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Understanding mathematical abstraction: A systematic literature review of its conceptualizations and research practices

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Abstract

Mathematical abstraction is a fundamental concept in learning mathematics and plays a significant role in students' mathematical understanding. While this classic topic has been studied by mathematics education experts for a long time, both conceptually and in practice, it has many interpretations. This study aims to systematically review literature published from 2016 to 2022 in the Scopus database on mathematical abstraction. This review focuses on definitions of mathematical abstraction, research methods, mathematical topics, and educational levels in studies of mathematical abstraction. Using a systematic literature review approach, 68 articles and conference papers were initially identified, and after applying the PRISMA flowchart, 23 documents were selected that focused on mathematical abstraction. The studies were then analyzed through content analysis. The results showed that mathematical abstraction is generally understood as a process of constructing students' mathematical knowledge by drawing on prior mathematical expertise or experience. The majority of studies conducted used qualitative research methods at the Junior High School level, aiming to describe the abstraction process among students, including the difficulties encountered. At the same time, the most chosen mathematical topic was geometry. One of the most interesting findings of this research is that research on mathematical abstraction focuses more on assessing abstraction abilities through students' problem-solving performance than on examining how learners construct mathematical concepts. This indicates the need for future research to explore the process of student concept development in greater depth, thereby strengthening theoretical and pedagogical understanding of mathematical abstraction.

Keywords:
Mathematics abstraction, Mathematics education, Mathematics learning, Systematic literature review

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1. INTRODUCTION

Mathematics plays a crucial role in life (Chin et al., 2022), making it a fundamental subject that students must master from elementary school to university level (Sachdeva & Eggen, 2021). The abstract nature of mathematical objects (Ding & Li, 2014; Khasanah et al., 2021; Wang & Cai, 2007) contributes to how students think in order to build their understanding of mathematics. Furthermore, this abstract nature requires students to engage in higher-level thinking, such as generalizing from experiences or concrete representations to higher conceptual structures (Dreyfus, 2014). Furthermore, when students are unable to do so, they tend to experience difficulties in connecting concrete representations with abstract mathematical ideas. This results in a lack of understanding of mathematical concepts and their application in a broader context. Therefore, it is very important to understand how the process of constructing mathematical knowledge occurs.

Mitchelmore and White (2004) assert that mathematics constitutes an independent system separate from the physical and social worlds, rendering mathematical objects unique. Thus, the learning process of mathematical concepts necessitates construction processes, including visualization and representation, to enhance students' comprehension. However, in classroom learning practices, students often find it difficult to shift from concrete to abstract representations, resulting in obstacles, misconceptions, or incomplete understanding of concepts. Therefore, understanding how abstraction develops is crucial, not only from a theoretical perspective but also to overcome the challenges that continue to arise in mathematics learning. This underscores the significant role of abstraction in the mathematics learning process.

There are several perspectives on mathematical abstraction. Dreyfus et al. (2015), Nurhasanah, Kusumah, Sabandar, et al. (2017), and Nurhasanah (2018) state that mathematical abstraction is vertically restructuring mathematical concepts based on previously acquired knowledge or experiences to derive new concepts. Meanwhile, Skemp (1986) states that abstraction occurs when students recognize similarities between the characteristics of a mathematical concept to be learned and their prior experiences or existing knowledge. Thus, Dreyfus and Nurhasanah et al. emphasize mathematical abstraction as a conceptual construction process, while Skemp focuses on the cognitive mechanism in recognizing similarities. Based on these views, mathematical abstraction can be seen as a mental process of constructing new concepts through the recognition of similarities between previous experiences or knowledge and the mathematical structures being constructed.

Piaget introduced two types of abstraction in mathematics and science education: empirical abstraction and reflective abstraction (Dreyfus, 2014; Scheiner, 2016). Empirical abstraction emphasizes individuals' experiences and observations of an object, where students identify similarities in objects and transform them into new concept (Mitchelmore & White, 2004). Skemp suggests that the empirical abstraction results can enable students to connect new experiences with prior knowledge, allowing concept development to build upon existing understanding (Mitchelmore & White, 2004). In mathematics learning, many students experience difficulties when they have to shift from concrete examples to a more general understanding of concepts. This shows that the empirical experiences gained in class do not always develop into meaningful abstractions. For this reason, empirical abstraction is very important because it helps bridge this process, whereby students use real experiences as a basis for recognizing patterns, generalizing, and constructing new mathematical concepts. By understanding how empirical abstraction occurs, educators can design more effective learning strategies, minimize misconceptions, and support the continuous development of conceptual understanding.

Reflective abstraction refers to advanced construction using existing structures to form new structures. From Piagets' psychological perspective, Piaget asserts that reflective abstraction is a process that can support significant mathematical logic construction (Nurhasanah, 2018). In reflective abstraction, Piaget explains four types of constructions: interiorization, coordination/composition, encapsulation, and generalization (Nurhasanah, 2018). Nurhasanah (2018) explained further that through internalization and coordination, students internalize and connect various thinking processes; through encapsulation, students transform actions into objects of thought; and through generalization, students expand the application of concepts. These four things show the stages of student thinking, which begin with an understanding of concrete things and move towards the formation of abstract concepts. Thus, reflective abstraction becomes an important basis for seeing the development of students' thinking abilities through a gradual construction process.

Abstraction is a mental process that occurs within students, making it challenging to observe directly, but its outcomes are clear. Nurhasanah (2018) identifies four levels of abstraction as modified results of mathematical abstraction indicators based on Battista (2007), Hong and Kim (2016), and Nurhasanah et al. (2013), consisting of perceptual abstraction, internalization, interiorization, and second level internalization. It is further explained that these four levels indicate the development of students' thinking, starting from the recognizing of mathematical attributes, symbolic representation, coordination between concepts, to the generalization of knowledge into new contexts. This shows that there is diversity in the ways students construct mathematical understanding, which can be an important point to consider.

