the checking steps that you did.*
S-R1	:	*Step 1: determine the intersection of the lines g1 and g2 as the center of the circle* *Step 2: determine the values of the coefficients x, y, and constants or (a, b, c).* *Step 3: determine the radius of the circle* *Step 4: draw up the equations of the circle*
R	:	*What do you think about the radius of this circle* $r^2 = x^2 + y^2$
S-F1	:	*The radius of the circle* $r^2 = x^2 + y^2$ *only applies to the equation of the circle centered on the point (0,0) and does not apply to the circle centered at the point (a, b)*

In the interview with the subject (S-R1), the teacher deliberately creates a social environment in the form of cognitive conflict, which aims to build the subject's thinking structure (S-R1) to be more complex. The forms of cognitive conflict created by the teacher can be in the form of denial, proof, reasoning, search questions, and others. Overall the subject (S-R1) can assimilate and accommodate information from outside (socio-cultural environment) that comes in contextually so as to allow cognitive balance. So this condition has facilitated the cognitive equilibrium process, avoiding cognitive disequilibrium. Smoothness in the process of assimilation and accommodation allows the subject (S-R1) to carry out the organization of knowledge as a whole by rearranging the internal schemes that they already have. Subjects feel challenged to answer the problems posed. The subject can explain the steps for completion precisely. Subjects can provide reasons for conclusions with logical reasoning. What happens to the subject (S-R1) is in line with the modern view of Rutherford (2011) which has added a new dimension to the socio-cultural realm (including environmental factors) of the accommodation and assimilation processes needed in the equilibration process for one's cognitive development.

According to Piaget's view, children (individuals) also change their schema according to the organization as has been done by the subject (S-R1). The organization is a person's tendency to regulate mental processes (thinking) by rearranging internal schemas and exploring the relationships and associations between schemas (Piaget, 1964; Zhiqing, 2015). Organizational processes aim at developing interconnected cognitive systems so that they become more effective. Piaget calls organization a high-level cognitive system (Aloqaili, 2012; Bormanaki & Khoshhal, 2017; Joubish & Khurram, 2011; Zhiqing, 2015).

To end cognitive conflict, it is necessary to have scaffolding both from the teacher and from peers who do not experience cognitive conflict, in addition to scaffolding the role of metacognition can also help to end cognitive conflict. With the existence of scaffolding and metacognition, there is a cognitive equilibrium (re-equilibrium) and reconceptualization of information so that there is a new balance of what was previously contradictory (cognitive conflict). Cognitive balance occurs because of the intervention (scaffolding) that is carried out deliberately by the teacher or other sources and metacognition, so that the assimilation and accommodation processes take place smoothly. Based on this, it can be said that cognitive disequilibrium or cognitive conflict needs to be conditioned so that an equilibrium occurs at a level higher than the previous equilibrium.

3.2. Discussion

Through text review analysis (document) of the responses of the subject's answers, the results of interviews, and Piaget's cognitive theory, the cognitive structure of the subject (S-R1; S-M1; S-F1) when solving the circle equation problem can be described in the scheme (see Figure 5).

