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THE BRIDGING UNDERSTANDING OF LANGUAGE AND MATHEMATICAL SYMBOLS BETWEEN TEACHERS AND STUDENTS: AN EFFORT TO INCREASE MATHEMATICAL LITERACY

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ABSTRACT

The research aims to analyze the gap between teachers' and students' understanding of language literacy and mathematical symbols. The study was designed with a concurrent triangulation strategy. The research respondents consisted of 20 teachers and 120 class VII students. Data collection through questionnaires, interviews, and cognitive tests. Qualitative data was analyzed descriptively, and quantitative data was analyzed inferentially. The results of the analysis of quantitative data show that there is a linear (significant) relationship between understanding language and mathematical symbols and mathematical literacy skills. The results of the qualitative data analysis describe that the teacher's understanding of language and mathematical symbols (high criterion) does not necessarily support the students' understanding of language and mathematical symbols. We confirm the suspicion that there is a gap in the ability of teachers and students to understand language and mathematical symbols. Students need to improve their understanding of mathematical language and symbols. The pattern of errors is based on the teacher's conception of learning in the previous class, so the process of transitioning the teacher's knowledge to students' understanding of mathematics experiences obstacles. The implication is that the process of transitioning meaning from mathematical symbols to written and spoken language must be carried out when the teacher introduces or teaches new topics to students, and the context in which mathematical symbols are used must be followed by clarification.

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1. INTRODUCTION

Mathematics is the language of science because with mathematics scientists can develop their knowledge and convey the results of their findings. Mathematical language and symbols have a single meaning so a mathematical sentence cannot be interpreted in various ways (Esuong et al., 2023). Mathematical language and symbols succeed in avoiding confusion of meaning because every sentence (term/variable) in mathematics already has a certain meaning.

Verbal language is only able to express qualitative statements. Meanwhile, mathematics has a quantitative nature, that is, it can provide more exact answers which enables problem-solving more quickly and accurately. Through language and mathematical symbols, the language used is simpler and the words used do not need to be as long-winded as ordinary language so that it can eliminate ambiguity and strengthen arguments (Fang et al., 2023).

Mathematics is a language that uses carefully defined terms and symbols. Learning mathematics requires communication skills using symbolic language (Wilkinson, 2019). Mathematical language is a language that attempts to eliminate the vague, compound, and emotional nature of verbal language (Mutodi & Mosimege, 2021). Mathematical symbols are created artificially and individually which constitute agreements that apply specifically to the problem being studied (Schoenfeld, 2016).

Mathematical language is a language of symbols, concepts, definitions, and theorems (Hassidov & Ilany, 2017). The language of mathematics does not develop naturally like the mother tongue that students learn, but the meaning of the language of symbols needs to be taught to students by mathematics teachers at school (Hassidov & Ilany, 2020). The advantage of mathematics as a language is that mathematics develops a numerical language that makes it possible to carry out quantitative measurements (Wilkinson, 2019). Learning mathematical terms and symbols can improve the ability to communicate about science, real-life situations, and mathematics itself (Wilkinson, 2019). The application of mathematics in learning aims to develop: (a) the ability to solve real-life problems; (b) communication skills through language and mathematical symbols; and (c) mathematical reasoning skills (Rohid et al., 2019).

The use and understanding of symbols is very important in learning mathematics. There are several things that teachers need to pay attention to when symbolizing, namely (a) symbolizing to make it easier for students to understand the material; (b) symbolism according to the functions of the symbolism itself; (c) the use of one symbol may only be used for one concept; (d) the teacher must explain the concept or idea contained in the symbol; and (e) the use of the symbol must comply with the rules and not give rise to misinterpretation of the symbol (Mutodi & Mosimege, 2021).

Mathematical concepts (in symbol language) are not easily interpreted in ordinary language (Ilany & Margolin, 2010; Purpura & Reid, 2016), Therefore mathematical symbolism was developed to express meaning that goes beyond what ordinary language can express (Mulwa, 2014). Mathematical symbolism can describe the relationship of parts to the whole, and establish continuous patterns of variation that cannot be represented precisely by natural language (Kung et al., 2019). Mathematical symbolism such as visual displays in graphs and diagrams can represent extensive and complex information that ordinary language cannot (OECD, 2022).

Symbolic reasoning abilities are included as important abilities in mathematical literacy. Symbolic reasoning means the ability to grasp the meaning of a symbol that represents an object or idea, without having to express it in the symbol itself (Garnelo & Shanahan, 2019). The early development of symbolic reasoning in young children enables them to use mathematical symbolization correctly in formal mathematics (Tobia et al., 2021). In learning mathematics there are various kinds of signs in it. One of the abilities that students must have is the knowledge of understanding a sign, which is called semiotics. Semiotics is the science that studies signs that lead to something else. Each sign will produce an interpretation for those who see it, this will allow students to have different visualizations of the sign they see (Godino et al., 2022; Monteiro, 2022). Monteiro (2022) revealed that different individuals can construct different interpretations of the same sign, thereby effectively creating different signs for the same object.

According to Schleppegrell (Ilany & Margolin, 2010), in solving mathematical problems students are not only required to be capable of solving procedures. However, students must be able to work in their own language (orally and in writing) to build their knowledge through multiple semiotic systems. The multiple semiotic systems include (a) mathematical symbolic notation; (b) written language; (c) spoken language; and (d) visual representation (graphs and diagrams) multiple semiotic systems (Bateman, 2021; Ilany & Margolin, 2010; O'Halloran, 2023).

The semiotic abilities that students have will be much more likely to enable students to visualize mathematical problems. This statement shows that representation starts from semiotics, so in mathematics, it can be called mathematical semiotic representation, where in this mathematical semiotic representation students learn things related to signs based on everyday life problems (Viseu et al., 2021).