In mathematics education, abstraction is crucial and supports the achievement of learning objectives. Given its significance, research on abstraction, particularly in mathematics education, is essential. Several studies have investigated mathematical abstraction, including in the context of mathematics education in Indonesia (Dreyfus, 2014; Dreyfus et al., 2015; Fitriani, 2018; Hong & Kim, 2016; Khasanah et al., 2021; Mitchelmore & White, 2004; Nurhasanah, 2018; Nurhasanah, Kusumah, & Sabandar, 2017; Nurhasanah, Kusumah, Sabandar, et al., 2017; Nurhasanah et al., 2018; Nurhasanah et al., 2013). However, the studies conducted have not mapped out a systematic review of mathematical abstraction. On the other hand, mathematical abstraction in mathematics education plays a very important role, so it is necessary to conduct a comprehensive review of studies related to mathematical abstraction, mainly related to trends and how understanding of mathematical abstraction concepts is applied in research practice. Thus, according to Putra et al. (2023), a literature review in the form of a Systematic Literature Review (SLR) is highly necessary. Systematic literature review is a form of secondary study with various approaches aimed at constructing, exploring, and summarizing a topic with predefined research questions (Munn et al., 2018). Therefore, SLR can provide information about research trends and offer comprehensive study results, leading to a better understanding (Lame, 2019).

Based on the explanation, the study was designed to address the gaps identified in previous studies and provide a deeper understanding of the conceptual and empirical developments related to mathematical abstraction. Therefore, this systematic literature review focuses on literature studies on mathematical abstraction by answering the following research questions: (1) what are the definitions of mathematical abstraction commonly proposed in research? (2) what are the research methods commonly used in mathematical abstraction research? (3) what mathematics topics are used for investigating mathematical abstraction in various studies? (4) at what educational level are studies related to mathematical abstraction conducted?

2. METHOD

This study was categorized as a Systematic Literature Review (SLR). Juandi (2021) stated that SLR was used to find, select, critically assess, and systematically and explicitly analyze research findings presented in scholarly publications. Kitchenham et al. (2009) indicated that SLR was beneficial for future research according to findings presented in previous publications. Further elaboration on these three stages is presented in the following Figure 1.

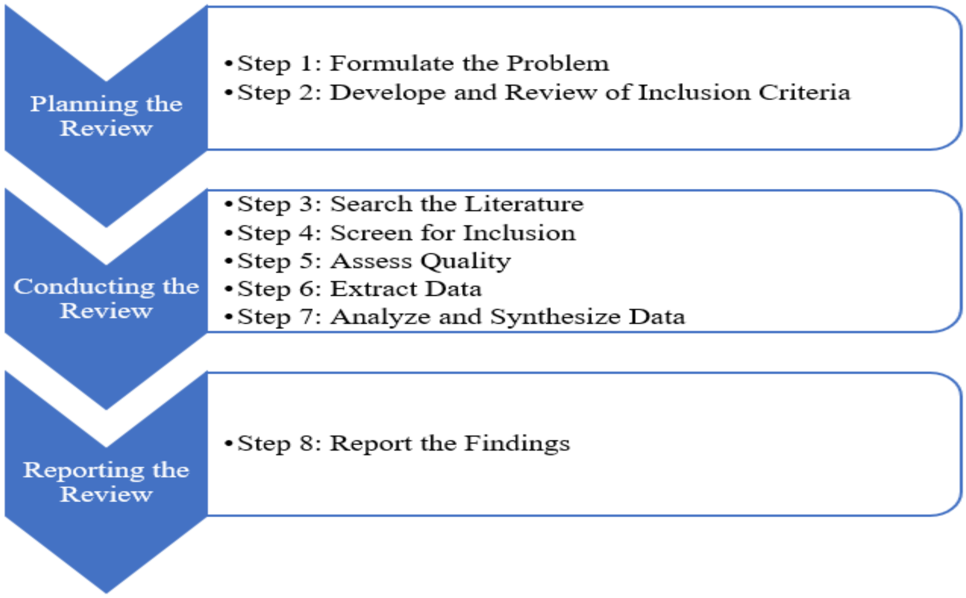

Figure 1. The systematic literature review process by Xiao and Watson (2019)

Xiao and Watson (2019) explained, as depicted in Figure 1, that the SLR process consists of three main stages: Planning the review, Conducting the review, and Reporting the review. The research procedures follow this model to increase transparency and replicability.

2.1. Planning the review

Xiao and Watson (2019) explained that in the planning stage, researchers identified the need to conduct the review, formulated specific research questions, and developed protocols for the review. Xiao and Watson (2019) stated that research questions were crucial in literature reviews and needed to be formulated specifically rather than broadly.

After formulating the research questions, the next step was to develop the review protocol. The review protocol aimed to explain the reviewe elements, including research objectives, research questions, inclusion criteria, search strategies, and screening procedures (Xiao & Watson, 2019). In this context, the researchers selectively determined keywords and databases for the search. Specifically, the researchers used the keywords (TITLE-ABS-KEY (mathematical abstraction) OR TITLE-ABS-KEY (abstraction) OR TITLE-ABS-KEY (abstracting) AND TITLE-ABS-KEY (mathematics education)) on the Scopus database for subsequent stages. The use of Scopus database is based on the relevance of research questions and the existence of leading journals to ensure the quality of selected articles. In addition, Scopus database is a comprehensive database in the academic field (Phuong et al., 2023), providing relevant and reliable information related to studies conducted and ease of downloading data directly. Finally, in the last stage, the researchers determined the inclusion and exclusion criteria to identify relevant literature aligned with the research objectives. Table 1 displays the indicators used in this study.

Table 1. Inclusion and exclusion criteria

Criteria	Inclusion	Exclusion
Document type	Article or conference paper	Book chapter, review, book, conference review, note, editorial, report, etc.
Source type	Journal or conference Proceeding	Book series, book, trade journal, undefined, report, etc.
Keyword of article	Mathematical abstraction, abstracting, abstraction	Other
Field of article study	Mathematics education	Other
Language	English	Other
Accessibility	Open acces or full-text articles	Articles requiring a payment or preview articles
Period	2016 – 2022	Other

2.2. Conducting the review

During the conducting the review stage, researchers searched for relevant literature using predetermined keywords in the Scopus database. The Scopus database was chosen because it is one of the credible databases for scholarly works, ensuring that articles indexed in Scopus have undergone a rigorous review process. Subsequently, article screening was performed based on previously determined inclusion criteria in conducting the review stage. In selecting relevant articles for further analysis, the researchers used the PRISMA (Preferred Reporting Items for Systematic Literature Review and Meta-Analysis) approach, using the Scopus database. Conde et al. (2020) stated that PRISMA provides certainty regarding the quality and reproducibility of the review process. The PRISMA framework used was an adaptation from Kholid et al. (2023), consisting of four primary stages: Identification, screening, eligibility, and inclusion, as presented in Figure 2.

