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CLUSTERS OF PREVALENT PATTERNS OF GEOMETRIC THINKING LEVELS AMONG MATHEMATICS STUDENTS

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ABSTRACT

Geometric thinking skills are the perceived abilities of an individual to think and reason in geometric contexts. These skills acquired by students in geometry remain poor and unsettling because of the misconceptions that hinder the students in learning the components of geometry. The study described the common unplaceable patterns in geometric thinking of 153 mathematics education students in a state university in Eastern Visayas, Philippines. Frequency Analysis was employed in the study to determine the number of occurrences of the patterns stressing the cause for students placed under level 0 or unplaceable. Van Hiele Achievement Test was used to gather the students' performance in geometry at all levels, namely: visualization, analysis, informal deduction, deduction, and rigor. The findings attested that only 13.1% of the students managed the third level of the Van Hiele Levels while 43.1% of them were unplaceable. Common patterns were drawn and describe to understand the consequences in geometric thinking ability at level 0. These observable patterns were grouped into core-remedial, topical-corrective, and close-corrective groups. The clusters will enable educational institutions to address the individual gaps in geometry.

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1. INTRODUCTION

Improving the students' performance in mathematics, specifically in geometry, is a challenge by educators because students find it complicated. Despite the efforts made by the academic community to address the students' performance in geometry in the discipline, it remained low and degrading. Concurrently, number of students do not attain any of the geometric levels denoting poor achievement in geometry (Mullis et al., 2016). The content of the students' performance calls the understanding of geometric thinking ability to determine the common patterns exist in level 0 or unplaceable and describe its' consequences. Learning geometry measures on the Van Hiele Levels of geometric thinking ability of the students which is helpful in analyzing the learners' performance (Alex & Mammen, 2012).

Result of the study of Atebe (2008) which indicated 41% of the learners is at level 0. This study showed the difficulty that the learners have in recognizing figures in nonstandard positions. This finding is supported by Alex and Mammen (2012), presented that majority of the learners were unplaceable which means none of the students acquired any levels. The assignment of learners into levels showed the percentage in level 0 was 48%. This seeks for the need to deliver instruction at a level appropriate to learners' level of thinking on the one hand and improving the quality of education starting from lower levels on the other hand.

Besides, Marchis (2012) stressed out that students have confusion in geometry due to idea definition. Proper idea definition produces a self-idea of the image. This concept image may not develop in a few understudies, and in others, it may not identify with the formal definition. There is the need to address these misinterpretations when educating to enable the learners to consider where the misconception between the verbal meaning and mental image originates from. Characterizing and distinguishing shapes inclination is given to a visual model than a formal description (Özerem, 2012). Students prefer to rely on a visual prototype rather than a verbal definition when classifying and identifying shapes. To obtain the mathematical knowledge required in everyday living, educating the techniques on how to solve problems, and inhibiting reasoning strategies are the objectives of Mathematics.

Troubles in learning geometry clarify cognitive improvement (Idris, 2009). Individual mental capacity is not just about visual discernment, breaking down components, knowing the connections between properties, building and appreciating proofs but also decision making, which is vital to accomplishing higher-level thinking in learning geometry. An individual with better visual perception has an advantage in geometric reasoning (Walker et al., 2011). Learners need help to uncover these misconceptions and thus, build on correct perceptions. Learners need to develop and build up the proper schema about the previous knowledge before taking the new higher lessons in the upper educational level. Teachers must provide learning experiences that fit the level of thinking of the students.

Concrete experiences of the learner in the primary level help to shorten the gap in abstract concept with the use of solid objects. In addition to that, visual assisted tools are being used to enhance the geometric thinking ability, and it functioned as a mental reference (Kamina & Iyer, 2009; Zanzali, 2000). Moreover, giving attention to the application of dialect is one of the pedagogical practices that support the development of the mathematical knowledge of the students (Schleppegrell, 2007). One of the main contributors to overall comprehension in many content areas, including mathematics, is vocabulary understanding.

These observations were evident in this study to anchor the Van Hiele Theory (van Hiele, 1999) after supporting numerous knowledge that emphasizes the mediocre achievement in geometry. The Van Hiele Model considers significant imperative models in educating geometry and geometric thoughts and ideas. This model has five phases in which each level represents the development of the thinking process in geometry. The improvement of the geometric thinking of the students will lead to the summarization of the learning which is vital in using in a real-life situation (Pegg & Tall, 2010). It is one of the theories that are effective in teaching geometry to students through the school stages (Mistretta, 2000). As per Van Hiele, the five levels of geometric reasoning are Visualization, Analysis, Abstraction, Deduction, and Rigor (Groth, 2005).