Mathematical literacy is very important for individuals to formulate and interpret mathematics in various contexts. So mathematical literacy is not solely related to mathematics lessons at school. Students' mathematical literacy abilities are not only numeracy skills but also the ability to reason mathematically and think logically and critically in solving problems that are needed to support other literacies (Kusmaryono & Kusumaningsih, 2023). The mathematical literacy indicator is described as a modeling process that includes four cognitive activities (OECD, 2018, 2022), namely: (a) simplifying real situations with mathematical models according to correct understanding, (b) using mathematical concepts, facts, procedures and reasoning in solving the problem, (c) interpreting problem-solving solutions in an authentic context, and (d) evaluating solutions (completing and concluding) problem-solving appropriately.

Literacy includes symbols, while symbols are closely related to mathematics because mathematical concepts also use symbols a lot. Indicators of mathematical literacy achievement also include the use of symbols. Students' achievements in using symbols are interpreting symbols, understanding the relationship with the language used, translating, understanding and using variables, and carrying out calculations (Esuong et al., 2023).

In mathematics, data reading skills and symbolic interpretation are very important in the process of understanding problems. Skills in reading data as initial information are needed to interpret and process information. Meanwhile, the interpretation of the symbols that appear becomes the basis for representing mathematical expressions (Matsumoto & Nakai, 2023). This means that the mathematical process can run after the symbolic representation has been completed so that symbolic interpretation skills will have an impact and support students' success in problem-solving (Mutodi & Mosimege, 2021).

This presentation interprets symbols as the main points in indicators of mathematical literacy achievement. Symbols must be interpreted so that students know the use of mathematical symbols, and also interpret mathematical concepts, especially for symbol literacy. Interpretation activities, which are often known as interpretation, will make it easier for students to describe information that students can change themselves in understanding mathematical concepts that involve symbols so that students can solve mathematical problems well. Mathematical literacy skills can help students make the right decisions. This is because students who are able to formulate and interpret mathematics in various contexts will find it easier to make decisions, and will be trained to be able to think with a high-level mindset.

Someone who can understand language and mathematical symbols means being able to read, write, and understand the meaning of mathematical symbols (Mutodi & Mosimege, 2021). This ability is not only limited to the ability to calculate but has an impact on increasing mathematical literacy abilities. Someone who understands mathematical language and symbols will easily find answers to the problems they face, so mathematical literacy skills also increase (Monteiro, 2022).

Several previous research results from experts have revealed that students' understanding of mathematical language and symbols is still limited (Amirbostaghi et al., 2021; Bardini & Pierce, 2015; Bermejo et al., 2021; Chin & Pierce, 2019; Hassidov & Ilany, 2017, 2020). Understanding language and mathematical symbols has an impact on students' mathematical literacy abilities, especially in problem-solving (OECD, 2022). However, their findings do not significantly reveal the gap in understanding mathematical language and symbols between teachers and students and the impact on mathematics learning. Teachers' abilities regarding language and mathematical symbols need to be revealed, considering that there is a potential gap between understanding language and mathematical symbols by teachers and students in learning at school.

Considering the importance of understanding mathematical language and symbols in mathematical literacy, teachers and students are required to have a good understanding of mathematical language and symbols. The aim of the study was to analyze the gap between teacher and student understanding of language literacy and mathematical symbols. It is hoped that the research will be useful for teachers, namely (a) adding information about the importance of understanding language literacy and mathematical symbols for teachers and students, (b) eliminating the factors that cause gaps in understanding language literacy and mathematical symbols between teachers and students, and (c) as a teacher's effort to improve students' mathematical literacy skills.

2. METHOD

The research design uses mixed methods with a concurrent triangulation strategy. In this strategy, researchers collect quantitative data and qualitative data at the same time at the research stage, then compare qualitative data with quantitative data to find out differences or combinations (Creswell & Clark, 2018).

The research respondents consisted of 20 teachers and 120 class VII students. The teachers have taught mathematics at the school with more than five years of teaching experience. The students consisted of 40 male students and 80 female students. They are between 11 and 12 years old and they learn mathematics in the early grades of junior high school. The selection of class VII students aims to support the discussion of whether the understanding of mathematical language and symbols obtained in elementary school is adequate or not. Meanwhile, the teacher's teaching experience of more than five years shows that the teacher has fulfilled the requirements for having an educational certificate as a professional teacher with proven competence (Ventista & Brown, 2023).

The research data was collected through questionnaires, interviews, and cognitive tests. The questionnaire instrument contains 20 closed questions with a Likert scale of 1 to 4. The questionnaire was given to respondents (teachers and students) to obtain data on understanding language and mathematical symbols. Before being used in this research, the questionnaire instrument had been tested to measure the validity and reliability of the questionnaire. For validity testing purposes, Pearson bivariate correlation analysis (productmoment correlation) with a significance level (2-tailed) of .05 is used as the testing standard (Sreedevi, 2022). The calculated r value is between .872 and .956 > r table .468 based on a significance test of .05, meaning that the questionnaire items are valid. Meanwhile, the calculation results for the overall reliability test of the questionnaire items have a Cronbach's alpha value greater than .700, namely .925, meaning sufficient reliability. Thus, it is concluded that the questionnaire instrument is declared valid and reliable, which means the instrument has met the requirements.

The interview instrument contains questions to obtain in-depth information about the impact of the teacher's understanding of language and mathematical symbols on students. The cognitive assessment is useful for seeing the extent to which students understand lesson material by using language understanding and mathematical symbols. Data from cognitive testing can also help analyze and interpret survey data more accurately (Wu et al., 2022). The cognitive assessment can improve the reliability and validity of questions and surveys and contribute to improving data quality (Bingham, 2023).