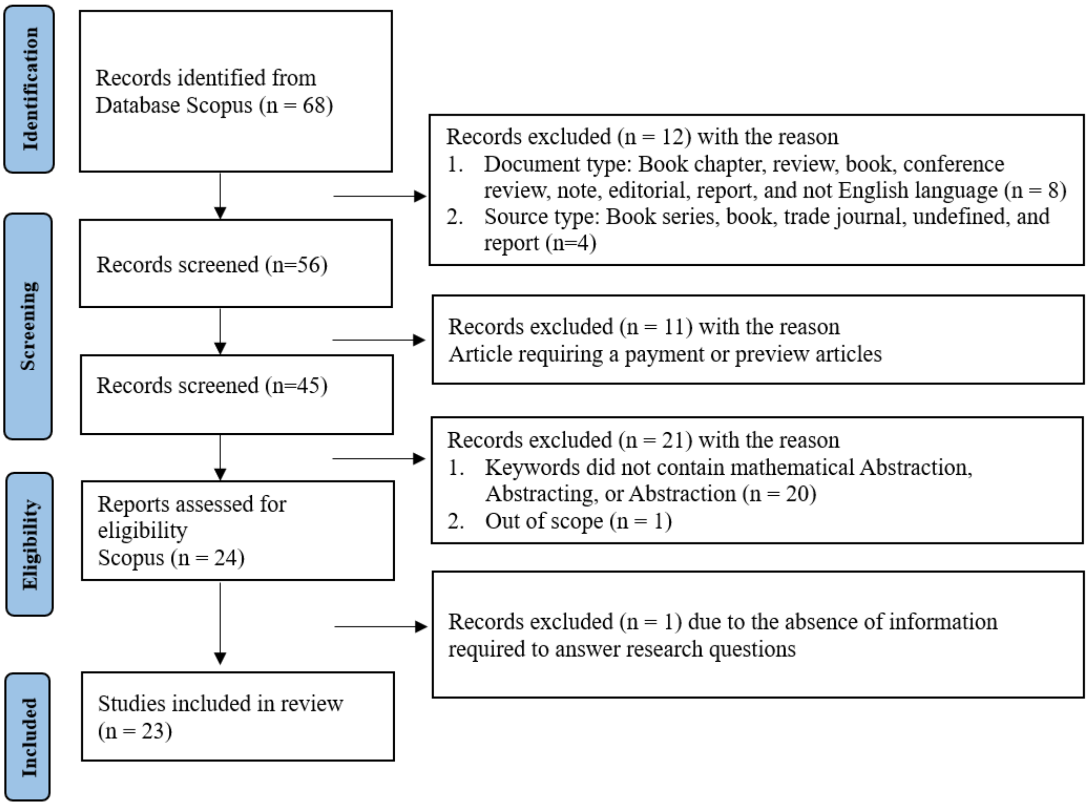

Figure 2. PRISMA flowchart protocol

Based on Figure 2, the first step was searching for articles in the Scopus database from 2016 to 2022 resulting in 68 articles. The search in the Scopus database was conducted in February 2024, hence the obtained articles are those still indexed in Scopus (Scopus coverage years to present). Next, out of the 68 articles obtained, screening was performed based on inclusion criteria presented in Table 1 and Figure 2 resulting in 56 articles. From the 56 articles obtained, another screening was conducted based on accessibility, selecting articles that are fully accessible (open access), resulting in 45 articles. The third step involved further screening from the 45 fully accessible articles, resulting in 24 eligible articles that met all inclusion criteria. However, from the 24 eligible articles, one article was found to lack the necessary information to answer the research question formulations, thus leaving 23 articles for further analysis to address the research questions.

The selection of these 23 articles resulted from the quality assessment stage in the SLR process, representing a refined filter to obtain complete articles for data extraction to address the formulated research questions. The final step of conducting the review involved data analysis and synthesis, where researchers organized the data to answer the research question formulations. Analysis of included articles was conducted through content analysis based on several dimensions, including author name, year of publication, research location, publication source, article title, research objectives, research methods and participants, and key research findings, including the abstraction concepts used in the research. The analysis was conducted manually without the use of software. Furthermore, a separate database was developed using MS Word and MS Excel to present the analysis results, making it easier for researchers to group them according to the research questions.

2.3. Reporting the review

The reporting the Review stage is the final stage of SLR. Xiao and Watson (2019) stated that researchers should report the findings of the literature search, the selection process, and quality assessment as presented in the flow diagram in Figure 2.

3. RESULTS AND DISCUSSION

3.1. Results

The Study conducted was a systematic literature review aimed at examining and analyzing research findings related to mathematical abstraction. Specifically, four aspects were investigated and analyzed: definitions of mathematical abstraction, research methods used, mathematical concepts involved in the research, and the educational level of the research subjects. Based on the analysis using the PRISMA approach, a total of 23 articles were obtained, including articles from journals and conference proceedings.