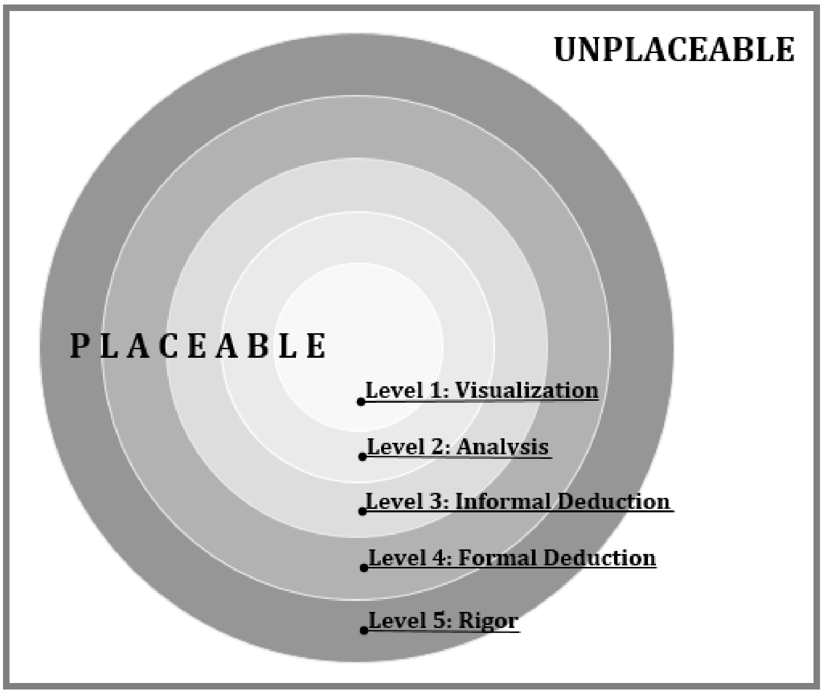

Figure 1. The procedural pattern of learning geometry

The illustration displays the levels of Van Hiele Model: the Visualization, Analysis, Informal Deduction, Deduction, and Rigor (see Figure 1). Furthermore, the patterns are formulated to identify the levels placed by the students, understand and describe the consequences of every unplaceable patterns in geometric thinking ability which is the main goal of the study. These consequences are necessary to address the concern that majority of the students are not able to reach even the first level of the model. These students are referred to be unplaceable into any of the levels of the Van Hiele Model.

One of the problems of the teachers is that the ability of teachers to present problems related to geometry is weak. They cannot transfer their knowledge appropriately about geometric thinking levels and claimed that course contents to be designed for practice are thought to improve the subject matter knowledge related to geometry (Erdogan, 2020). Focusing on the geometric thinking skills of those students who were not able to be classified in any of the levels is indispensable in order to help the struggling educators to identify the possible interventions that is useful in improving the geometric levels by imparting the precise way of presenting knowledge to the learner.

Levels are hierarchical, it is needed to fully acquire the previous levels in order to reach a higher level. Failing to do so leads to not acquiring any of the Van Hiele Model. These levels are associated to geometric experiences (Van de Walle et al., 2014). Exploring the reasons why students failed to attain any of the Van Hiele Model. will result to providing teachers the possible solutions to the main concern by understanding what happened in the levelling of the said ability. Teaching geometry is the focus of the improvement of logical thinking and a vital factor of mathematical understanding (van Hiele, 1999). Educators have a critical role in teaching and learning geometry.

2. METHOD

This study utilized frequency analysis. It deals with the number of occurrences or frequency of the patterns stressing the cause for students placed under unplaceable. It describes the data set and provide a fair idea of what patterns the students are acquiring. The method used was Complete Enumeration in the conduct of the study.

The locale of the study was all campuses of Southern Leyte State University. All mathematics students enrolled in Academic Year 2018-2019 were considered to be the participants of the study towards exploring the placement of learners' geometric ability (see Table 1).