The test instrument consists of 4 cognitive test items which are prepared with reference to indicators of mathematical literacy abilities (OECD, 2018, 2022). The test items used in this research have been validated by a team of experts and tested for their level of validity and reliability. The results of the test set of test items obtained a face validity test value of α = .462 and a content validity test of α = .555. Because the test results for face validity and content validity have a probability value (Sig.) of more than α = .05 it is concluded that the validation results are uniform for the research instrument. In Cronbach's alpha test, it was .743 > 0.05, included in the high-reliability category. So, it is concluded that the instruments used in this research are reliable and trustworthy.

The qualitative data analysis combines deductive and inductive coding with the following steps: (a) The researcher determines a list of codes deductively; (b) The researcher reads all the data and codes any common phrases, ideas, or categories that emerge; (c) The researcher must comb through the data line by line, refining the code list and adding detail; (d) Researchers group codes into categories and develop themes. Codes can be grouped based on similarities or if they relate to the same topic or general concept; (e) The researcher modified inductively and added to the list as the analysis progressed; and (f) The researcher then looks at the categories, paying close attention to any themes or patterns that emerge across the data set. Within these themes is the overall research narrative. Quantitative data was analyzed inferentially through correlation tests between research variables and analysis of variance (ANOVA) tests. The correlation test uses Pearson Correlation with significance at the .05 level (2-tailed). The ANOVA to test the linear regression model so that it meets the linearity criteria. In the final stage, researchers combined the two groups of data. The merging technique involves changing one type of data into comparable qualitative or quantitative groups. The aim of merging data is to get a complete picture of research findings and answer the problem formulation. To increase credibility, the findings are discussed in a discussion forum with experts to obtain better conclusions from the existing data group.

This research procedure was carried out in the following stages: (a) compiling a research instrument; testing the validity and reliability of the instrument; (b) determining the research respondents (subjects); (c) distributing questionnaires on understanding mathematical language and symbols (teachers and students); (d) conduct interviews; (e) mathematics literacy test (students); (f) statistical data testing and data analysis; (g) Forum Group Discussion and interpretation; and (h) drawing conclusions.

3. RESULT AND DISCUSSION

3.1. Results

Preliminary research data was obtained through a questionnaire to determine teachers' and students' understanding of language and mathematical symbols in terms of multiple semiotic systems. In the following, Tables 1 and 2 are presented as a comparison of the understanding of language and mathematical symbols between teachers and students.

Table 1. Teachers understanding of language and mathematical symbols

Multiple Semiotic System	Comprehension Level (%)			Criteria
	Low	Medium	High	
Mathematical symbolic notation	10%	30%	60%	High
Spoken language	10%	50%	40%	Medium
Written language	10%	40%	50%	High
Visual representation	5%	20%	75%	High
Average	8.75%	35.0%	56.25%	High

The data in Table 1 can be explained that more than 56% (mean score), of teachers have a good or high-level understanding of language and mathematical symbols. However, almost 44% of teachers have an understanding of mathematical language and symbols at low and intermediate levels. Based on the results of Table 1, the researcher conducted interviews with teacher representatives. The purpose of the interview is to obtain more in-depth information about understanding language and mathematical symbols. Below is an excerpt from an interview with the teacher.

1st Interview Snippet

R-Q1	:	*Do you introduce more complex mathematical symbols to students?*
Teacher-01	:	*Introduction of mathematical symbols according to the context of the material students are studying.*
Teacher-02	:	*I introduce mathematical symbols according to the mathematical problems and situations faced by students.*
R-Q2	:	*Does your understanding of mathematical language and symbols help you enough to learn mathematics in class?*
Teacher-01	:	*Yes, it's quite helpful for learning mathematics in class*
Teacher-02	:	*Yes, but the transition to understanding symbols and mathematical language requires students to understand their meaning.*
R-Q3	:	*Are students ready to accept the transition from mathematical language and symbolization to new concepts?*
Teacher-03	:	*Students need to be prepared to understand new concepts and their meanings.*
Teacher-04	:	*Some students are not ready to make the transition, but there are students whose symbolic understanding is more advanced than other students.*
R-Q4	:	*Does your ability in language and mathematical symbols influence your students' ability in language and mathematical symbols?*
Teacher-03	:	*We realize that there is an influence on students, in reality, not one hundred percent of knowledge can be transferred to students.*
Teacher-04	:	*In terms of cognitive development, there must be an influence on students. However, what we understand is not necessarily what students understand.*

R-Q5	:	*What are the factors causing students' weak language and mathematical symbol skills?*
Teacher-05	:	*We realize that not one hundred percent of knowledge can be transferred to students, so I encourage students to manipulate symbols only.*
Teacher-06	:	*Cognitive development of students who have not yet reached the formal stage.*

The results of interviews with teachers (1st Interview Snippet) can be interpreted as indicating that students' initial abilities regarding understanding language and mathematical symbols are not sufficient. Students are not ready to transition from concrete to abstract understanding. This occurs due to students' cognitive development factors which have not yet reached the formal stage. However, teachers have tried to introduce mathematical symbols according to the context of mathematical problems and situations faced by students.

Table 2. Students understanding of language and mathematical symbols

Multiple Semiotic System	Comprehension Level (%)			Criteria
	Low	Medium	High	
Mathematical symbolic notation	60%	30%	10%	Low
Spoken language	40%	50%	10%	Medium
Written language	30%	60%	10%	Medium
Visual representation (Graphs and diagrams)	10%	60%	30%	Medium
Average	35.0%	50.0%	15.0 %	Medium

The data in Table 2 shows that only 15% (mean score) of students have an understanding of language and mathematical symbols at a high level. Meanwhile, 85% of students have an understanding of mathematical language and symbols at low and intermediate levels. Of course, the percentage data in Table 2 is very different and inversely proportional to the data in Table 1.

The results of Table 2 were followed up by researchers by conducting interviews with students. The purpose of the interview is to get more in-depth information about understanding language and mathematical symbols. The following shows excerpts of interviews with students.

