Infinity Journal of Mathematics Education
Volume 14, No. 1, 2025
p-ISSN 2089-6867
e–ISSN 2460-9285
https://doi.org/10.22460/infinity.v14i1.p85-108

Improving mathematical proof based on computational thinking components for prospective teachers in abstract algebra courses

Elah Nurlaelah[1*], Aneu Pebrianti[1], Muhammad Taqiyuddin[2], Jarnawi Afgani Dahlan[1], Dian Usdiyana[1]

[1]Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia
[2]Department of Mathematics, University of Auckland, New Zealand
[*]Correspondence: elah_nurlaelah@upi.edu

Received: Aug 2, 2024 | **Revised**: Sep 13, 2024 | **Accepted**: Sep 19, 2024 | **Published Online**: Oct 6, 2024

Abstract

Understanding and constructing mathematical proofs is fundamental for students in abstract algebra courses. The computational thinking approach can aid the process of compiling mathematical proofs. This study examined the impact of integrating computational thinking components in constructing mathematical proofs. The researcher employed a sequential explanatory approach to ascertain the enhancement of algebraic proof capability based on computational thinking through the t- test. A total of 32 prospective teachers in mathematics education programs were provided with worksheets for seven meetings, which were combined with computational thinking components. Quantitative data were collected from initial and subsequent test instruments. Moreover, three prospective teachers were examined through case studies to investigate their mathematical proof capability using computational thinking components, including decomposition, abstraction, pattern recognition, and algorithmic thinking. The study's findings indicated that CT intervention enhanced students' logical reasoning, proof-writing abilities, and overall engagement with abstract algebra concepts. The findings illustrate that integrating computational thinking into learning strategies can provide a framework for developing higher-order thinking skills, especially in proving, which are essential for studies in mathematics education programs.

Keywords:
Abstract algebra, Computational thinking, Mathematical proof, Prospective teacher, Worksheet design

How to Cite:
Nurlaelah, E., Pebrianti, A., Taqiyuddin, M., Dahlan, J. A., & Usdiyana, D. (2025). Improving mathematical proof based on computational thinking components for prospective teachers in abstract algebra courses. *Infinity Journal*, *14*(1), 85-108. https://doi.org/10.22460/infinity.v14i1.p85-108

1. INTRODUCTION

Mathematics is a logical and deductive science because the process of seeking truth (generalization) in mathematics differs from the process of seeking truth in natural and other sciences. Arnawa et al. (2020) suggested that the study of mathematics at the university level must follow these stages: understanding definitions, understanding theorems or lemmas and their proofs, and doing practice questions to strengthen understanding of definitions, lemmas, and theorems in solving problems. Many mathematical concepts must be proven because proof is essential for developing, building, and communicating mathematics (Stylianides, 2007). Logical arguments built through mathematical proofs can demonstrate the correctness of mathematical statements (McCarthy, 2021). A proof is a logical argument that is known to be true. According to Pythagoras, a mathematical proof verifies the truth of a statement, provides reasons for the truth of the statement, systematizes findings using concepts, axioms and theorems, leads to new findings, disseminates mathematical concepts, and poses intellectual challenges (Hanna & Barbeau, 2002).

According to Hanna and Barbeau (2010), a mathematical proof can be approached from two different perspectives. First, solution finding involves a set of deductive strategies that prioritize the syntax of the proof. The other perspective prioritizes ideas that lead to a more meaningful understanding. Each perspective from which the mathematical proof is approached on three components: hypotheses, conclusions, and constructed ideas (Arbaugh et al., 2018).