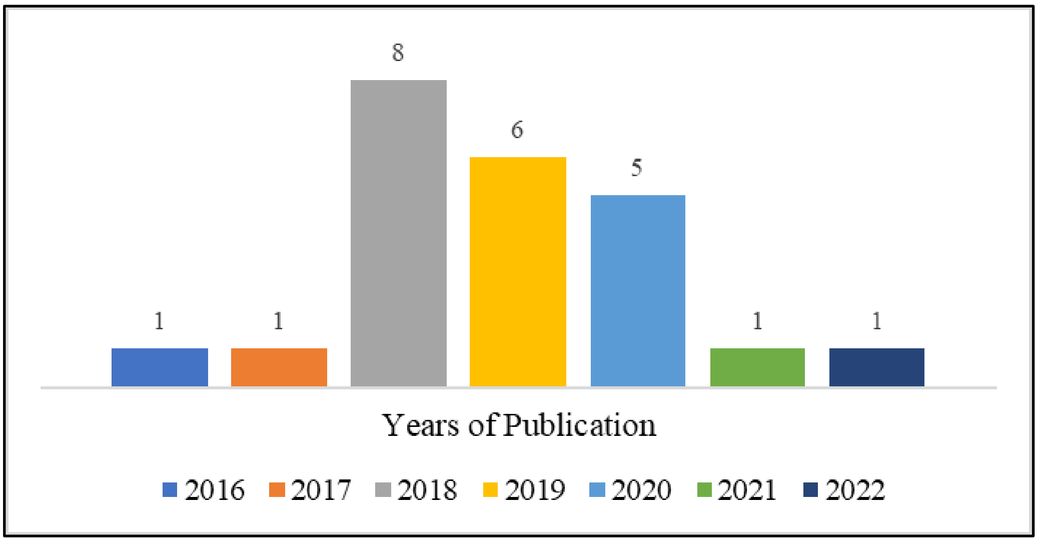

Figure 3. Number of publications on mathematical abstraction between 2016 to 2022

From the 23 articles reviewed, Figure 3 shows that the number of articles related to mathematical abstraction published in both journals and Scopus-indexed proceedings increased to eight articles in 2018 from the years 2016 and 2017. However, in the subsequent years, from 2019 to 2022, there was a decrease. Furthermore, based on the countries where the research was conducted, Figure 4 indicates that from 2016 to 2022, research on mathematical abstraction was predominantly conducted in Indonesia, with a total of 18 studies, while in Turkey there were three studies, and in South Korea and England there was one study each. This dominance provides an overview and hope for a greater emphasis on mathematical abstraction and conceptual understanding in the Indonesian mathematics curriculum. The distribution of research by country also indicates that all 23 articles were written by authors who did not collaborate across countries or were conducted by authors affiliated with their respective countries.

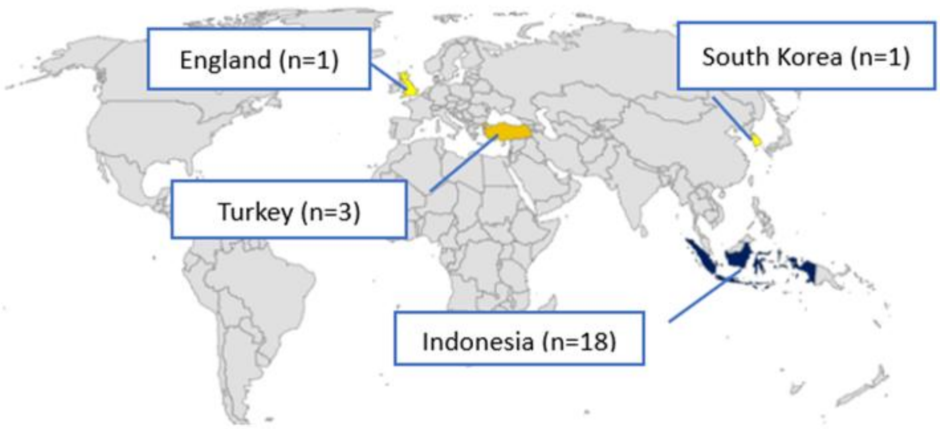

Figure 4. Distribution of studies on mathematical abstraction in terms of country between 2016 to 2022

The detailed information of the 23 papers analyzed in this study are described in Table 2.

Table 2. Data of authors, year, papaer type and research results on mathematical abstraction from 2016 to 2022

Authors & Year	Journal/Conference Proceeding	Research Results
Hong and Kim (2016)	Eurasia Journal of Mathematics, Science and Technology Education	The level of mathematical abstraction and the form of mathematical abstraction can be improved through a problem-solving learning approach using unstructured problems.
Nurhasanah, Kusumah, Sabandar, et al. (2017)	Journal of Physics: Conference Series	Prior knowledge of Cartesian coordinates played an important role in developing the concept of parallel coordinates, with the abstraction process dominated by empirical abstraction influenced by group social interaction.
Fitriani et al. (2018)	Journal of Physics: Conference Series	Students in general still do not have abstraction skills on the concept of Curved Spaces in Three Dimensional.
Panjaitan (2018)	Journal of Physics: Conference Series	The abstraction profile of students with field independent cognitive style is different from students with field dependent cognitive style in solving math problems.
Subroto and Suryadi (2018)	Journal of Physics: Conference Series	the existence of epistemological obstacles in the form of false intuitions and excessive generalizations in the process of recognizing, building-with, and constructing concepts.
Hakim and Nurlaelah (2018)	Journal of Physics: Conference Series	The important core of the mathematical mindset model is abstraction, conceptual and procedural

Authors & Year	Journal/Conference Proceeding	Research Results
		knowledge, all of which occur simultaneously to build a mathematical mindset.
Putra et al. (2018)	Journal of Physics: Conference Series	Students understand the concept, but their abstraction is still weak and dependent on numerical illustrations.
Nurhasanah et al. (2018)	Journal of Physics: Conference Series	There is a positive correlation between the level of abstraction and student achievement, namely, the higher the level of abstraction in parallel coordinates, the higher the student's achievement in analytical geometry.
Priatna et al. (2018)	Journal of Physics: Conference Series	The GeoGebra-assisted reciprocal teaching strategy is more effective in improving abstraction skills, especially among students with high initial abilities.
Dewi et al. (2018)	Journal of Physics: Conference Series	Students are not yet able to perform abstraction in problem solving; although they can recognize the characteristics of objects, they are not yet able to see the conceptual relationships between objects.
Iswari et al. (2019)	Journal of Physics: Conference Series	Students have been able to show abstraction activities which include observation of patterns, specialization, generalization, conjecturing, and testing conjectures.
Cahyani et al. (2019)	Journal of Physics: Conference Series	Students accomplish the reflective abstraction indicator by solving quadratic equations in different ways.
Fitriani and Nurfauziah (2019)	Journal of Physics: Conference Series	There is a significant difference in mathematical abstraction between male students and female students in several indicators after using the scientific approach with the help of VBA Excel.
Murtianto et al. (2019)	Infinity Journal	Learning with Mathematica software is more effective at improving mathematical abstraction skills than conventional methods.
Worthington et al. (2019)	Educational Studies in Mathematics	Childrens' strategies as they communicate their thinking, which shows the importance of symbolic number knowledge in acquiring the abstract graphical language of mathematics.
Sumen (2019)	Journal on Mathematics Education	Students have difficulty abstracting the concepts of whole, half, and quarter because they do not yet understand division into equal parts.
Harry et al. (2020)	Proceedings of the 7th Mathematics, Science, and Computer Science Education International	There is a relationship between mathematical abstraction level and cognitive style; reflective-impulsive students show the highest achievement at all abstraction levels, while abstraction errors are caused by mistakes in understanding, transformation, processing, and coding.