2nd Interview Snippet

R-Q6	:	*Do you understand the meaning of all mathematical symbols?*
Student-45	:	*I don't understand unfamiliar mathematical symbols.*
Student-07	:	*I don't understand the meaning of some mathematical symbols.*
R-Q7	:	*Does understanding mathematical symbols help you transition into spoken and written language?*
Student-16	:	*I have tried, but I fail to understand the meaning in written language.*
Student-37	:	*I didn't find a solution to this problem.*
R-Q8	:	*What are the factors that cause your weak language and mathematical symbol skills?*
Student-16	:	*What I understand at school is easy to forget when studying at home, especially mathematical rules and formulas.*
Student-37	:	*There is a lot of symbolic language and mathematical formulas that I have to understand, and it's not easy.*
Student-07	:	*I still have difficulty in visual perception (linear inequality graph)*
Student-104	:	*This material (symbols) has not been taught when studying mathematics in elementary school.*

The results of the interview (2nd Interview Snippet) can be explained that students have difficulty recognizing (new) symbols in mathematics. Students fail to understand the meaning of symbols in written language. Students feel that there is still a lot of symbolic language and mathematical formulas that they have to understand, and it is not easy, because it has not been taught in elementary school.

The Data on students' mathematical literacy abilities were obtained from test results. Mathematical literacy ability data is presented in the form of statistical descriptions and the percentage of answers to the test items (see Tables 3 and 4).

Table 3. Statistical description of students' mathematical literacy ability

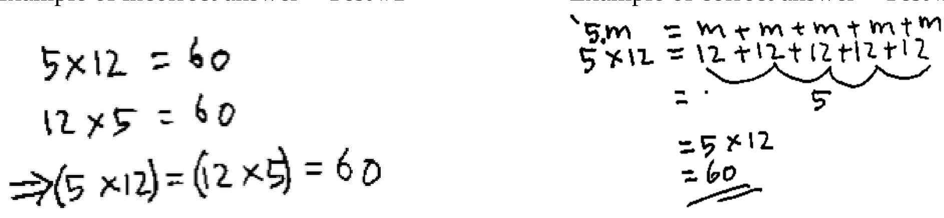
	N	Range	Minimum	Maximum	Mean	Std. Deviation
Math Literacy	120	75.00	25.00	100.00	72.917	17.401
Valid N (listwise)	120					

Table 4. Percentage of answers to the math literacy test

Test Item	Indicator Mathematical Literacy	N	Answer Percentage (%)	
			Correct	Incorrect
1	Design problem-solving strategies using mathematical concepts, facts, procedures, and reasoning	120	96 (80%)	24 (20%)
2	Simplifying real situations with mathematical models according to proper understanding	120	72 (60%)	48 (40%)
3	Finishing and concluding proper problem solving	120	96 (80%)	24 (20%)
4	Interpret solution results in an authentic context	120	90 (75%)	30 (25%)
	Average	120	(73.75%)	(26.25%)

Tables 3 and 4 show that the achievement of students' mathematical literacy test results is not satisfactory (mean score 72.917). Students' mathematical literacy ability has not reached 80.0 and the percentage of answering the test correctly is only 73.75%. Next, we conducted interviews to highlight students' answers in completing the math literacy test. We selected students using the purposive sampling method to get information about the problem-solving process for each test item. The following shows examples of student answers to test items number 1 to 4.

Example of incorrect answer – Test #1 | Example of correct answer – Test #1



(a) Student Answers (S-90) | (b) Student Answers (S-07)

Figure 1. Example of student answer on test number 1

In Figure 1a, it can be seen that students (S-90) still have difficulty interpreting the equal sign with "=" which is related to the concept of equality. In Figure 1b, student (S-90) only uses the commutative property to support his idea. Students (S-07) can compare the expressions on both sides of the equal sign "=" correctly.

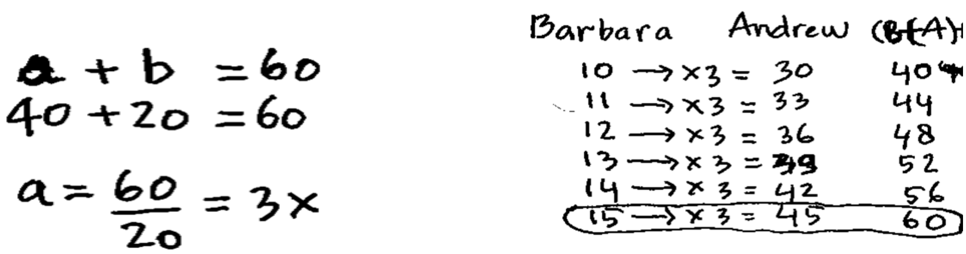
Based on a sample of student answers on test item number 1 (see Figures 1a and 1b), the researcher conducted interviews to confirm student answers. Below is an excerpt from the researcher's interview with students.

3rd Interview Snippet

R-Q9	:	*How do you transfer spoken language to written language to solve this problem?*
Student-90	:	*The number 5 is multiplied by 12. So there are 12 factors for the number 5.*
Student-07	:	*I remember my mother in the kitchen. Mother asked me to take five mangoes (5 mangoes). I put them on the table sequentially, namely: Mango + Mango + Mango + Mango + Mango = 5 Mango*
Student-45	:	*I agree that 5 x 12 = 12 + 12 + 12 + 12 + 12 = 60. But I don't know.*
Student-07	:	*For 5 x m = 5m* *Then 5m = m + m + m + m + m (to m by 5 factors)*
R-Q10	:	*How do you plan a problem-solving strategy?*
Student-90	:	*I understand that 5 x12 is the same as 12 x 5*
Student-07	:	*Multiplication is the repeated addition of the numbers being multiplied.* *Here 5 is the multiplier and 12 is the number being multiplied.*
Student-90	:	*Actually, I agree that the nature of the operation of multiplying numbers is commutative so that 5 x12 = 12 x 5 the result is 60.*
Student-07	:	*I understand that 5 x m = 5m* *So, 5m = m + m + m + m + m (m is 5 factor)* *So, 5 x 12 = 12 + 12 + 12 + 12 + 12 = 60*

The results of the interview (3rd Interview Snippet) can be described that students understand the language of symbols (algebra) using the transition from everyday experience to understanding mathematics. Students use their understanding to plan problem-solving strategies. In this case, students succeeded in using knowledge of horizontal mathematics correctly.

