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# Mathematics teachers' practices in light of the effective teaching practices

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## Abstract

Recognising the pivotal role of effective teaching practices in enhancing student learning outcomes, the National Council of Teachers of Mathematics (NCTM) has outlined a comprehensive framework of key strategies. However, empirical research on these practices remains limited. Therefore, this study aimed to investigate the extent to which mathematics teachers apply NCTM-identified effective teaching practices and examine potential differences attributable to teachers' gender and teaching experience. Adopting a quantitative approach and employing a descriptive survey, the study systematically assessed eight designated practices using a validated observation checklist in the Jazan Education Administration, observing 38 mathematics teachers during routine classes. The results indicated a moderate level of implementation overall, with ten practices demonstrating high application. Statistical analyses found no significant differences in practice implementation based on gender or teaching experience at the .05 level. The limitations included the study's relatively small sample size, potentially limiting generalisability, and the observational nature of the checklist, which may not fully capture nuanced teaching dynamics. Future research could focus on developing training initiatives to enhance educators' application of NCTM-recommended practices and undertake qualitative inquiries to understand barriers to integration and teachers' attitudes towards these practices. Finally, this study contributes to the discourse on effective mathematics teaching by providing empirical insights into current practices and highlighting avenues for further investigation and professional development in educational settings.

**Keywords**:
Effective teaching, Mathematics teachers, NCTM, Teaching practices

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## 1. INTRODUCTION

Multiple factors contribute to efforts aimed at enhancing educational environments across various systems (Barrenechea et al., 2023), with the teacher's role being especially significant. In mathematics education, teachers' reliance on traditional teaching methods, an emphasis on lower-level cognitive skills, and a disconnection from students' everyday experiences have been identified as key factors leading to low achievement and negative attitudes towards the subject (Al-Harbi & Al-Ruwais, 2022; Wakhata et al., 2024; Yin et al., 2020). Consequently, initiatives to improve educational settings stress the pivotal role of teachers in addressing these issues, highlighting the need to shift away from traditional methodologies and better integrate students' daily experiences into the learning process.

Mathematics students' low achievement and underperformance in international assessments has been attributed to ineffective teaching practices stemming from mathematics teachers' inadequate grasp of mathematical content and teaching methodologies (Gningue et al., 2013). Furthermore, Walkowiak et al. (2014) found that deficiencies in teacher practices account for approximately 12–14% of students' low mathematical achievement. On the other hand, Al-Shammari and Al-Arini (2019) also underscored the substantial influence of teachers' instructional performance on students' learning outcomes, attributing 60% of the success of the educational process to this factor and thereby emphasising the pivotal role of teachers in any educational reform initiative. Therefore, teachers' inadequate knowledge and practices significantly affect students' mathematical achievement and the success of educational reform.

To aid learning, Wernet et al. (2015) emphasised the importance of designing mathematical tasks and activities that are realistic and contextualised, fostering students' mathematical thinking and understanding of the practical application of mathematics. Such tasks should provide opportunities for students to deepen their mathematical comprehension, appreciate the potency of mathematics and recognise its relevance to real-world scenarios. Furthermore, the design of tasks that integrate interdependent activities, logical analysis, and both abstract and contextual elements aims to enhance problem-solving skills by emphasizing the importance of linking learning to real-life contexts and the external environment (Prahmana et al., 2024; Voon et al., 2022). Additionally, the significance of nurturing students' critical thinking skills, respecting individual differences and affording students ample time to engage in independent or collaborative problem-solving tasks. Designing realistic, contextualised mathematical tasks fosters critical thinking, problem-solving skills and understanding of the practical applications of mathematics.

Recognising the pivotal role of effective teaching practices in enhancing student learning outcomes, the National Council of Teachers of Mathematics (NCTM, 2014) outlined a comprehensive framework encompassing several core strategies. These include establishing clear mathematics goals to guide learning endeavours, employing tasks designed to foster reasoning and problem-solving skills, utilising and interconnecting various mathematical representations, facilitating meaningful mathematical discourse within the classroom, posing purposeful questions to stimulate critical thinking, fostering procedural fluency grounded in conceptual understanding, supporting students' productive engagement with challenging mathematical concepts, and eliciting and leveraging evidence

of student thinking to inform instructional decisions. Collectively, these practices form a robust foundation for promoting deeper conceptual understanding and proficiency in mathematics among learners.