Authors & Year	Journal/Conference Proceeding	Research Results
	Seminar, MSCEIS 2019	
Camci and Tanışlı (2020)	EGITIM VE BILIM-EDUCATION AND SCIENCE	Low-ability students who use reflective abstraction in measuring the volume of rectangular prisms, with the abstraction process influenced by social and sociomathematical norms.
Kadarisma et al. (2020)	Infinity Journal	The higher the students' abstraction abilities, the lower the level of misconceptions they experience.
Dewi et al. (2020)	Journal of Physics: Conference Series	The Design local wisdom-based Means-end Analysis model based on local wisdom is effective for mathematical communication, but mathematical abstraction remains weak because teachers and students find it difficult to relate it to cultural contexts.
Hutagalung et al. (2020)	Journal of Physics: Conference Series	Students with low abilities understand triangles concretely and require teacher guidance to achieve higher conceptual abstraction.
Nurrahmah et al. (2021)	Journal of Physics: Conference Series	Low mathematical abstraction ability in notation and symbol manipulation, mainly due to weak comprehension of story problems.
Kilicoglu and Kaplan (2022)	Athens Journal of Education	The abstraction process occurs at the application and conceptual knowledge levels and can be effectively categorized using Bloom's revised taxonomy.

Subsequently, the analysis results of the 23 articles used based on the research questions posed are presented.

Definition of mathematical abstraction

The understanding of mathematical abstraction in research is crucial as it serves as the theoretical foundation used to convey the results or assess the achievement of the research outcomes or objectives. The analysis of these 23 publications indicates various definitions of mathematical abstraction used or stated in these publications. However, from the various definitions of abstraction used, two main themes can be identified, namely: (1) abstraction as a process of concept construction, which includes the view of abstraction as a process of constructing concepts by utilizing students' prior experiences or knowledge related to concepts or theories; and (2) abstraction as a part of mathematical thinking process, which emphasizes mental activities in reorganizing mathematical concepts vertically into new structures, thinking activities to classify objects based on similar properties, and becoming part of advanced thinking activities with an emphasis on generalization, synthesis, and the results of the thinking process itself.

Based on the various definitions of mathematical abstraction presented earlier, it can be generally understood that mathematical abstraction refers to a mathematical process of concept construction that occurs in students' minds as part of mathematical thinking. However, the study results of the 23 articles indicate that not all research explicitly states the intended definition of abstraction. Moreover, in some of the reviewed articles (Dewi et al., 2018; Murtianto et al., 2019; Subroto & Suryadi, 2018), mathematical abstraction is defined as abstraction ability, which is described as follows: (1) abstraction ability is the ability to reorganize existing mathematical knowledge into new mathematical structures; (2) abstraction ability is a process of depicting a specific situation into a concept that can be thought through a construction process; and (3) abstraction ability is the ability to identify manipulated objects and transform problems (ideas) into notation or symbols. Additionally, the study of one article by Camci and Tanışlı (2020) explicitly did not state the definition of mathematical abstraction but presented the concept of abstraction based on Piaget's theory, where abstraction is distinguished into empirical and reflective abstractions. Thus, in that article, the definition of mathematical abstraction used refers to the definitions of empirical and reflective abstractions.

Predominant methods in abstraction research

Based on the analysis of the overall articles used, several research methods were identified, whether explicitly stated or mentioned generally. The research methods broadly consisted of quantitative research, qualitative research, mixed methods, and development research. Table 3 displays the distribution of articles based on the research methods used.

Table 3. Distribution of research by research method

Research Method	Design	Frequency	Total
Qualitative	Descriptive Study	3	16
	Case Study	8	
	Explorative Study	1	
	Ethnographic Study	1	
	Teaching Experiment	1	
	Not explicitly stated	2	
Quantitative	Quasi-Experimental	3	3
Mixed Method	Sequential Explanatory	1	1
Research and Development	Plomp Model	1	2
	Not explicitly stated	1	
Design Research	–	1	1

Table 3 shows that out of the 23 articles used, qualitative research method was predominantly used by researchers in studying mathematical abstraction, with a total of 16 articles. This is based on the research objectives, as most studies aimed to observe or describe mathematical abstraction, including the process and outcomes such as students' mathematical abstraction abilities, including classifying the levels of students' mathematical abstraction. Through qualitative research methods, researchers have the chance to get comprehensive data on how students think through interactions and learning activities, not just focusing on test results. Specifically, research from Subroto and Suryadi (2018) and Nurhasanah, Kusumah, Sabandar, et al. (2017) used a specific methodology to examine the abstraction process proposed by Dreyfus et al. (2015), known as the Abstraction in Context (AiC) framework, which provides a model for studying abstraction processes. This model is named RBC+C (Recognizing, Building-with, and Construction) followed by the subsequent C, Consolidation. Kilicoglu and Kaplan (2022) employed the theoretical APOS framework to study abstraction specifically in topics such as abstract algebra, calculus, statistics, and others for mathematics at the college level, as proposed by Dubinsky and McDonald (2001).