Example of incorrect answer – Test #2 | Example of correct answer – Test #2



(a) Student Answers (S-16) | (b) Student Answers (S-37)

Figure 2. Example of student answer on test number 2

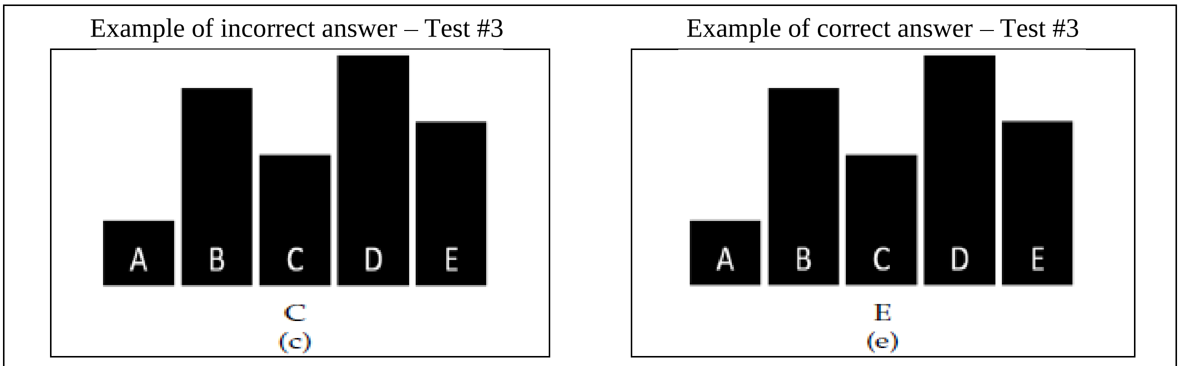
Figure 2a shows students (S-16) only compiling algebraic equations at the beginning. The value a = 3x found has not been substituted into the first equation. However, student (S-37) determined Barbara and Andre's age list with initial estimates of b = 10 and a = 30 until the sum of ages b and a was found to be equal to 60.

Based on a sample of student answers on test item number 2 (see Figures 2a and 2b), the researcher conducted interviews to confirm student answers. Below is an excerpt from the researcher's interview with students.

4th Interview Snippet

R-Q11	:	*What is your strategy to solve this problem?*
Student-16	:	*I need to understand the language of mathematics in this problem.*
Student-37	:	*I will do a trial until I find the right answer*
R-Q12	:	*How would you simplify this problem into a mathematical model?*
Student-16	:	*I thought that the age of A + B = 60, so I spontaneously got the appropriate number, namely 40 + 20 = 60, and A = 3x.*
Student-37	:	*I understand that Andre's age is three times Barbara's age in the ratio A : B = 3 : 1, then I put together a table of A and B's ages to get the number 60.*
R-Q13	:	*Why don't you think with the following reasoning.* *If Andre's age is A and Barbara's age is B* *Then A = 3B, so A + B = 60* *3B + B = 60* *4B = 60* *B = 60 : 4 = 15* *and A = 3B = 3 x 15 = 45* *So, Andre's age is 45 years and Barbara's age is 15 years*
Student-37	:	*I compiled a mathematical model in the form of an algebraic equation A + B = 60, where A = 3B so 3B + B = 60.*
Student-16	:	*Maybe I'm too late to think of the correct model of the equation.*

The results of the interview (4th Interview Snippet) show that students (S-16) still use trial-and-error methods to solve problems. The problem-solving strategies used by students (S-16) have a high risk of errors. Meanwhile, students (S-37) use appropriate algebraic equations as a problem-solving strategy.



(a) Student Answers (S-49) (b) Student Answers (S-11)

Figure 3. Example of student answer on test number 3

Test item number 3 asks students to indicate the position on the podium where the third-place swimmer will stand. As seen in Figure 3a, the student's answer (S-49) is not correct. Meanwhile, the student's answer (S-11) is correct. Is the student's error (S-49) due to a lack of understanding of the diagram? Based on a sample of student answers on test item number 3 (see Figures 3a and 3b), the researcher conducted interviews to confirm student answers. Below is an excerpt from the researcher's interview with students.

5th Interview Snippet

R-Q14	:	*How do you understand the problem in test item number 3?*
Student-49	:	*It's just a matter of order from high level to low level.*
Student-11	:	*I will order the podiums according to their height.*
R-Q15	:	*How do you come up with ideas as solutions to problems?*
Student-49	:	*I have to determine the positions of first, second, and third place.*
Student-11	:	*Third place means the podium is lower than first and second place.*