It is necessary to establish clear learning objectives and leverage all available resources to discern appropriate educational pathways that can foster effective mathematics learning (NCTM, 2014). This entails elucidating learning trajectories and delineating specific mathematics objectives tailored to students' needs. Another crucial facet of mathematics instruction involves strategically selecting and deploying tasks that cultivate reasoning abilities and problem-solving abilities. Developing effective mathematical tasks involves adhering to several guiding principles, such as promoting student interaction, integrating social and cultural contexts, generating student interest, using accessible language, and fostering active engagement among both teachers and students (Schoenfeld, 2022). Such tasks are pivotal in honing students' mathematical thinking abilities and deepening their comprehension of mathematical concepts and procedures (Lithner, 2017). Figure 1 illustrates the symbiotic relationship between mathematics tasks and learning outcomes.

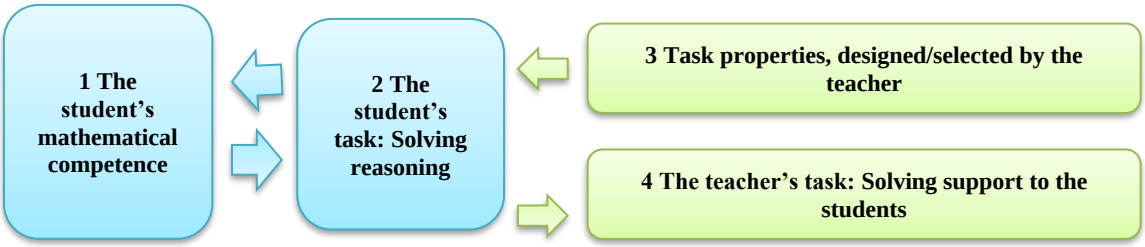


**Figure 1**. The relationship between mathematical tasks and learning (Lithner, 2017)

Mathematical representations play a crucial role as tools for communication and cognitive processing in mathematics education (NCTM, 2014). They aid students in recognising fundamental mathematical elements across diverse contexts, thereby enriching their comprehension of mathematical concepts and procedures (Anwar et al., 2016; NCTM, 2014; Sari & Karyati, 2022). Moreover, these representations facilitate the clarification of relationships between mathematical concepts and empower students to apply their mathematical knowledge in real-world problem-solving scenarios. Another pivotal aspect of effective mathematics instruction lies in fostering meaningful mathematical dialogue among students, which enables educators to cultivate a shared understanding of mathematical concepts and deepen comprehension of mathematical relationships (Ballard, 2017; NCTM, 2014). Grounded in social constructivist theory, meaningful mathematical dialogue supports students in bridging prior knowledge and new information, thereby facilitating a holistic grasp of mathematical concepts (Anderson-Pence, 2017).

Purposeful questioning is a principal teaching practice in mathematics education, enabling educators to gauge and enhance students' reasoning and sense-making abilities concerning critical mathematical concepts and relationships (NCTM, 2014). The quality of questions posed in the classroom significantly influences students' cognitive engagement, with well-crafted questions spanning various cognitive domains such as analysis, synthesis

and evaluation, fostering profound understanding and involvement (McCarthy et al., 2016). Moreover, building procedural fluency from a foundation of conceptual understanding as a fundamental tenet of effective mathematics instruction. Conceptual understanding is the bedrock of procedural fluency and the two mutually reinforce each other (NCTM, 2014; Rittle-Johnson et al., 2015). While the precise interplay between conceptual understanding and procedural fluency remains debatable, evidence suggests that advancements in one domain often yield improvements in the other, underscoring their interconnectedness and reciprocal influence (Rittle-Johnson et al., 2015). Consequently, effective mathematics instruction involves enabling educators to assess and enhance students' reasoning and sense-making abilities, utilizing well-crafted questions across cognitive domains to promote deep understanding and active engagement, while emphasizing the interconnectedness and mutual reinforcement between conceptual understanding and procedural fluency.

Supporting productive struggle in mathematics learning is paramount as a means of fostering students' growth and development. As defined by Hiebert and Grouws (2007), productive struggle entails students' structured efforts to comprehend mathematics and uncover underlying insights. It is a process that demands both time and emotional investment from students. Cultivating an environment conducive to productive struggle involves offering opportunities for student engagement, embracing mistakes as integral to the learning process and guiding students in uncovering and learning from their errors (Khalil et al., 2023).

Furthermore, effective mathematics instruction entails eliciting and leveraging evidence of student thinking. By actively soliciting students' thoughts and reasoning, educators can gauge their progress and adjust instruction to support and extend their learning (NCTM, 2014). The ability to interpret students' ideas and reasoning is pivotal in understanding their cognitive processes and making informed instructional decisions (Shaughnessy & Boerst, 2018). Moreover, reasoning and interpreting ideas are essential skills for formative assessment, which can profoundly influence student learning outcomes (Wiliam, 2009). Eliciting and interpreting students' thinking constitute indispensable elements of effective mathematics instruction, contributing significantly to the cultivation and responsiveness of student understanding (Gupta et al., 2018; Shaughnessy & Boerst, 2018). Therefore, effective mathematics instruction involves actively eliciting and interpreting student thinking to gauge progress, inform instructional adjustments, and foster formative assessment practices that enhance learning outcomes and cultivate student understanding.