There are several types of proofs in mathematics: direct proof, indirect proof (contrapositive and contradiction proof) and induction proof. Each method of proof in mathematics is different; each proof technique has different epistemic, cognitive, and practical strengths and weaknesses (D'Alessandro, 2019). It is essential to use clear and concise methods of proof that are widely accepted and understood by the mathematical community. (1) Direct proof establishes the conclusion by logically combining the axioms, definitions, and earlier theorems, for example, to prove that the sum of two even integers is always even. (2) An indirect proof begins with the presumed negation of the proportion to be demonstrated and demonstrates that it results in an opposing condition. There are two principal forms of indirect proof: proof by contradiction involves assuming the opposite of the statement to be proven and then showing that this assumption leads to a contradiction. For example, to prove that the square root of 2 is irrational, one can assume that it is rational and then derive a contradiction; proof by contrapositive involves proving the contrapositive of the statement to be proven. For example, to prove that P implies Q, one can prove that not Q implies not P. (3) Proof by mathematical induction is used to prove true statements for all natural numbers. It involves proving the base case and then showing that if the statement is true for a given number, it must also be true for the following number.

The capacity to demonstrate proficiency in proof is regarded as a crucial element of mathematical comprehension and the reinforcement of mathematical principles (Stylianides, 2007). Moreover, proof ability can be employed to facilitate a more profound comprehension of the subject matter (Hanna & de Villiers, 2008). Killpatric posits that this concept understanding ability represents one of the fundamental competencies in mathematics. Furthermore, proof provides a basis for students to learn new knowledge more deeply because it enables students to make sense of things systematically, not just through the authority of the teacher or textbook (Ball & Bass, 2003). As such, proof occupies a place of much importance in mathematics (Gabriel et al., 2020). From a curricular perspective, the importance of mathematical proof skills requires that early education mathematics curricula provide authentic experiences related to proof (Bruner, 1974; Stylianides, 2016). In other words, special attention should be directed to how mathematical proof skills may be achieved by students.

One of the courses that requires students to make a lot of proofs is the abstract algebra course. Abstract algebra, also known as modern algebra (Albert, 2018) or structural algebra (Kieran, 2018), is the manipulation of abstract symbols (Wagner & Parker, 1993) in the context of solving equations. Abstract algebra is a compulsory course for mathematics students in undergraduate mathematics programs. This course revolves around the study and complex analysis of different algebraic structures, namely groups (Halbeisen et al., 2007), rings (Rowen, 2018), and fields (Gouvêa, 2012).

However, many students struggle with this course, first, because of some problems and misconceptions related to the basic concepts of abstract algebra (Feudel & Unger, 2024) and, secondly, because of weak mathematical proof skills (Bergwall, 2019). At least they tend to struggle with two significant aspects of proof: the first one, they wrestled with making assumptions when part of an if-then statement is not satisfied and understanding that a statement and its converse are not equivalent (Putra et al., 2023), and on the other hand, they grapple with understanding the role of examples, counterexamples, and specific cases (Bergwall, 2019).

A method or way of teaching that can be used to improve mathematical proof skills is needed (Jeannotte & Kieran, 2017; Selden & Selden, 2008). This means that teaching mathematical proof can be incorporated into direct instruction, for example, in abstract algebra courses (Valenta & Enge, 2022). In addition, lecturers have an important role in helping students to prove by introducing new rules that encourage students to do so. Thus, what must be done in the learning process to help students develop the ability to prove? From the mathematics learning process perspective, mathematics teaching as a communication activity is expected to bring the mathematical discourse of students into that of mathematicians' (Tabach & Nachlieli, 2016; Valenta & Enge, 2022). Computational Thinking (CT) may be employed as a methodology to attain the standards of scientific thinking (Orban & Teeling-Smith, 2020).

CT directs students to help solve problems, in this case, with proof in mathematics using certain stages. Palts and Pedaste (2020) described it as a problem-solving approach. It is as prevalent and valuable for computational scientists as fundamental for anyone. The educational benefits of computational thinking are due to the use of abstractions and reasoning skills, which enhance and reinforce intellectual abilities and, therefore, are transferable to different domains (Rodríguez-Martínez et al., 2020). CT can be integrated into the teaching and learning process to help students explore new concepts in several stages (Kallia et al., 2021; Palts & Pedaste, 2020; Waterman et al., 2020).