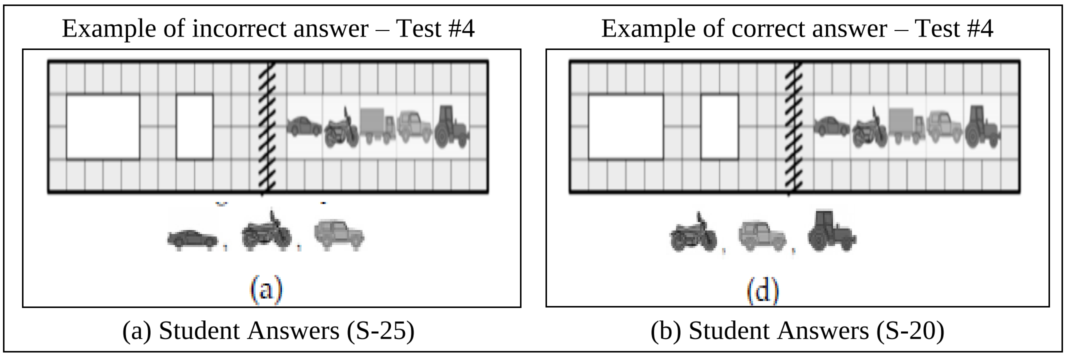


Figure 4. Example of student answer on test number 4

The question in point 4 reads, "What image will you see through the windows when you close the book?" The student's answer (S-25) in Figure 4a shows an invisible image. It seems that the student (S-25) did not consider the symmetrical nature of the book, so the answer was wrong. Meanwhile, in student answers (S-20) Figure 4b, students have used the symmetrical nature of the book to see the image that appears.

Based on a sample of student answers on test item number 4 (see Figures 4a and 4b), the researcher conducted interviews to confirm student answers. Below is an excerpt from the researcher's interview with students.

6th Interview Snippet

R-Q16	:	Do you understand the problem in test item number 4?
Student-25	:	I really understand and I know what I have to do.
Student-20	:	I think this requires accuracy in answering test items.
R-Q16	:	*In your opinion, what form of mathematical representation is easier for you to understand in solving mathematical problems?*
Student-25	:	*I find it easier to understand representations in the form of pictures and tables than words or written language.*
Student-20	:	*I understand the explanation in the form of pictures and tables. With spoken language, I have to record the teacher's explanation and it takes longer to understand it.*
R-Q18	:	*How would you evaluate the solutions to the problems you found?*
Student-25	:	*I thought many times to decide on this solution.*
Student-20	:	*This seems to be a matter of thoroughness and accuracy in making decisions. I have to understand the test item command very well.*
R-Q19	:	*What is the reason for your wrong answer?*
Student-25	:	*I perfectly understand that the pages of the book must be closed. But without me noticing that the cover has shifted to the left.*

As a result of the interview (6th Interview Snippet), both students (S-20 and S-25) tried very carefully to determine the answer. They had used precision and accuracy in answering the question, but the student (S-25) realized too late that the book cover had shifted to the left so the answer was wrong.

At the quantitative data analysis stage, we conducted a correlation test between understanding of language and mathematical symbols and students' mathematical literacy. The purpose of the correlation test is to investigate the relationship between the two variables. The results of the correlation test of the two variables are presented in Table 5.

Table 5. Correlation between research variables

Correlation		Understanding of language and math symbols	Math Literacy
Understanding of language and math symbols	Pearson Correlation	1	.781**
	Sig. (2-tailed)		.001
	N	120	120
Math literacy	Pearson Correlation	.781**	1
	Sig. (2-tailed)	.001	
	N	120	120

**Correlation is significant at the .05 level (2-tailed).

Based on the output of Table 5, it is known that the sig (2-tailed) significance value is .001 < .05, which means there is a significant correlation between the variables of understanding language and mathematical symbols and the variable of mathematical literacy. Meanwhile, based on (Pearson Correction), the calculated r value was .781 > r table .179, and it was concluded that there was a significant correlation between the variables of understanding language and mathematical symbols and the variable of mathematical literacy. Because the calculated r (Pearson Correlation) is positive, it means that the relationship between the two variables is positive, or in other words, the greater the understanding of mathematical language and symbols, the greater the students' mathematical literacy.

Table 6. Description of statistics in ANOVA test

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	58.351	1	58.351	27.694	.001b
	Residual	28.683	119	2.865		
	Total	87.034	120			

a. Dependent Variable: Math Literacy
b. Predictors: (Constant), Language and Symbols Math

Table 6 shows that the calculated F-value is 27.694 with a probability value (Sig.) = .001 less than .05 which means that the linear regression model meets the linearity criteria.

Table 7. Independent variable regression coefficient

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
(Constant)	25.879	8.109		12.913	.000
Language and Symbols Math	.749	.556	.456	5.487	.001

a. Dependent Variable: Math Literacy

Based on the results of Table 7, the linear regression equation Y = 25.879 + .749X is obtained. In the model summary output, the values are R = .813 and R^2 = .661. This means that 66.1% of the variance that occurs in the mathematical literacy ability variable is influenced by the variable understanding of language and mathematical symbols and 33.9% is influenced by other factors outside the independent variable.

3.2. Discussion

Relating to the understanding of language and mathematical symbols by teachers, in general, teachers have good or high knowledge of language and mathematical symbolization (see Table 1). However, the understanding of mathematical symbols that are introduced to students is still limited. Teachers are of the opinion that not up to one hundred percent of knowledge can be transferred to students (R-Q4). This is because the introduction of mathematical symbols must be in accordance with the context and situation of mathematics being studied by students. Therefore, understanding more complex mathematical symbols is a challenge for teachers to add to learning. Given that the transition from symbolization and the language of mathematics to new, diverse concepts requires students' readiness to understand their meaning (OECD, 2018).

In Table 1, there are 8.75% of teachers who do not fully understand the use of mathematical symbols. Even though the percentage is small, if teachers misunderstand the use of mathematical symbols, they will pass on the wrong information to their students, leading to incorrect use in the future. It is therefore important to teach the proper use of mathematical symbols from the primary school level. We suggest that understanding meaning always precedes symbolization.

Relating to the understanding of mathematical language and symbols by students, based on the results of Table 2 and interview excerpts, it is revealed that students do not understand mathematical symbols because they are not familiar. They have tried to associate it with the mathematical concepts they have. However, mathematical symbols prevent students from understanding mathematical concepts so they cannot find solutions to problems. According to experts, trying to navigate mathematical symbols and their meanings is a complex process. Thus, this means that algebraic topics and rules must be manipulated into newer mathematical ideas (Edo & Tasik, 2022; Mutodi & Mosimege, 2021).