One compelling rationale for promoting effective teaching practices is the evolution of modern mathematics curricula, which increasingly incorporate activities and problems necessitating higher-order thinking skills, such as analysis, discussion, comparison, justification, evaluation, and the articulation of multiple written ideas and solutions (Khalil et al., 2023). Numerous studies have underscored the criticality of equipping mathematics teachers with practical skills. For instance, effective teachers must skilfully employ a variety of teaching practices, such as fostering dialogue and discussion among students to provide multiple opportunities for expression, enhancing students' articulation of their problem-solving rationale, cultivating an appreciation for peers' ideas, and guiding students in realistic

reasoning processes (Pepkolaj et al., 2024; Shahrill et al., 2023). Courtney and Caniglia (2021) emphasised the significance of offering practical pre- and in-service training to mathematics educators on effective teaching methodologies to bolster students' conceptual understanding, skills and problem-solving approaches in mathematics. Similarly, Ash-Shalhoub et al. (2022) advocated providing training programmes rooted in the eight practices of effective teaching delineated by the NCTM, highlighting the imperative for mathematics teachers to embrace and implement these practices.

The Achievement Network's (2022) “Learning Together” initiative proposes numerous strategies for mathematics educators to implement in their classrooms. First, teachers are encouraged to foster classroom discussions to enable students to construct their own understanding through dialogue, explanation, justification and critical thinking. Before making abstract presentations, educators should prioritise ensuring students' conceptual understanding, using diagrams, visual models and other forms of representation. In addition, teachers need to identify and address common conceptual errors among students. To cultivate procedural fluency, instructors are advised to establish connections between mathematical concepts and solution algorithms, while also guiding students to justify their solution steps and providing constructive feedback. Moreover, educators should create opportunities for students to share their problem-solving approaches, explanations and justifications, whether individually or collaboratively, to promote a comprehensive understanding of mathematical concepts and processes.

The topic of effective teaching practices has gained significant attention from researchers, with numerous studies investigating their implementation by male and female mathematics educators. Ar-Rashidi (2021) examined the use of effective teaching practices, including evaluation, clarity of instruction and classroom management, among middle school mathematics teachers in Hail, Saudi Arabia. Furthermore, integrating student struggle into tasks and instructional practices, rather than avoiding it, can play a productive role in fostering mathematical engagement and enhancing learning with comprehension (Warshauer, 2015). On the other hand, Grosser-Clarkson and Neel (2020) focused on training pre-service mathematics teachers in diverse, effective teaching methods, such as stimulating mathematical thinking, posing meaningful questions and facilitating mathematical communication. Finally, Al-Imam et al. (2022) presented a procedural approach to fostering meaningful classroom dialogue, both orally and in writing, to enhance students' mathematical comprehension.

Khalil et al. (2023) introduced an educational programme grounded in constructivist theory to bolster mathematical achievement and support productive struggle among middle school students. Finally, Mohamed et al. (2023) reported a widespread awareness of effective teaching practices outlined by the NCTM among mathematics educators in Saudi Arabia and Egypt, with no discernible differences in awareness levels among Saudi teachers based on various demographic variables, such as qualifications, teaching experience or school stage. Therefore, previous research has highlighted the need for further investigation into mathematics teachers' practices in the Kingdom of Saudi Arabia and the identification of suitable avenues for development, particularly concerning the eight effective teaching practices outlined by the NCTM. Therefore, this study aimed to respond to this call by

examining mathematics educators' practices in the region and exploring avenues for enhancement in line with the principles delineated by the NCTM

### 1.1. Study Problem

Despite the Kingdom of Saudi Arabia's implementation of the "Development of Mathematics and Natural Sciences" project, which is based on translating and adapting the McGraw Hill series and the accompanying plan for the professional development of mathematics teachers, many studies, including that of Al-Qurashi (2021), have identified poor teaching practices among mathematics teachers, suggesting that professional development programmes have not been effective in helping teachers develop their teaching practices. This calls for a shift in training to focus on effective teaching practices, especially given the recommendation made by many conferences to pay attention to teacher preparation and professional development with a view to enhancing practice and performance. These include the Conference on Teacher Preparation and Training in Light of Development Demands and Recent Advances, held at Umm Al-Qura University in 2016, and the Conference on Teachers and the Age of Knowledge: Opportunities and Challenges, themed "A Teacher Renewed for a Changing World", which was held at King Khalid University, also in 2016. Several studies have also highlighted the importance of developing mathematics teachers' teaching practices in line with recent trends (see, e.g., Al-Harbi & Al-Ruwais, 2022; Al-Otaibi & Al-Matham, 2022; Ar-Rashidi, 2021).