Wing (2006) sees CT as pivotal to any activity involving human analysis, not just computer programming. Computerless CT skills can drive CT capabilities in non-computing environments (Tang et al., 2020). This approach allows students to experience and learn computational thinking without a computer but with other learning media or materials (Kuo & Hsu, 2020).

The integration of CT into the curriculum in several schools in different countries has had a positive impact on students' problem-solving, data analysis, and modeling skills (Aydeniz, 2018; Kong, 2016). CT involves a systematic thinking process, from the formulation of the problem to the solution, where the solution is presented in a form that can be effectively executed by an information processing agent (Wing, 2006). In this study, four CT components were used, namely decomposition, pattern recognition, abstraction, and algorithmic thinking (Emara et al., 2021). Decomposition refers to simplifying a complex problem by breaking it down into smaller problems to make the overall problem easier to manage and solve (Csizmadia et al., 2015; Dong et al., 2019; Grover & Pea, 2023). Pattern recognition refers to identifying patterns, regularities, and trends in data, processes, or problems (Dong et al., 2019). Abstraction is the process of making artifacts easier to understand by reducing unnecessary details and the number of variables, leading to simpler solutions (Cansu & Cansu, 2019). Finally, algorithmic thinking pertains to the development of step-by-step instructions for solving the same problem (Csizmadia et al., 2015).

Research that identifies and explores mathematical proof skills has been linked to several other mathematical contexts. To ensure the novelty of this research, the VOSviewer tool was used, with data extracted from Scopus. The keywords used were 'proof' and 'mathematics.' The inclusion criteria were that the research must be published anywhere between 1935 and 2024 within the mathematics or social sciences subject areas and written in English. A total of 6,740 articles were found to satisfy these criteria. The VOSviewer visualization of the results is provided in Figure 1.

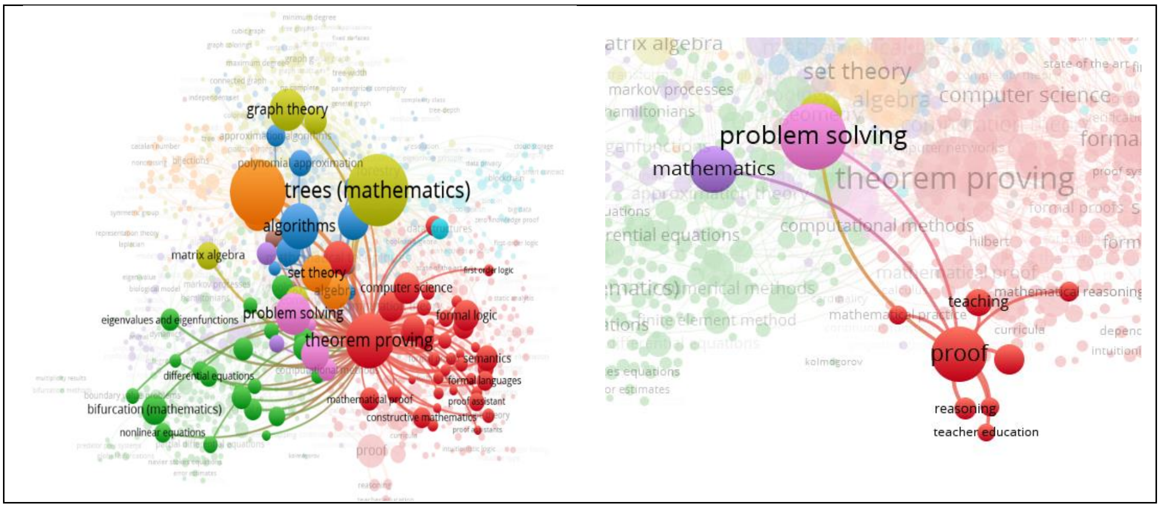


Figure 1. Linkages between the keywords "proof" and "computational thinking" visualized by VOSviewer

Two main keywords, "theorem proving" and "proof," appear in the two figures (see Figure 1). The former is related in terms of research context with "problem-solving," "graph theory," "matrix algebra," etc., while the latter with "problem-solving," "reasoning," "teacher education," etc. No research work links "students' mathematical proof skills" with "computational thinking" or "abstract algebra."