Each student can have different mathematical literacy abilities based on their learning experiences. As per the student's experience (S.07) when answering the researcher's question (R-Q9) about 5 x 12, it is indicated that the path to this goal starts from home and class, supported by family and community. The way teachers teach is also confirmed to influence students' abilities in mathematical literacy (R-Q1 and R-Q4). Teachers should teach well so that students gain conceptual understanding. This is the only way for students to be able to apply the mathematics they have learned in real life. In addition, school mathematics content should reflect relevance to social life (Gravemeijer et al., 2017).

In the case of solving questions point 1 and point 2, which is related to the equal sign "=", showing that students' understanding of each sign (symbol) will produce an interpretation for those who see it. In this case, it will allow students to have different visualizations of the signs (symbols) they see. Meanwhile, according to experts (Amirbostaghi et al., 2021; Bermejo et al., 2021; Esuong et al., 2023) reveal that different individuals can build different interpretations of the same sign (symbol) so that they can effectively create a sign. different for the same object. In the case of solving questions 3 and 4, which relate to pictures and diagrams, it was found that students still experienced difficulties when working through pictorial (visual) expressions that reflected the sequence. Meanwhile, previous researchers revealed that students experienced difficulties when working through symbolic expressions and arithmetic sentence equations that did not reflect the order of calculations (Chesney et al., 2018).

The humans (students) interpret mathematics by building their conceptions (Kooloos et al., 2022). Some students may not realize that problematic conceptions arise if the connection with previous understandings does not take context into account. Students often attach imprecise meanings to conceptions. The accumulation of problematic conceptions can hinder future mathematics learning. Based on the students' answer data there is evidence that the respondents' incorrect answers (S-16 and S-90 in Figures 1a and 2a) are caused by conceptions developed in other contexts but become a problem if applied without improvement and in a changing mathematical domain. Typically, mathematical symbols look the same but, in a new context, their meanings and sometimes their syntax have changed (Chin & Pierce, 2019).

Relating to the effect of understanding language and mathematical symbols on mathematical literacy skills, the relationship between the variables of understanding language and mathematical symbols to the mathematical literacy variables (see Table 5) shows a unidirectional relationship. The contribution of the variable understanding of language and mathematical symbols is .749 to the variable of mathematical literacy ability. The linear regression model meets the linearity criteria (see Table 6). If the understanding of language and mathematical symbols is higher, the ability of mathematical literacy will increase. Conversely, if the understanding of language and mathematical symbols is low, then the ability of mathematical literacy will be low.

In general, the mathematical literacy skills of class VII students are in the range of 25.00 – 100.00 with a range of 75.00 (see Table 3). This interval is in the moderate category when compared to the maximum score of 100. This data shows that there is a significant gap in mathematical literacy skills between students of class VII. This result is in accordance with the results of the 2015 PISA survey which shows that there is a very large difference in the abilities of students with the highest and lowest scores in a country, namely 300 points (equivalent to seven years of schooling) (OECD, 2018).

The student's answer to test item number 1 (see Figure 1a), shows that the student has a low ability to interpret mathematical ideas and compose mathematical models in writing. This is influenced because students do not understand the basic concept of algebraic multiplication. Students' ability to express mathematical ideas to others both in oral and written form was demonstrated by the student (S-07) who expressed mathematical ideas from practical experience in the kitchen. Student (S-07 can apply the multiplication concept correctly and correctly (see Figure 1b). At this level students (S-07) can understand symbolic transformations that change the form of expression without having to change the relationship of similarity which is symbolized by the equal sign "=" (Matthews & Fuchs, 2020).

The student's answer to test item number 2 (see Figure 2a), shows that the student has not been able to use mathematical symbolic communication skills to express everyday events in mathematical language. Students only managed to carry out the procedure and get the correct answer. The students were more creative in problem-solving methods than understanding concepts. Students carrying out their own procedures (see Figure 2b) only lead to short-term success. But they will fail to cope with more complex long-term tasks. Mathematical symbols are a basic form of mathematical communication. The students must understand the meaning of mathematical symbols correctly. Because, failure of students to understand mathematical symbols will hinder the achievement of learning objectives (Bardini & Pierce, 2015).

The student answers to test item number 3 can be seen in Figures 4a and 4b. The student's answer in Figure 4a shows that the student made an error. Students understand mathematical situations and linguistic situations but are not optimal in evaluating ideas (during the reflection process). Furthermore, test item number 4 measures students' ability to explain ideas, situations, and mathematical relationships in writing with real objects, or pictures. Students have understood the problem and have ideas for finding solutions (R-Q18). But in the third step, when carrying out the plan the student experiences an error. The student understands that the page must be closed, but the cover shifts the image (R-Q19).

Based on student answers in Figures 1 to 4 and Table 4 as well as interview support, we describe several student difficulties. Students experience difficulty in solving problems related to mathematical concepts. Students lack an understanding of mathematical literacy and gain experience in solving problems that require high-level reasoning. For students, studying abstract mathematical concepts is difficult. This shows that class VII students (11-12 years old) are just entering the level of formal thinking so they have not yet achieved strong abstract mathematical thinking (Kusmaryono et al., 2021). Meanwhile, it is found in R-Q5 (Teacher.05) that the teacher only encourages students to manipulate symbols without a proper conceptual foundation, thereby limiting students' progress to a higher understanding of mathematics. Therefore, the teacher needs to guide and foster students' mathematical symbolic development in a reasonable way to accommodate the cognitive demands of mathematics.