Several researchers have argued the need to provide teachers with greater support and opportunities to incorporate challenging problem-solving tasks into their classrooms (Ash-Shalhoub et al., 2022; Fredagsvik, 2023; Khalil, 2024; McCormick, 2018). They have also pointed to the importance of enhancing teachers' knowledge and skills to enable them to plan and facilitate mathematical discussions effectively with students, as well as noting that training on selecting and designing mathematical tasks that offer ample opportunities for students to engage in critical thinking and develop proofs is required in teacher training programmes. These studies have identified students' perceptions of their abilities as a significant obstacle to integrating challenging tasks. Hence, there is a call for further research to evaluate teaching practices that foster productive struggle (Khalil, 2024) and to assess the extent of mathematics teachers' proficiency in the eight practices that support all students in learning mathematics (Ash-Shalhoub et al., 2022).

To assess the current situation in the Kingdom of Saudi Arabia, a pilot study was conducted involving interviews with mathematics teachers to evaluate their understanding of trends in mathematics education, particularly focusing on the eight practices of effective teaching, and to measure the extent to which these practices were integrated into their instructional methods. The interviews highlighted a perceived lack of knowledge regarding contemporary trends and identified the need for intentional incorporation of these practices in mathematics teaching. The results of the pilot study contributed to the development of a checklist comprising observable criteria for each of the eight practices of effective teaching outlined by the NCTM (2014), drawing on constructivist principles and used to assess the application of these practices within the classroom setting.

### 1.2. Research Questions

The study sought to answer the following two questions: (1) To what extent do mathematics teachers apply effective teaching practices in the classroom? and (2) Are there statistically significant differences at the 0.05 level in mathematics teachers' application of effective teaching practices attributable to gender and/or teaching experience?

### 1.3. Study Contributions

This study makes several significant contributions, both theoretical and practical, as follows: (1) It provides a list of actions representing the effective teaching practices defined by the NCTM (2014); (2) It develops an observation tool for evaluating mathematics teachers' performance consistent with the latest trends in the field of teaching and learning mathematics; and (3) It provides feedback to officials in the Ministry of Education on mathematics teachers' practices and performance.

### 1.4. Study Scope

The study scope entailed identifying the level of application of effective teaching practices as defined by the NCTM (2014) by mathematics teachers in public schools in the Jazan region in Saudi Arabia. The study was conducted in the third semester of the academic year 2022.

## 2. METHOD

This quantitative study employed a descriptive survey, consistent with the research aims, to identify the extent of mathematics teachers' application of effective teaching practices and any statistical differences attributable to gender or teaching experience. The study used an observation checklist to assess the mathematics teachers' level of application of effective teaching practices.

Having developed the observation checklist, authorisation to visit schools and observe teachers was obtained from the Jazan Education Administration. Following this, an online questionnaire was administered with collaboration from the Mathematics Department, inviting teachers to participate voluntarily in the study.

### 2.1. Research Sample

The study population comprised teachers of mathematics within the Jazan region in Saudi Arabia, which is affiliated with the Jazan Education Administration (Samta Office of Education), in the academic year 2022–2023. The sample consisted of 38 teachers, representing 10% of the mathematics teachers in the Samta Office of Education, selected through random sampling. Table 1 shows the sample distribution.

**Table 1**. Number of respondents and distribution according to gender and teaching experience

| Variable | Category | N | Percentage |
|---|---|---|---|
| Gender | Male | 28 | 74 |
| | Female | 10 | 26 |
| Teaching Experience | Less than 10 years | 11 | 29 |
| | More than 10 years | 27 | 71 |

### 2.2. Research Instruments

The study used an observation checklist to identify mathematics teachers' level of application of effective teaching practices. The development of the checklist comprised several steps, as follows:

1) Reading the educational literature and previous studies that dealt with effective teaching practices, including Baker et al. (2020) and Al-Khidr and Ar-Rasheedi (2021).
2) Determining a set of observable actions for the checklist, representing each dimension of the eight effective teaching practices.
3) Presenting the observation checklist to a group of 12 experts in the field of mathematics education.
4) Writing the final form of the checklist.