Based on the results of the above exploration, this study aimed to explore students' achievement of mathematical proof skills in the Abstract Algebra course with an intervention through several components of computational thinking in the learning process. There are three research questions posed in this study: (1) How did the design of worksheets examining students' proof processes in the Abstract Algebra course based on CT components? (2) Is there an increase in students' mathematical proof skills after learning the Abstract Algebra based on CT component course? and (3) How did the process of proof applying computational thinking stages in the Abstract Algebra course go?

2. METHOD

2.1. Research Design

This study used a sequential explanatory research design, where quantitative data analysis was supported by qualitative data (Creswell & Creswell, 2017). The quantitative research method was applied to a quasi-experimental with one sample:

$O \ X_1 \ \ O \ X_2$ (Davison & Smith, 2018)

Where O is the treatment,
X1 is the first test, and
X2 is the second test

This research began with implementing learning based on the stages of computational thinking. After seven meetings of teaching and learning activities, a test was conducted to see students' achievement of mathematical proof skills. Another seven learning meetings were conducted afterward, followed by another test to review the students' achievement of mathematical proof skills a second time. Quantitative data were collected from students' results of the first and second tests.

Moreover, a qualitative methodology was employed utilizing a case study approach. Of the 32 prospective teachers who completed the test, three representative responses were selected, which could be indicative of other responses with a similar process. Semi-structured interviews were conducted to align with the student's written responses. In order to gain more in-depth information, and identify the thought processes of the students on whom CT stages were implemented to help with their mathematical proof.

2.2. Population and Sample

The participants in this study were 32 students of the Mathematics Education study program at Universitas Pendidikan Indonesia. They studied the ring theory topic, part of the Abstract Algebra course, in the odd semester of 2023. They were in their fifth semester of studying at the university. Before taking this course, they took the number theory, linear algebra, and group theory courses. Three of the students who had completed the assignment were selected to participate in interviews, they chose because of specific answer (Etikan et al., 2015). This selection was grounded on the selected students' representativeness of responses with similar response characteristics. In other words, it was expected that they represented students with similar stages of thinking as theirs.

2.3. Instruments

The quantitative instruments used in this study were tests, while the qualitative instruments were interview guides. Prior to undertaking the test, the students engaged in learning activities using the worksheet were prepared in compliance with the components of computational thinking. In the context of mathematical proof (Ambarwati et al., 2017; Intan et al., 2022), computational thinking was expected to assist students in navigating the components.

The mathematical proof indicators utilised in the student assessment are delineated in Table 1 (A'idah, 2022), accompanied by the CT components that are tasked with assisting in the resolution of the mathematical proof problem.

Table 1. The mathematical proof indicator intersects with component of CT

Indicators Mathematical Proof	CT Components
Understanding the problem and simplifying it	Decomposition
Determine ideas based on patterns from given statements	Pattern recognition
Provide reasons for each step of a given mathematical proof	Abstraction
Assessing true/false statements in each step of the proof	Algorithmic thinking

Moreover, qualitative data were gathered through semi-structured interviews. The interview instrument was employed to ascertain the students' methodology for solving mathematical proof problems. Consequently, the questions were structured according to the following proof indicators: (a) does recording the known information facilitate the resolution of the problem?; (b) following the recording of the known information, what is the subsequent step?; (c) how do you identify the solution pattern?; (d) How do you manipulate numbers to find the solution?