Mathematics concepts used as the object of studies on abstraction

Research on mathematical abstraction in the scope of Mathematics Education is often conducted on specific topics or materials within mathematics. Based on the reviewed articles, it is found that research on mathematical abstraction is carried out on mathematical topics such as geometry, number theory, algebra, statistics, and mathematics in general. Additionally, one article was found that did not specify or mention the mathematical topic used in the research conducted. Table 4 presents the distribution of topics used in literature.

Table 4. Distribution of research in terms of topics used

Topic	Sub-Topic	Frequency	Total
Geometry	Three-Dimensional Space Curved Shapes	2	11
	Area of Plane Figures	2	
	Volume of Rectangular Prism	1	
	Parallel Coordinates	2	
	Concepts of Triangles, Quadrilaterals, Plane Geometry, and Curved-Side Geometry	1	
	Lines and Angles	1	
	Concepts and Area of Triangles	1	
	Analytical Geometry	1	
Number	Sequences and Series	2	3
	Fractions (Whole-Half-Quarter)	1	
Algebra	Equation	2	4
	Group Theory (Abstract Algebra)	2	
Statistics	Mathematics Statistics	1	1
General Mathematics	Quantity, Transformation, Measurement, Space, etc.	3	3
Not explicitly stated		1	1

Based on Table 4, it is found that the mathematical topic most commonly used in research on mathematical abstraction is the topic of geometry, both related to concepts in plane geometry and solid geometry. The dominance of geometry topics with visual characteristics and representations provides researchers with the opportunity to observe students' thinking processes concretely in constructing new concepts based on visual representations. This can facilitate the identification of abstraction processes through the transformation of visual ideas into conceptual ideas. In addition to the mathematical topics listed in Table 4, one research was found where the mathematical topic used in the study was not specified.

Educational level

The study of 23 research articles on mathematical abstraction was conducted across all educational levels, ranging from preschool to university level. Figure 5 presents the distribution of mathematical abstraction research based on educational levels.

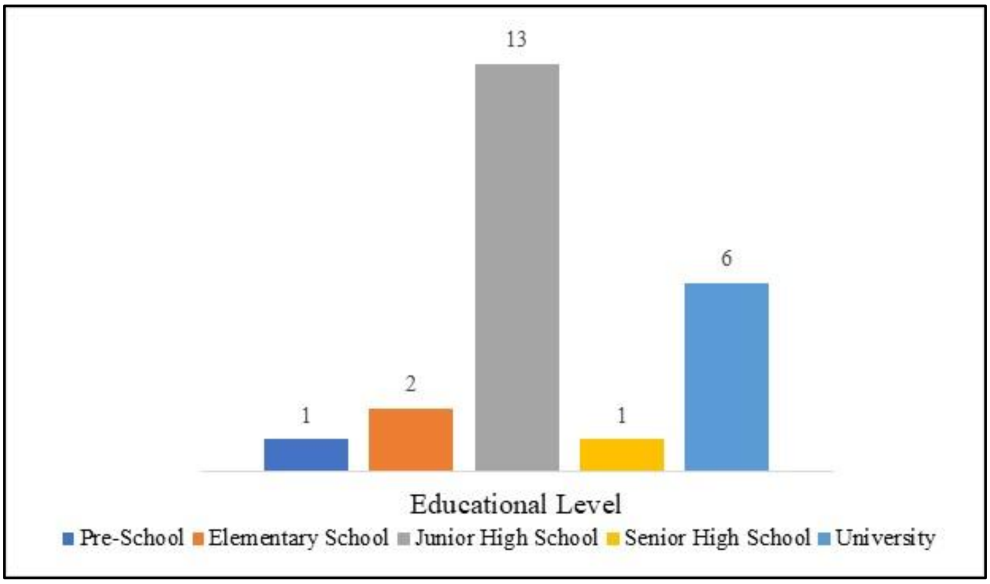

Figure 5. Distribution of abstraction research based on the educational level

From on Figure 5, research on mathematical abstraction is primarily conducted at the secondary education level, specifically at the Junior High School level, with a total of 13 studies. Following this, at the university level, there are six studies. The remaining studies are distributed with one study each at the Pre-School, Elementary School, and Senior High School levels. Specifically, at the university level, research is conducted at the undergraduate level to examine mathematical abstraction among students majoring in mathematics education as prospective mathematics teachers.

3.2. Discussion

The systematic literature review of 23 articles on mathematical abstraction from 2016 to 2022 has provided an overview of the research development in this area. Specifically, the discussion focuses on the use of definitions of mathematical abstraction, research methods employed, mathematical topics involved in the studies, and the educational levels examined in the research.

Definition of mathematical abstraction

In terms of defining mathematical abstraction, several studies explicitly state the definitions of mathematical abstraction they utilize. Among the 23 reviewed articles, seven definitions are identified, with two definitions being widely referenced or used by most researchers on the topic of abstraction. The first definition asserts that abstraction is the process of constructing concepts by leveraging students' prior experiences or knowledge related to concepts or theories. Meanwhile, the second definition states that abstraction is a part of mathematical thinking, involving mental activities in the vertical reorganization of existing mathematical concepts into new mathematical structures. Based on these two definitions, it is stated that abstraction is a process that is part of mathematical thinking in constructing mathematical concepts based on previous mathematical experiences or knowledge. In addition to definitions that characterize it as a process, these other definition of mathematical abstraction as a skill. Referring to the use of the first definition of mathematical abstraction it is found that the definition of abstraction as a process of constructing mathematical concepts is fundamentally a general definition that can be based on abstraction indicators. This definition of abstraction is based on the view of Ferrari (2003), who states that abstraction is closely related to the formation of new mathematical concepts.

The second definition of abstraction widely used in these studies is specifically defined as the vertical reorganization of existing mathematical concepts into new mathematical structures (Hakim & Nurlaelah, 2018; Kilicoglu & Kaplan, 2022; Nurhasanah, Kusumah, Sabandar, et al., 2017; Nurhasanah et al., 2018; Panjaitan, 2018; Sumen, 2019). This definition generally indicates the process of constructing mathematical concepts (reorganization of concepts) vertically (vertical mathematization), thus specifically representing the definition of mathematical abstraction. This is consistent with Dreyfus et al. (2015) as an adaptation of the ideas of Freudenthal (1991), stating that mathematical abstraction is a process of vertically reorganizing several previously acquired student mathematical constructions to construct new mathematical concepts.