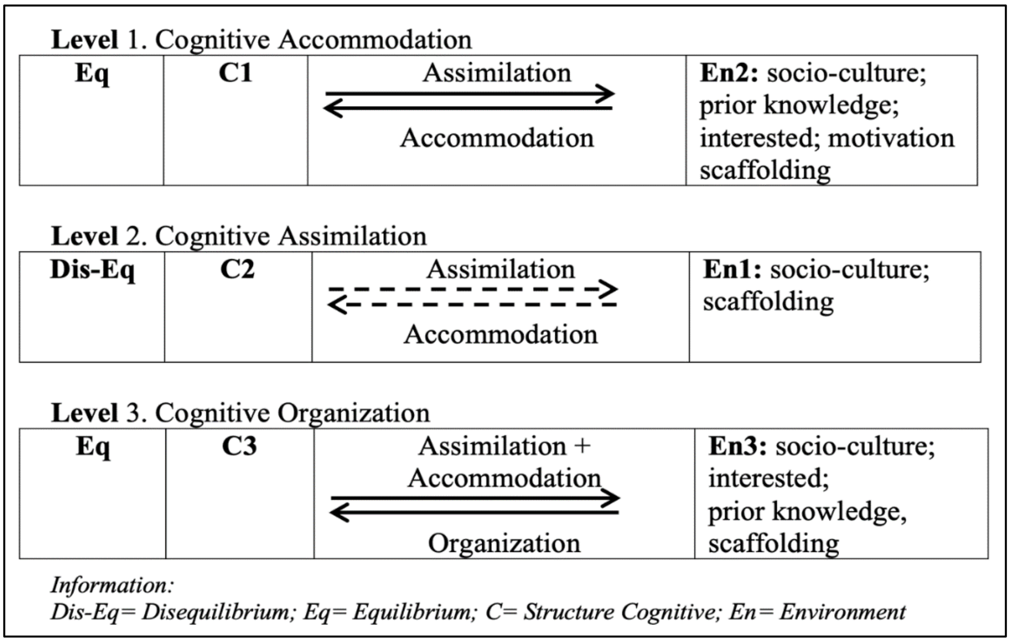

Figure 5. The cognitive structures in the realm of structural and cultural

Referring to Figure 5, it can be explained how equilibration occurs. At level 1 (low level), cognitive balance (Eq) occurs, so there is no cognitive conflict even though assimilation and accommodation occur. At this level (subject to S-M1) new information is assimilated and accommodated properly, in other words, incoming information can be captured, knowledge can be understood according to the schemata that have been in the child's mind. At level 2 (middle level) there is a cognitive imbalance (Dis-Eq) or cognitive conflict occurs due to a lack of data so that the information that is entered is incomplete and does not match the cognitive structure (schemata) that is owned so that when the information comes in, it is imperfect assimilation. At level 2, it appears that the role of the social environment (En) is in the form of scaffolding assistance from both the teacher and from peers who are free of cognitive conflicts. At level 2 conditions (experienced by the S-F1 subject), individuals reflect re-concept (re-conceptualize) the incoming information (knowledge) so that a new equilibrium that was previously conflicting (cognitive conflict) occurs. At level 3 (high level), cognitive balance occurs because of the ability of cognitive organization (subject S-R1) which is carried out on purpose and is realized by the subject so that the assimilation and accommodation processes take place perfectly. Cognitive organization ability is owned (subject S-R1) because the subject's initial knowledge is complete. Cognitive disequilibrium or cognitive conflict can be overcome and conditioned by the subject by regulating mental processes properly so that equilibrium occurs at a level higher than the previous equilibrium.

According to a Neo-Piaget follower that assimilation and accommodation models in the Cognitive (structural) and Cultural (behavioral) realms (Rutherford, 2011), we can understand from the diagram in Figure 5 that: (1) Cognitive accommodation requires a process in which individual cognitive constructs change through interaction with the environment (prior knowledge, interests, and motivation) for external suitability; (2) Cognitive assimilation requires a process in which individual cognitive constructs grow from interaction with the environment (get scaffolding) for the purposes of internal conformity; and (3) Cognitive organization requires a process whereby individual or group cognitive constructs adapt (intelligently) to the environment by rearranging internal schemes and exploring the relationships and associations between schemes for external conformity by involving socio-cultural environments, interests, and initial knowledge.

In the equilibration process, the subject (student) will take advantage of their cognitive abilities in an effort to find the truth and justify their opinion. This means that their cognitive abilities have the opportunity to be empowered, or strengthened, especially if the student is still trying. For example, students will use their understanding of mathematical concepts or experiences to make the right decisions. In situations like this, students can get clarity from their environment (Chang et al., 2009), scaffolding, among others, from teachers or students who are smarter (Vygotsky, 1987). In other words, during equilibration where cognitive conflicts occur, a person must respond appropriately or positively by refreshing and empowering their cognitive abilities (Bormanaki & Khoshhal, 2017; Zhiqing, 2015).

Piaget's learning theory provides an explanation that students (S-F1 subjects) are involved in reconstructive generalization because they try to assimilate experiences with their schemes (Dorko, 2019), adapt new knowledge (S-M1 subjects) in case of disturbances and imbalances, and modify and rearrange schemes (subject S-R1) to balance back so that the development of cognitive structures occurs (Bormanaki & Khoshhal, 2017; Dorko, 2019; Grokholskyi et al., 2020). In the context of mathematics learning, it has been shown that the construction of cognitive structures (knowledge) in the construction of cognitive structures (knowledge) has a strong relationship between the process of assimilation and accommodation and the process of equilibrium (equilibration) in the development of students' cognitive structures. So the main key to intellectual development is the organizational process and the adaptation of schemes that children get (Kholiq, 2020). Thus, Piaget's theory has been shown to have an influence in organizing and constructing the thinking structure of children (students) in the practice of learning mathematics.

4. CONCLUSION

Based on the findings and discussion it can be concluded that the role of equilibration in the formation of cognitive structures is to guide students to compile previously and new knowledge (schemes) and to form or enrich thinking structures so that they become more complex. In schema formation (cognitive structure), equilibration has influenced the process of assimilation and accommodation by considering the socio-cultural (environmental) dimension in mathematics learning. The most important thing about equilibration is the resolution of a cognitive conflict and the formation of cognitive equilibrium. The role of the teacher when students experience cognitive conflicts in the structure of students' thinking is to provide a conducive learning environment by providing scaffolding to students so that cognitive equilibrium can occur at a higher level. Further studies suggest that when selecting topics for equilibration-based activities, teachers should provide an effective learning environment to support the formation of cognitive structures to become richer.

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