2.4. Procedure

The study began with the delivery of lectures. At the start of each lecture, students were assigned to complete a worksheet designed to test their understanding of certain concepts they were assigned to learn independently in the previous week. For example, if the lecture to be delivered in a meeting this week concerned the definition and examples of a ring, students were assigned to learn them independently one week before this week.

After the first seven learning sessions in which students completed tasks following the CT components, the first test was administered to measure their abilities. After another seven sessions, the second test was given. The first test question contains materials on the definitions of ring, subring, integral region, and field. The second test question contains materials on ring homomorphisms, ring polynomials, ideals, and the fundamental theorem of homomorphism. The assessment of students' responses was based on the rubric presented in Table 2.

Table 2. CT-based mathematical proof ability assessment rubric

CT Component	Indicator Mathematical Proof	Score
Decomposition	The student wrote known and questioned components completely in their language	2
	The student did not write down all known and questioned components	1
	The student left the answer sheet blank	0
Pattern recognition	The student could find the right pattern to simplify and solve problems	3
	The student found inappropriate patterns to simplify and solve problems	2
	The student did not complete the argument that had been formed	1
	The student left the answer sheet blank	0
Abstraction	The student could identify general principles that produced patterns or regularities	3
	The student could identify some general principles that produced patterns or regularities	2
	The student identified general principles but did not arrive at the correct answer	1
	The student left the answer sheet blank	0
Algorithmic thinking	The student developed and carried out procedures following the directed concepts and performed calculations well and correctly	2
	The student developed and carried out procedures incorrectly or performed calculations incorrectly	1
	The student left the answer sheet blank	0

In addition, representative student responses were selected for further analysis. Three students with representative responses were interviewed. The results of the interview became the source of data to identify the students' thinking patterns. The flow chart in this study is shown in Figure 2.

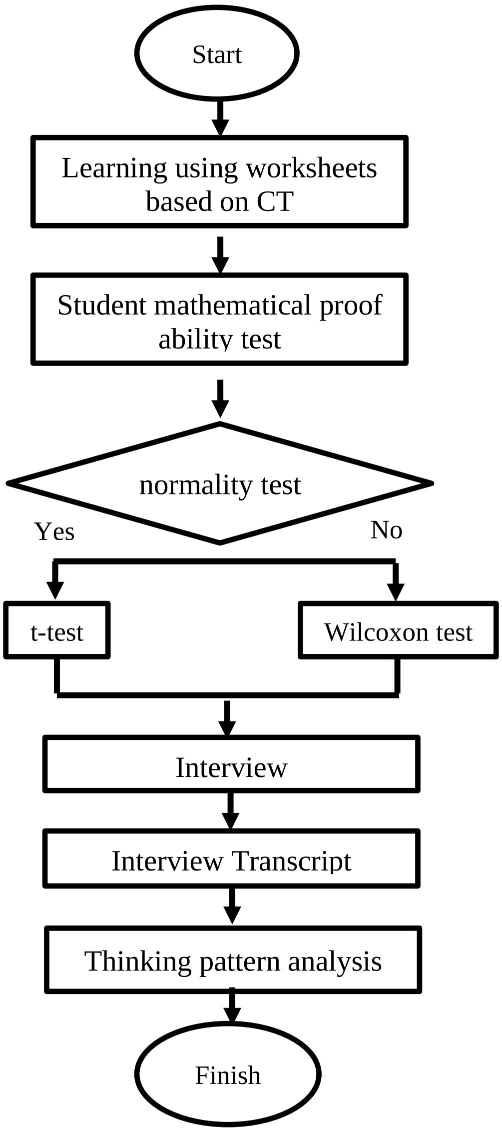


Figure 2. Flowchart of research procedure

2.5. Data Analysis

Data were collected from the first and second tests and interviews and analyzed for relevance and adequacy. The data from the first and second tests were analyzed by conducting a dependent t-test or Wilcoxon signed-rank test to see the impact of CT on achieving mathematical proof skills, and achievement of students of the Mathematics Education program and observe their improvement after the second test in comparison to their outcomes in the first test.