In addition to the two definitions mentioned above, another definition of abstraction used refers to the similarity of ideas in classifying an object according to the similarity of properties from newly formed experiences. This definition is based on Skemp (1986) that abstraction is a result of the abstract process to realize a new experience or knowledge that has been classified previously. Thus, this definition emphasizes the existence of similarity as the basis for the process of forming new concepts. Therefore, this definition leads us to the understanding of the type of abstraction known as empiric abstraction, as stated by Piaget (1977). The variation in mathematical abstraction definitions in the articles indicates that abstraction is a complex process with many meanings. Nurhasanah (2018) states that abstraction has many interpretations, in mathematics and mathematics education. It is not surprising that Ferrari (2003) states that abstraction is a highly fundamental process in both mathematics and mathematics education.

In addition, these findings also indicate variations in approaches to the definition of mathematical abstraction. These variations refer to Dreyfus et al. (2015) view of abstraction, which emphasizes abstraction as a vertical reorganization of mathematical concepts based on prior knowledge, and Piaget (1977) view, which emphasizes empirical abstraction. The existence of these variations indicates two different orientations, such as indicators, units of analysis, or contexts. This can be a new avenue for understanding mathematical abstraction, including how the two approaches can be integrated, which can be explored through pedagogical interventions that facilitate the shift from concrete experiences to abstract conceptual knowledge.

Based on the abstraction definitions revealed in the reviewed articles, one interesting finding is that in some articles (Cahyani et al., 2019; Dewi et al., 2018; Harry et al., 2020; Hutagalung et al., 2020; Iswari et al., 2019; Murtianto et al., 2019; Nurrahmah et al., 2021; Priatna et al., 2018), where abstraction is defined as a process, there is no explanation of how this abstraction process occurs in the research findings. The research findings mainly discuss the abstraction ability of the subjects studied. This indicates that in these studies, the abstraction process is examined from the perspective of abstraction ability formulated using indicators based on theories of abstraction processes, such as the RBC (Recognizing, Building-with, and Construction) or RBC+C (Recognizing, Building-with, and Construction + Consolidation) model as proposed by Dreyfus et al. (2015). This is related to the methodological framework within the abstraction research. In this context, there is already a methodological framework related to the abstraction process proposed by Dreyfus et al. (2015) through the RBC or RBC+C model to understand how the abstraction process occurs. However, at the implementation level in some studies, this methodology is not used to observe the abstraction process, ultimately referring to the abstraction ability examined through indicators obtained from RBC or RBC+C.

Predominant methods in abstraction research

The research methodology employed in studies on the topic of abstraction is tailored to the issues and theoretical frameworks used by researchers. Qualitative research is often chosen because it examines processes, and as stated by Creswell (2012) and Moleong (2007), suitable when the collected data are words, processes, and other non-quantifiable aspects. Referring to Table 3, it is found that out of 23 studies, 16 utilize qualitative research methods with various research designs. The choice of research methods is closely related to the research objectives, where the majority aim to describe mathematical abstraction, both processes and outcomes such as students' mathematical abstraction abilities, including classifying levels of students' mathematical abstraction. In addition to qualitative research methods, several articles employ quantitative research with quasi-experimental designs, where researchers conduct experiments involving the application of teaching models to observe their impact on students' mathematical abstraction. The teaching approaches used include the Scientific Approach with the assistance of VBA Excel (Fitriani & Nurfauziah, 2019), reciprocal teaching strategy with the assistance of GeoGebra (Priatna et al., 2018), and learning using Mathematica software (Murtianto et al., 2019). Thus, mathematical abstraction in these three studies is the dependent variable as students' mathematical abstraction abilities are assessed based on indicators. Among the 23 articles reviewed, other research methods used include mixed methods, research and development, and design research.

In several articles reviewed, the achievement or process of student abstraction is observed through the attainment of abstraction indicators, indicating that the research methods used are not specifically aimed at examining how the abstraction process occurs. This finding is intriguing as it suggests that, by conceptualizing abstraction as a process, the description of students' mathematical abstraction or research subjects refers to abstraction indicators. However, in some articles (Nurhasanah, Kusumah, Sabandar, et al., 2017; Subroto & Suryadi, 2018), specific methods are explicitly described for observing the process of mathematical abstraction using the Abstraction in Context (AiC) Framework with the use of the RBC (Recognizing, Building-with, and Construction) or RBC+C model with +C referring to Consolidation as proposed by Dreyfus et al. (2015).

Dreyfus et al. (2015) assert that AiC is a theoretical framework used to study the process of constructing abstract mathematical concepts by students that occurs within a specific mathematical context. Within the AiC framework, the process of abstraction performed by students can be explained or depicted through a series of epistemic actions consisting of Recognizing, Building-with, and Constructing (RBC)+C, which includes Consolidation. Thus, through the RBC model, the process of mathematical abstraction occurring in the research subjects can be effectively illustrated.

Mathematical concepts used in abstraction research

The results of the literature review on 23 articles related to the mathematical topics used in abstraction research did not focus on just one topic of mathematics. The diversity in the use of mathematical topics to identify mathematical abstractions suggests that mathematical abstraction can be identified or developed not only through a single mathematical topic. Moreover, this diversity indicates the interconnection among mathematical topics, allowing for their construction through structured activities.

The selection of geometry as a domination used mathematical topic in abstraction research is based on its significance in enhancing logical thinking and the ability to generalize, contributing to a solid understanding of arithmetic, algebra, and calculus, as well as individual mental development (Novita et al., 2018). The prevalence of geometry as a research topic in abstraction, particularly in Indonesia, is rooted in the country's research trends in mathematical abstraction, which initially focused on geometry. In a qualitative context, the topic of geometry may indicate that abstraction is more easily observed in a visual and constructivist context. On the other hand, the choice of geometry in abstraction research is closely linked to van Hiele's theory of geometric thinking.