Table 1. Distribution of respondents

Year level	Number of Students	Percentage
Freshmen	46	30%
Sophomore	36	24%
Junior	33	21%
Senior	38	25%
Total	153	100%

Distribution of the questionnaires to all students majoring Mathematics in the master list provided by the university followed. The Achievement test for the Van Hiele was composed of five questions in each level. In every query, students will choose the best answer from the options. Students must reach three points in each level to grasp the level need to attain. However, your previous score in the lower level is less than to 3 points, and you achieved a score that is <3 points in the next level, students can be classified as unplaceable. It is impossible for the students to obtain the higher levels without accomplishing the lower levels. Hence, students should have consistent score that is <3 without failing the levels in between to grasp any of the levels.

Mean, Frequency, and Weighted Average. This was used to determine the mean rating of the sample and the exact number of each pattern exist. It is necessary for analysis and interpretation of any data and it indicates how well the data is.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1. Van Hiele Geometric Thinking Ability

Van Hiele Geometric Thinking designates how an individual acquires learning in the field of geometry that proposes five levels of geometric thinking. Each level utilizes its symbols and language and students can pass through this level "step-by-step."

Table 2. Geometric thinking ability according to Van Hiele levels

VAN HIELE MODEL	YEAR LEVEL					
	Freshmen Level			Sophomore Level		
	f	<cf	%	f	<cf	%
Unplaceable	24	46	52.2	16	36	44.5
1	6	22	13.0	7	20	19.4
2	7	16	15.3	7	6	19.4
3	6	9	13.0	6	3	16.7
4	0	3	0	0	0	0
5	3	3	6.5	0	0	0

VAN HIELE MODEL	YEAR LEVEL					
	Junior Level			Senior Level		
	f	<cf	%	f	<cf	%
Unplaceable	17	33	51.3	9	38	23.7
1	9	16	27.4	9	29	23.7
2	7	7	21.3	8	20	21.1
3	0	0	0	8	12	21.1
4	0	0	0	0	4	0
5	0	0	0	4	4	10.4

VAN HIELE MODEL	OVER ALL		
	f	<cf	%
Unplaceable	66	153	43.1
1	31	87	20.3
2	29	56	19.0
3	20	27	13.1
4	0	7	0.00
5	7	7	4.5

Legend: 0 - Unplaceable, 1- Visualization, 2- Analysis, 3- Informal Deduction, 4- Deduction, 5- Rigor

Table 2 shows the achievement of the students across year levels in terms of their geometric thinking ability. Amongst respondents of the study, 20.3% were only level 1; 19.0% are level 2; and 13.1% are level 3. There is a decreasing number od students who were able to reach a higher level in the Van Hiele Model. Looking at the distribution of students per geometric level, only 56.9% managed to attain visualization level. In the analysis level, only 36.6% achieved this level. Meanwhile, in the informal deduction level, 17.6% were left to reached the third level. Among the respondents, there is only 4.5% who were able to developed the two highest level of geometric thinking which is deduction and rigor.

On the other hand, a staggering 43.1% of the students were classified to be level 0 or unplaceable. These students were not able to reach visualization level which is the basic foundation of the geometric skills.

The huge percentage of students found to be unplaceable initiated the query on what were the achievement made in the long period of time being exposed to geometry and other related areas in mathematics. Patterns were observed in the study as presented below.

3.1.2. Common Patterns of Respondents under Unplaceable

Outcomes of the international tests revealed that the leaners' geometry knowledge and skills were not at the preferred level. Geometry is the lowest performance area among all fields (Mullis et al., 2016). The data below represent the pattern of the scores of the students under the undesirable level called unplaceable. Each digit of the pattern represents each of the levels of the Van Hiele Achievement Test in an orderly manner.

Table 3. Patterns of geometric thinking skills of unplaceable

Patterns	Mathematics Students	
	f	%
00000	40	26
01000	48	30

Patterns	Mathematics Students	
	f	%
00100	17	11
01100	15	10
11101	17	11
11001	7	4
10100	7	4
10010	7	4

*0- Level not attained, 1- Attained level

Core-Corrective Group

The core-corrective group include the patterns 00000, 01000, 00100 and 01100 (see Table 3). This group barely succeeded the foundation of the geometric skills and abilities. In 00000, it means that none of the levels were partially attained. Different aspects of the mathematics achievement of students can cause failure in achieving any of the levels (Meng & Sam, 2013). This means that students were really having a hard time visualizing concept, reasoning, synthesizing three to two dimensional objects, formulating assumptions, creating logical inferences, and also proof skills in geometry. Also, this is affected by the influence of teaching-learning process. Researches derive conclusions that students' level of geometric thinking depends on how the instruction is delivered (Abu & Abidin, 2013; Alex & Mammen, 2012).