From the 32 student responses, representative answers were selected. Selected representative responses were analyzed to examine the proof processes of the students. Answer sheets and interview transcripts were analyzed based on completeness in the context of mathematical proof skills through each CT components. Interviews were carried out to clarify the students' thought processes that they demonstrated in writing. Interview transcripts were written to allow for repeated reading and easy understanding and to match the interview results with the written answers. In addition, the researcher clarified the students' answers to the questions in the worksheets they worked on previously, which served as the main source of data on the student's learning process in the classroom. This was done to create good credibility in qualitative research (Fraenkel et al., 1993). Data from tests, interviews, and documentation were analyzed in the context of mathematical proof skills.

3. RESULTS AND DISCUSSION

Research data from the first and second tests will be presented in the form of descriptive statistical data. These data were analyzed using a dependent t-test or Wilcoxon signed-rank test. Students' mathematical proof skills demonstrated through worksheet answers were examined following CT components.

3.1. Results

The design of worksheets examining students' proof processes in the Abstract Algebra course based on CT components

The Abstract Algebra course taken by Mathematics Education students comprises four topics, each supported by a module corresponding to the stages of CT. These modules facilitate students' ability to prove. CT provides a structured approach to building predictive models, conducting investigations, and analyzing data. In this context, CT-steps-based worksheets guided students in solving problems with a more systematic thinking process. In other words, these worksheets guided students to identify the most appropriate way to describe patterns and processes. A problem regarding the concept of homomorphism, one of the topics raised in the Abstract Algebra course, is depicted in Figure 3.

PERMASALAHAN Diketahui C adalah himpunan bilangan kompleks, (C, +, x) adalah suatu ring. γ: C → C yang didefinsikan oleh γ (a+bi) = a-bi. Buktikan bahwa pemetaan γ merupakan suatu isomorfisma! Langkah yang harus dilakukan: a. Apa yang diketahui dan apa yang harus diselesaikan? b. Konsep apa yang diperlukan untuk membuktikan masalah itu? c. Langkah apa yang diperlukan untuk menyelesaikan masalah tersebut? d. Lakukan langkah yang disajikan pada bagian c!	**Translation** PROBLEM Let C be a set of complex numbers and (C, +, x) be a ring. γ: C → C is defined by γ (a + bi) = a - bi. Prove that γ is an isomorphism! a. What is known and what needs to be solved? b. What concepts are needed to prove the problem? c. What steps are needed to solve the problem? d. Carry out the steps presented in section c!

Figure 3. Intervention with an abstract algebra worksheet based on CT components

Each proof problem contained a guiding principle that encouraged students to develop more focused solutions. Each affirmation was tailored to the CT components. More details can be found in Table 3.

Table 3. Adjustment of CT components to the stages used in the worksheet

CT components	Context at each CT component	Guide to the problem
Decomposition	Troubleshooting and finding the core problem	What is known and what needs to be solved?
Pattern recognition	Looking for patterns in a problem to solve	What concepts are needed to prove the problem?
Abstraction	Identifying general principles that create patterns	What steps are needed to solve the problem?
Algorithmic thinking	Using the steps/information to solve the problem	Carry out the steps presented in section c!

The CT components applied through questions assist students in decomposing intricate problems into more straightforward components, thereby facilitating deeper comprehension, description, or analysis of them. For illustration, an exemplary worksheet is depicted in Figure 3. The question "What is known and what needs to be solved?" was designed to assist students in conducting preliminary investigations by identifying pivotal elements within the problem. In turn, these essential elements were organized into a systematic structure. The question "What concepts are needed to prove the problem?" was designed to assist students in anticipating the concepts pertinent to the problem. Students were encouraged to investigate the concepts and their relevance to the problem to be proven. This intervention also aimed to help students identify emerging patterns and trends and find alternative solutions. The next question "What steps are needed to solve the problem?" directed students' focus to the constituent elements of the solution. This was exemplified by the specific steps that led to the desired outcome. The final instruction "Carry out the steps presented in part c to solve the problem!" required that students navigate the solution within the established framework. At this juncture, students must be able to cope with the manipulation of the mathematical context.