The final version of the observation checklist comprised the following dimensions:

1) Practice 1. Establishing mathematics goals to focus learning (four actions).
2) Practice 2. Implementing tasks that promote reasoning and problem-solving (three actions).
3) Practice 3. Using and connecting mathematical representations (four actions).
4) Practice 4. Facilitating meaningful mathematical dialogue (four actions).
5) Practice 5. Posing purposeful questions (four actions).
6) Practice 6. Building procedural fluency from conceptual understanding (four actions).
7) Practice 7. Supporting productive struggle in learning mathematics (four actions).
8) Practice 8. Eliciting and using evidence of students' thinking (four actions).

### 2.3. Validity and Reliability

To establish the content validity of the observation checklist, a panel of 10 experts reviewed and evaluated each item. The item-level content validity index (I-CVI) for the items ranged from 0.85 to 1.0, indicating a high level of agreement among the experts. Furthermore, the scale-level content validity index (S-CVI) for the entire checklist was 0.95, suggesting excellent overall content validity. The S-CVI was computed by first determining the I-CVI for each individual item on the checklist, as recommended by Polit et al. (2007). For example, for the first item, 9 out of the 10 experts agreed on its relevance, resulting in an I-CVI of 0.90 for that item. This process was repeated for all the items and the mean I-CVI value across all items was then calculated to arrive at the final S-CVI of 0.95 for the entire observation checklist.

To assess the reliability of the observation instrument, the study employed an exploratory sample comprising 12 male and female teachers from outside the main study sample. First, two observers evaluated the implementation of the teaching practices by four teachers from the exploratory sample and the level of agreement was calculated, resulting in a high agreement rate of 91%. Second, reliability was determined using Cronbach's alpha coefficient, which demonstrated a 96% agreement level, indicating high reliability. Table 2 presents the reliability coefficients calculated for each of the study dimensions.

**Table 2**. Cronbach's alpha coefficients for the eight observation practice dimensions

| No | Practice | Cronbach's α |
|---|---|---|
| 1 | Establishing mathematics goals to focus learning | 0.752 |
| 2 | Implementing tasks that promote reasoning and problem-solving | 0.779 |
| 3 | Using and connecting mathematical representations | 0.770 |
| 4 | Facilitating meaningful mathematical dialogue | 0.784 |
| 5 | Posing purposeful questions | 0.815 |
| 6 | Building procedural fluency from conceptual understanding | 0.717 |
| 7 | Supporting productive struggle in learning mathematics | 0.749 |
| 8 | Eliciting and using evidence of students' thinking | 0.890 |
| | Overall | 0.96 |

As illustrated in Table 2, the values fall within the range of 0.717–0.890, indicating acceptable to good reliability. The observers used a three-point scale to rate teachers' performance in employing actions related to each of the eight practice dimensions: 1 = low, 2 = medium and 3 = high. The distribution of ratings for the observation checklist is presented in Table 3.

**Table 3**. Distribution of rating categories for the observation tool

| Level of Implementation | Range of Means |
|---|---|
| High | 2.34–3 |
| Medium | 1.67–2.33 |
| Low | 1–1.66 |

## 3. RESULTS AND DISCUSSION

### 3.1. Results

#### *3.1.1. Mathematics Teachers' Implementation of Effective Teaching Practices*

To address RQ1, concerning the extent to which mathematics teachers apply effective teaching practices in the classroom, the study calculated the means and standard deviations for each practice related to the eight dimensions. Table 4 presents the results, indicating the overall performance of mathematics teachers in implementing effective teaching practices.

**Table 4**. Means and standard deviations for mathematics teachers' actions related to effective teaching practices

| Practice Dimension | Actions | *M* | *SD* | Level | Overall Mean and Level |
|---|---|---|---|---|---|
| Establishing mathematics goals to focus learning | Clarifies the goals of the lesson for students at the beginning of the session to understand what needs to be learned from the mathematical content | 2.51 | 0.64 | High | 2.23 Medium |
| | Diversifies lesson goals to deepen students' mathematical knowledge | 2.18 | 0.68 | Medium | |
| | Relates the goals of the lesson to the daily lives of students and to other subjects | 2.26 | 0.72 | Medium | |