In the 01000 and 00100 patterns (see Table 3), respondents succeeded in the second phase but failed in the rest of the levels. It was because students considered the properties of the figures which is not needed in the visualization level and not enough in attaining other levels. Students are unable to visualize both solids and positions of solids that they cannot see, neither the pattern of movements. There is also a lack of coordination between geometric presentation and visualization.

In the 01100 patterns (see Table 3), students attained the mid-levels; however, previous and succeeding levels failed to reach and considered as unclassified. It was because students used the properties possessed by each of the shapes, thus failed in obtaining the first level. Considering characteristics acquired by the figures were not enough to reach into the higher levels. This stresses that learners have a hard time to produce a formally deduced definitions and properties in order to prove and provide a logical concept which is necessary to reach in its desired level.

Topical-Corrective Group

The patterns 11101 and 11001 (see Table 3) were classified to be topical-corrective group. This group managed to complete the foundation skills but not the higher levels of the model. In these patterns, 11101 and 11001 (see Table 3), students failed between the levels, thus considered it as unplaceable. In the first pattern, failing in deduction level (Level 4) but reaching into the higher-level means failing to attain any of the Van Hiele levels. It was due to lack of knowledge in proving and understanding axioms, corollaries, and postulates. In the second pattern, students used their instincts rather than logic in creating conceptual statements and properties of the geometric figures and not considering the necessary and sufficient conditions in the Euclidean and non-Euclidean geometry. Both patterns failed in attaining the formal deduction level. In this level, students failed to provide logical reasons or proofs of the following mathematical structure of the elements. Students' mathematical understanding of proofs, axioms and postulates and corollaries is at risk. It implies that students are struggling in terms of proving. Thus, students need to build a strong foundation of the basic understanding of geometry to achieve the higher levels. Recently, a growing number of studies have provided evidence that students have difficulty with mathematical proofs (Ko & Knuth, 2009).

Close-Corrective Group

This group of patterns managed to achieved the foundation skills in geometry which is visualization, however, failed to achieved even partially and struggling to complete the levels of the Van Hiele Model.

10100 and 10010 patterns were categorized as unplaceable. Students attained the first levels but failed in reaching the second and third level. It was unplaceable since students must achieve the second level to surpass to the next levels. Students cannot skip into the higher levels unless attained the previous levels. It was because students only focused on their visual perception, and nonverbal thinking but neglecting the properties, relationships, proofs and manipulating axioms. In this level, they can measure, fold and cut paper, use geometric software, etc. It implies that undertakers only focused on their visual recognition, considered as difficult in applying the properties and unable to perceive relationship between figures and components of each shape used to justify and support for the students to answer. The pattern reveals that difficulty in understanding the properties and perceiving the relationships between properties and figures can affect the students' performance in the higher level of mathematics.

After identifying patterns, we classified all the existing patterns into three groups, topical- corrective, close-corrective, and core- remedial groups.

3.2. Discussion

As a result of the Van Hiele geometric achievement test, it is concluded that majority of the undertakers are under unplaceable. Thus, exploring the patterns existed in the said level give us the keys in observing and providing insights on remedial actions in geometry that assess the geometric thinking skills of the students in order to improve the levels of their geometric ability.

Topical- corrective group, these are the group of students that have higher chances in learning the levels of geometric thinking however, teachers should take consideration in providing exercises or activities that can help improve in proving and logical reasoning. The group that already captured visualization level but finds it difficult for them attain the succeeding levels are placed in close-corrective group because they cannot break down information into concepts. Under core-remedial group, students need recalibration of the geometrical concepts from simple to complex. Students placed in this group failed to visualize geometric figures. Thus, students were having a hard time understanding geometry.

All these remedial actions provide us analysis of errors observed in every unplaceable patterns. Each group introduced differentiated instruction based from the individual group needs to improve the geometric levels. These actions will contribute plenty intervention strategies and techniques to educators that can be integrated in their geometry lessons.

4. CONCLUSION

Individual gaps in the performance of geometry in the early levels of education is important to nurture because of the relevance of the topics in geometry in higher education.

The progress of a student determined the ability of them to perform well in any mathematics related disciple. The levels of Van Hiele's geometric thinking showed that many of the student were not able to attain any of the levels namely visualization, analysis, informal deduction, deduction, and rigor.

Appropriate strategies were needed to ensure full understanding by the students are needed by all mathematics educators especially in the basic education to address the issue of geometric thinking performance. It will showcase the distinction of every student and the required remedial action based from their performance.

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