The concept of mathematical abstraction is extensively studied in the field of geometry, closely related to specific theories in geometry learning, as proposed by van Hiele regarding levels of abstract thinking in geometry learning (Crowley, 1987). Additionally, Clements and Battista (1992) elucidated the process of reasoning in geometry learning, leading many subsequent researchers to associate the process of abstraction with thinking processes in geometry learning. van Hiele's theory suggests that in the learning model, students' progress through levels of thinking until they reach formal geometric thinking (Teppo, 1991). van Hiele delineates three levels of thinking: visual (level 1), where students recognize geometric objects globally; descriptive (level 2), where students identify geometric objects through their geometric properties; and theoretical (level 3), which involves deductive reasoning to prove geometric relationships (van Hiele, 1986). Another perspective (van de Walle et al., 1998) suggests that van Hiele's levels of geometric thinking consist of five levels: Level 0 (Visualization), Level 1 (Analysis), Level 2 (Informal Deductive), Level 3 (Deduction), and Level 4 (Rigor). Referring to the definitions of each level, whether three or five, indicates that van Hiele's stages or levels of geometric thinking are structured or formulated according to the level of abstraction of geometric concepts being studied. Thus, the selection of geometry topics in abstraction research becomes relevant.

In learning geometry, geometric figures can be easily interpreted in the real world, including through software, thereby assisting students in the abstraction process. This notion is supported by Saitta and Zucker (2013), who states that the process of abstraction in geometry begins with the observation or physical measurement of shapes, which then moves towards abstract geometric axioms, whether from Euclidean or Non-Euclidean geometry. On the other hand, abstraction is part of mathematical thinking, so through geometry, the development of students' thinking activities can be facilitated, potentially providing a good understanding of the process or ability of mathematical abstraction. However, this does not mean that the process of mathematical abstraction can only be facilitated through geometry topics; other mathematical topics also allow for abstraction processes. This can be seen in several research articles on abstraction and in previous studies that have used other mathematical topics such as algebra (Cahyani et al., 2019; Kilicoglu & Kaplan, 2022; Putra et al., 2018; Subroto & Suryadi, 2018), number theory (Iswari et al., 2019; Mason, 1989), statistics (Nurrahmah et al., 2021), as well as mathematics in general.

Education level

Mathematical abstraction is part of the process or mental activity of students' thinking, resulting in differences in the abstraction process among different educational levels. A literature review of literature related to the educational levels used in abstraction research indicates that out of the 23 articles reviewed, research on mathematical abstraction is conducted at the Junior High School level. From the perspective of Piaget's Cognitive Development Theory, Piaget (2000) states that individuals over 12 years old are at the formal operational stage. In Indonesia, the age of students at the Junior High School education level is already at that age. At this stage, cognitive skills move from simple abstract thinking to complex, logical thinking skills develop further, including the use of hypotheses in problem-solving (Zhang, 2023). In this condition, Piaget suggests that students can be trained in creative thinking processes, abstract reasoning, and imagining the consequences of specific actions (Zhou & Brown, 2015). This enables the ideal development of the abstraction process. Referring to Ferrari (2003), generalization is one of the main components of abstraction, which can be observed when students reorganize mathematical knowledge. Thus, the abstraction process can proceed at this stage of cognitive development because cognitive abilities at this age allow for such conditions.

At the formal operational stage, there is a great opportunity for reflective abstraction to occur. In this case, students are already able to reflect on and organize concrete experiences into complex knowledge structures (Piaget, 1977). In addition, at the formal operational stage, students begin to shift from empirical abstraction based on concrete experiences to reflective abstraction. Thus, teachers must be able to choose the right learning strategies so that the transition from concrete experiences to conceptual thinking can be facilitated properly.

Mathematical abstraction does not only occur at the formal operational cognitive development stage, such as at the Junior High School level. Several studies from the reviewed 23 articles also found that the abstraction process occurs at the primary school level, where students are in the concrete operational cognitive development stage, including pre-school. Some studies also indicate that the abstraction process can be carried out in students at the primary school or pre-school level (Sumen, 2019; van Oers & Poland, 2007; Worthington et al., 2019). van Oers and Poland (2007) state that primary school students (young children) also provide opportunities to develop quality mathematical abstract thinking. Thus, the abstraction process can also be observed in students younger than those in the formal operational cognitive development stage. This also shows that the abstraction process is continuous, so that a learning approach that supports a gradual transition from empirical abstraction to reflective abstraction is very important to implement, not only in secondary schools but also starting from elementary schools.

4. CONCLUSION

Mathematical abstraction stands as a crucial component of mathematics education, representing a facet of mathematical thinking that illustrates the process of organizing mathematical concepts possessed by students into new mathematical knowledge or concepts. This study showed diverse presentations regarding the definition of mathematical abstraction. However, despite varied definitions, they converge on the same interpretation: abstraction is a process of constructing mathematical concepts in students using previously acquired mathematical knowledge. Nevertheless, some studies examined this construction process through indicators of abstraction abilities formulated based on theories related to the abstraction process.

Five research methods were identified based on the research objectives in each paper, with qualitative methods were commonly used to describe the abstraction process. Explicit research designs, such as descriptive studies, case studies, exploratory studies, ethnographic studies, and teaching experiments, were mentioned in several papers. Additionally, some studies utilized specific models to examine the abstraction process, such as the RBC or RBC+C model and the APOS theory. Besides qualitative methods, other papers reviewed used quantitative research methods, including experimental research, mixed methods, research and development, and design research. Regarding the mathematical topics used in research, geometry emerged as the dominant topic, while other studies explored algebra, number theory, statistics, and general mathematics. Furthermore, mathematical abstraction is not only carried out on students in higher education, but can also begin at the elementary education level, including preschool.

These findings indicate that mathematical abstraction tends to be understood as a dynamic process in knowledge construction. This study reveals two main patterns—abstraction as a process and as a ability—thus requiring an integrated framework that links the dynamics of concept construction with measurable indicators. However, this review has limitations, such as the use of the Scopus database alone, a limited publication time frame, and specific inclusion criteria. Therefore, further research is recommended to expand the literature sources, examine different cultural or curricular contexts, and integrate qualitative and quantitative methods more evenly.

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