| Practice Dimension | Actions | *M* | *SD* | Level | Overall Mean and Level |
|---|---|---|---|---|---|
| | Encourages students to reflect on the degree of achievement of learning goals as they progress through the lesson | 1.97 | 0.81 | Medium | |
| Implementing tasks that promote reasoning and problem-solving | Designs mathematical tasks and activities that enhance students' problem-solving skills | 2.10 | 0.72 | Medium | 2.08 Medium |
| | Designs mathematical activities that enhance students' ability to model real-life problems using mathematical knowledge | 1.90 | 0.85 | Medium | |
| | Trains students to choose the appropriate strategy to solve a mathematical problem | 2.05 | 0.79 | Medium | |
| Using and connecting mathematical representations | Designs mathematical activities and situations that require students to use various mathematical representations to solve them | 1.80 | 0.73 | Medium | 1.90 Medium |
| | Highlights integration and interdependence between mathematical knowledge through conceptual maps and the use of various visual and electronic models | 1.92 | 0.74 | | |
| | Assesses students' ability to meaningfully use various mathematical representations to solve life problems and to organise their mathematical thinking | 1.85 | 0.63 | | |
| | Encourages students to infer mathematical knowledge from multiple representations and connect them to infer new combinations | 2.03 | 0.78 | | |
| Facilitating meaningful mathematical dialogue | Conducts meaningful dialogues and discussions in the mathematics classroom | 2.46 | 0.64 | High | 2.46 High |
| | Encourages students to use the language of mathematics to present and explain their thinking and various solution methods and strategies | 2.59 | 0.50 | High | |
| | Encourages students to extract common mathematical concepts and the resulting multiple solution procedures through meaningful dialogue among them | 2.33 | 0.70 | Medium | |
| | Motivates students to justify their answers while solving problems | 2.44 | 0.72 | High | |
| Posing purposeful questions | Asks questions that guide and support students' thinking | 2.59 | 0.55 | High | 2.49 High |
| | Varies class questions to build meaning of a range of mathematical knowledge and relationships | 2.49 | 0.51 | High | |
| | Poses purposeful questions that make math lessons more meaningful | 2.56 | 0.60 | High | |

| Practice Dimension | Actions | *M* | *SD* | Level | Overall Mean and Level |
|---|---|---|---|---|---|
| | Poses classroom questions that motivate students to infer mathematical relationships | 2.31 | 0.80 | Medium | |
| Building procedural fluency from conceptual understanding | Gives students enough time to think about different ways of solving mathematical problems | 2.29 | 0.56 | Medium | 2.27 Medium |
| | Encourages students to use a variety of methods to solve mathematical tasks and problems and justify their choices | 2.18 | 0.61 | Medium | |
| | Gives examples and non-examples of mathematical concepts to achieve conceptual comprehension among students | 2.45 | 0.72 | High | |
| | Balances conceptual knowledge and procedural knowledge while explaining | 2.16 | 0.75 | Medium | |
| Supporting productive struggle in learning mathematics | Gives students ample time to work hard and try to solve problems while solving mathematical tasks | 2.46 | 0.60 | High | 2.24 Medium |
| | Encourages students to reflect on and evaluate their own thinking and solution strategies | 2.10 | 0.79 | Medium | |
| | Supports students to overcome the educational challenges they face while learning mathematical content | 2.13 | 0.77 | Medium | |
| | Provides students with opportunities to make an effort to acquire mathematical knowledge and think deeply to solve problems on their own | 2.26 | 0.68 | Medium | |
| Eliciting and using evidence of students' thinking | Defines the indicators by which students' progress toward learning goals can be judged | 1.82 | 0.85 | Medium | 1.87 Medium |
| | Collects evidence about students' understanding of mathematical content during learning to plan next educational steps | 1.85 | 0.84 | Medium | |
| | Seeks to identify the level of students and evaluate their ways of thinking to make appropriate future decisions | 1.87 | 0.69 | Medium | |
| | Uses a variety of assessment methods to ensure that students have mathematical thinking skills | 1.95 | 0.79 | Medium | |
| Overall Mean | 2.17 | | | Medium | |

For the first practice dimension, "Establishing mathematics goals to focus learning", the teachers' application was medium overall, except for one action: "Clarifies the goals of the lesson for students at the beginning of the session to understand what needs to be learned from the mathematical content", which achieved a high level (M = 2.51). This suggests that while teachers generally set clear learning goals, there is room for improvement in consistently communicating these objectives to students.

The second, third and eighth practice dimensions, namely "Implementing tasks that promote reasoning and problem-solving", "Using and connecting mathematical representations" and "Eliciting and using evidence of students' thinking", respectively, exhibited medium levels of implementation for all individual actions and thus overall. This indicates that teachers may be struggling to design tasks that optimally foster reasoning and problem-solving, effectively utilise and interconnect multiple representations, and systematically gather and leverage evidence of student thinking.

In contrast, for the fourth practice dimension, "Facilitating meaningful mathematical dialogue", and the fifth practice dimension, "Posing purposeful questions", overall implementation was high. Most individual actions for these dimensions also achieved high means, except for one action each, which exhibited medium performance. This suggests that teachers excel at creating opportunities for meaningful mathematical discourse and posing purposeful questions to stimulate critical thinking.

The overall means for the sixth practice dimension, "Building procedural fluency from conceptual understanding", and the seventh dimension, "Supporting productive struggle in learning mathematics", indicated a medium level of application. Only select actions within these dimensions achieved high levels. This indicates that teachers may be less adept at fostering procedural fluency grounded in conceptual understanding and providing the appropriate level of support for students to engage productively with challenging mathematical concepts.