Relating to bridging the gap in understanding language and mathematical symbols between teachers and students, interview findings show that there is a pattern of errors in students' understanding of mathematical language and symbols and the error pattern is based on the teacher's conception of learning in previous classes. The concepts about language and symbols received by teachers in previous classes have formed false thinking structures (pseudo-false thinking) (Kusmaryono et al., 2020; Nizaruddin & Kusmaryono, 2023). Students do not fully understand that mathematical symbols should only relate to the nature of mathematical objects. Moreover, even if they use the symbols correctly, the reasons behind their use are often wrong.

The results of the interviews indicated that there were several reasons why students (S.20 and S.25) made mistakes in answering tests (R-Q18 and R-Q19), including (a) students' lack of accuracy in reading and writing mathematical symbols; and (b) students' lack of accuracy in interpreting problems into mathematical models. This is a problem related to mathematical literacy, namely understanding written and spoken mathematical language. Oral mathematical literacy skills are still weak (Tables 2 and 4), such as expressing something in words and discussing it with others. Students' skills in written mathematical language were also low (Tables 2 and 4). A skill that stands out is that students can convey mathematical ideas in the form of tables, graphs, or equations (Tables 2 and 4).

The data in Table 2 shows that the teacher's ability to understand mathematical language and symbols in terms of the multi-semiotic system is in the high category. However (see Table 1), students' understanding ability of mathematical language and symbols does not exceed or is still far below the teacher's understanding ability. We can see a comparison of the percentage data in Table 1 and Table 2 which are very different and inversely related. A good teacher's ability (high level) in understanding language and mathematical symbols does not guarantee that students' abilities are also high, because this is influenced by several factors in the learning process (Mukuka et al., 2023).

The results of interviews with teachers and students also indicated that the teacher's competence (knowledge) which was transferred to students never exceeded the competence possessed by the teacher. This shows that teacher competence is actualized in learning mathematics in the classroom, where the teacher is a learning model for students, so their success is reflected in students' mathematics learning outcomes (Kliziene et al., 2022).

Based on this analysis, we confirm the notion that there is a gap in the ability to understand language and mathematical symbols between teachers and students.

The discrepancies occur when students have little experience with this process, for example when students are working on math test items about mathematical concepts (test items number 1 and 2), where students need to translate into an understanding of the language of mathematics. The results of the data in Table 3 show that students are familiar with test items where the mathematical concepts are clear such as graphic or table presentations and students do not need effort to first identify the mathematical concepts in these test items.

Discrepancies also arise when solving story problems, especially about understanding mathematical language and natural language. Our main argument is that there needs to be a bridge between the language of mathematics which requires looking at components of mathematics, and natural language which demands textual literacy for the text as a whole. In other words, there is a bridge between the mathematical component and the literal component. When knowledge gaps in the language of mathematics are large, natural language must provide what is missing, and in a clear and explicit manner. However, when the knowledge gap in the language of mathematics is small, natural language need not provide what is missing. So, it can be said that the process of transitioning the teacher's knowledge to students' understanding of mathematics is experiencing obstacles.

This transition process of meaning from mathematical symbols to written and spoken language must be carried out when the teacher introduces or teaches a new topic and the context in which mathematical symbols are used must be followed by clarification. For example, in the case of test item number 2, the transition from understanding comparisons to algebraic equations requires students to have a strong ability to interpret symbols. Therefore, teachers need to prioritize guiding students on how students construct the meaning of symbols as mathematical concepts and algebraic thinking processes.

Understanding mathematical language and symbols is not only useful when learning mathematics, but mathematical language and symbols support technological development. The application of mathematical logic in computer science involves a lot of mathematical language and symbols as a basis for learning programming languages, data structures, artificial intelligence, and databases. In the medical and pharmaceutical fields, it is used as a tool to measure a patient's body temperature and what dosage of medication should be used. Even in the fields of construction, mechanical, and electrical engineering, all use mathematical language and symbol systems. So, there is no doubt that the language and symbols (science) of mathematics have a very important role in the progress of science and technology. Therefore, from the beginning of learning mathematics, students must be strengthened with an understanding of mathematical language and symbols that are useful in solving problems in the world of work in the future.

4. CONCLUSION

Quantitative data analysis shows that there is a linear (significant) relationship between understanding language and mathematical symbols on mathematical literacy abilities. Qualitative data analysis describes the teacher's understanding of mathematical language and symbols (high criteria) but does not necessarily support the student's understanding of mathematical language and symbols. We confirm the suspicion that there is a gap in the ability to understand language and mathematical symbols between teachers and students. There is a pattern of errors in students' understanding of mathematical language and symbols. Error patterns are based on the teacher's conception of learning in previous classes so the process of transitioning teacher knowledge to students' mathematical understanding experiences obstacles. The implications of this research are (a) the transition process of meaning from mathematical symbols to written and spoken language must be carried out when teachers introduce or teach new topics to students and the context in which mathematical symbols are used must be followed by clarification; (b) before mathematics learning progresses to higher level thinking, teachers must ensure that students are able to use and manipulate mathematical concepts and understand the meaning of assigned symbols, (c) teachers need to guide and foster students' mathematical symbolic development in a reasonable manner to accommodate cognitive demands mathematics, and (d) educational practitioners and researchers need to see the importance of integrating meaning across various semiotic modes in mathematics classroom discourse. Considering the mutual contextualization and joint application of language, gestures, and visual representations can greatly change how actions emerge.

The limitation of this research is that the number of samples (respondents) is still considered small (20 teachers and 120 students), so it is considered a research limitation which results in doubts about generalizing. Although some researchers consider that the number of respondents in experimental research is between 30 and 50 and more than 100 samples in survey research are considered large enough samples (Delice, 2010; Memon et al., 2020). Analysis of this research data is certainly limited because it is based on cross-sectional data. This analysis cannot definitively conclude that there is a cause-and-effect relationship between the variables. In future research, other researchers need to see whether the above findings can be replicated using different data and analysis approaches.

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