Overall, while the study identified a predominantly medium level of implementation across various practice dimensions, several actions stood out with high levels, highlighting areas of strength among mathematics teachers in terms of effective teaching practices. This points to potential areas for targeted professional development and support to enhance teachers' holistic application of the NCTM framework and drive more consistent and comprehensive implementation of these research-based strategies.

### *3.1.2. Differences in the Level of Implementation of Effective Teaching Practices According to Gender and Teaching Experience*

To address RQ2, concerning potential differences in the application of effective teaching practices among mathematics teachers based on gender and/or teaching experience, the study employed the Mann–Whitney U test. Table 5 displays the results, indicating that there were no statistically significant differences at the 0.05 level in the implementation of effective teaching practices among mathematics teachers attributed to either gender or teaching experience.

**Table 5**. Independent samples Mann–Whitney U test of differences according to gender and teaching experience

| Variable | Category | No | *M* | Total | *U*-value | Sig. | Statistical Significance |
|---|---|---|---|---|---|---|---|
| Gender | Male | 28 | 21.25 | 595.00 | 91.00 | 0.109 | Not significant |
| | Female | 10 | 14.60 | 146.00 | | | |
| Teaching experience | Less than 10 years | 11 | 20.77 | 228.5 | 134.50 | 0.657 | Not significant |
| | More than 10 years | 27 | 18.98 | 512.50 | | | |

### 3.2. Discussion

This study aimed to investigate mathematics teachers' practices to identify the extent of implementation of the practical teaching strategies advocated by the United States NCTM (2014) in the classroom and explore potential differences in teaching practices based on gender or teaching experience. The study was motivated by the Kingdom of Saudi Arabia's endeavours to enhance mathematics education and improve its performance in international assessments, such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). The country has invested significant efforts in designing teacher development programs to bolster teaching practices.

The results indicated that the implementation of effective teaching practices by mathematics teachers was predominantly at the medium level, although specific observable actions reached a high level. Regarding differences in teaching practices based on gender or teaching experience, no statistically significant disparities were found at the 0.05 level. This lack of differentiation could be attributed to the standardised in-service training programmes for all mathematics teachers in Saudi Arabia, reflected in their teaching practices.

The observation that teachers implemented practices to a largely medium extent may be due to the alignment of mathematics curricula in Saudi Arabia with the NCTM standards, which incorporate associated ideas and principles. Therefore, mathematics teachers may naturally integrate some practices congruent with these standards, either intentionally or as part of following the lesson structure given in the curriculum. This finding resonates with Hull et al.'s (2014) study, underlining the importance of mathematics teachers engaging in effective teaching practices.

The findings of this study align with several previous studies regarding the levels of effective teaching practices identified (e.g. Al-Maliki & As-Salouli, 2018; Ar-Rashidi, 2021; As-Salahi, 2019), although there are variations for several reasons. For instance, As-Salahi's (2019) study focused on mathematics supervisors rather than mathematics teachers, whereas this study directly assessed the performance of mathematics teachers in the classroom. Furthermore, while this study examined teaching practices broadly based on the NCTM's (2014) Effective Teaching Standards, As-Salahi's (2019) study concentrated solely on practices supporting students' understanding of mathematical concepts.

Similarly, this study and Ar-Rashidi's (2021) study shared the goal of evaluating mathematics teachers' implementation of effective practices using an observation checklist, with some results coinciding regarding evaluation practices, clarity of teaching, classroom management and teaching skills. However, discrepancies emerged, particularly in terms of practices related to the classroom environment and differentiated teaching. Zohar and Ben-Ari (2022) found that while specific characteristics of teachers' knowledge and learning processes are identified, metacognition is seldom satisfactorily addressed in large-scale efforts to teach higher-order thinking, and meta-strategic knowledge is largely neglected in professional development programs for teaching inquiry learning.

From a thorough analysis across each dimension of the teaching practices, it is evident that there was a high level of implementation of the practice of posing purposeful questions. This finding is consistent with the studies conducted by both Al-Otaibi and Al-Matham (2022) and Wilburne et al. (2018), which noted that the ability to pose purposeful

questions is an important part of teachers' practices, stimulating students' thinking and providing opportunities for them to explain and justify their solutions. Similarly, teaching practices that facilitate meaningful mathematical dialogue also received high scores. This is consistent with Grosser-Clarkson and Neel (2020) findings, which identified a high level of proficiency among mathematics teachers in fostering meaningful mathematical dialogue. Additionally, Al-Otaibi and Al-Matham (2022) documented a high level of proficiency in classroom dialogue skills aligned with the NCTM (2014) standards among mathematics educators in Saudi Arabia. However, Al-Maliki and As-Salouli's (2018) study rated classroom discussion and dialogue activities as moderate.

The high level of implementation of these practices among mathematics teachers in posing purposeful questions and facilitating meaningful mathematical dialogue further corroborates the findings of Al-Olayan (2022) and As-Surayhi (2022). This can be attributed to the emphasis placed on mathematical communication skills in mathematics textbooks in Saudi Arabia, which encourages teachers to prioritise these skills and integrate them into their teaching practices. Furthermore, Saudi schools implement active learning programmes that emphasise the importance of fostering dialogue and discussion, as well as posing purposeful questions, to stimulate students' thinking in mathematics classrooms.

This study also revealed a medium level of teaching practices aimed at building procedural fluency from conceptual understanding. This finding is consistent with the results of Al-Qurashi (2021) and As-Salahi (2019), but not with those of Al-Shammari and Al-Arini (2019) or Al-Otaibi and Al-Matham (2022). The discrepancy may arise from teachers prioritising practices geared towards training students to solve mathematical tasks by following specific steps to achieve accurate results.

Teaching practices designed to support productive struggle in learning mathematics also received a medium score in this study, aligning with the findings of Al-Otaibi and Al-Matham (2022) and Wilburne et al. (2018). These studies demonstrated that mathematics teachers can identify procedures related to practices supporting productive struggle in learning mathematics. In Saudi Arabia, mathematics educators undergo training to develop remedial plans to assist low-achieving students in overcoming learning challenges related to mathematical content. Additionally, they adopt strategies such as "the young teacher" approach to encourage students to help their peers reflect on their solutions and overcome difficulties.

The divergence in findings concerning teaching practices centred on "establishing mathematics goals to focus learning", "implementing tasks that promote reasoning and problem-solving" and "using and connecting mathematical representations" between this study and that of Al-Otaibi and Al-Matham (2022) may be attributed to variations in the focus of professional development initiatives. While Al-Otaibi and Al-Matham (2022) identified low scores for these practices, our investigation yielded moderate scores, consistent with the outcomes reported by Shelton et al. (2020) and Bal (2014). Specifically, Shelton et al. (2020) documented a moderate level of engagement in promoting reasoning and problem-solving, while Bal (2014) indicated high levels among mathematics educators in using diverse mathematical representations. This disparity suggests that professional development programmes in Saudi Arabia prioritise the cultivation of mathematical

reasoning, problem-solving competencies and the effective use of mathematical representations.

The medium level of implementation of teaching practices supporting the elicitation and use of evidence of students' thinking observed in this study is consistent with the findings of Al-Otaibi and Al-Matham (2022). Mathematics educators in Saudi Arabia undergo training in continuous formative assessment techniques and employ achievement portfolios to systematically gather tangible evidence of students' performance. Moreover, they demonstrate positive attitudes toward these assessment practices, recognising their significance in gaining insights into students' cognitive processes, as noted by Albalawi (2018). Al-Essa (2019) further identified satisfactory formative assessment skills among female mathematics teachers in the region. However, Wilburne et al. (2018) found that eliciting and utilising evidence of students' thinking were among the aspects of mathematics teachers' instructional practices less emphasised.

## 4. CONCLUSION

The outcomes of this investigation underscore the critical need for comprehensive training courses tailored to mathematics educators across diverse educational tiers, aimed at enhancing their understanding and implementation of effective pedagogical strategies. In addition, educational resources, such as the Teacher's Textbook, need to be updated to provide instructors with the necessary support in implementing appropriate instructional methodologies. This study has developed a diagnostic tool intended for mathematics supervisors to assess the extent to which effective teaching methodologies are being implemented.

This research employed a quantitative approach, using an observation checklist to evaluate the implementation levels of effective pedagogical practices among 38 male and female mathematics educators in Saudi Arabia. While informative, the study's scope was limited by the sample size. Future scholarly endeavours in this field could focus on designing and implementing a training programme specifically aimed at improving the application of the eight effective teaching practices outlined by the National Council of Teachers of Mathematics (NCTM). Also, qualitative investigations could explore the barriers mathematics instructors face in adopting these practices and explore their perceptions regarding their integration into pedagogical frameworks.

Moving forward, efforts should be directed towards refining educational practices through continuous professional development and targeted interventions. By addressing these gaps in research and practice and further exploring the perspectives of mathematics educators, stakeholders could better support the effective implementation of modern teaching methods. This approach promises to foster a conducive learning environment that enhances students' mathematical proficiency and critical thinking abilities in Saudi Arabia and similar educational contexts globally.

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