Quantitative Result. Students' mathematical proof skills achievement in the Abstract Algebra course after studying with CT worksheets

A total of 32 students participated in the first and second tests. The evaluation criteria were based on a rubric. The students demonstrated a commendable performance in both the first and second tests (see Table 4). This indicates that the students possessed the requisite abilities to engage with the learning process of abstract algebra in accordance with the CT components. Furthermore, the overall score of the students exhibited an upward trend from the first test to the second one. The results demonstrated a notable improvement in the students' outcomes, as illustrated in Table 4.

Table 4. Comparison of the first and second test results

		Statistic	Std. Error
First test	Mean	68.66	39.61
	Std. deviation	22.45	
	Minimum	20.00	
	Maximum	97.50	
	Range	77.50	
	Interquartile range	25.00	
	Skewness	-.933	.414
	Kurtosis	-.048	.809
Second test	Mean	80.09	17.28
	Std. deviation	37.79	
	Minimum	48.75	
	Maximum	98.75	
	Range	40.00	
	Interquartile range	15.41	
	Skewness	-.384	.414
	Kurtosis	-.447	.809

The descriptive statistics in Table 4 illustrate that the mean score on the second test was higher than the mean score on the first test, with an average difference of 11.42. The distribution of first and second test score data exhibited a broad range of values, with the mode exceeding the mean. Additionally, there was an increase in the minimum score, with a difference of 28.75. This suggests that students had enhanced mathematical proof skills. The significance of the difference between the first and second test results was identified using a paired samples t-test, and the results are presented in Table 5.

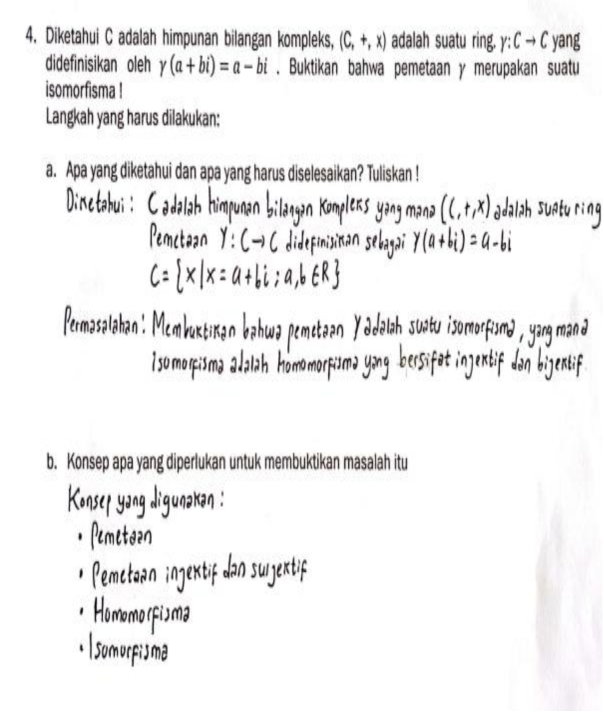
Tabel 5. T-paired test result

		Paired Differences					t	df	Sig. (2-tailed)
		Mean Difference	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	The second test - The first test	11.41	14.34	2.53	6.24	16.59	4.50	31	0.000

Based on the paired sample test output (see Table 5), the Sig (2-tailed) value is 0.000 <0.05, so Ho is rejected and Ha is accepted. It can be concluded that there is an average difference between the learning outcomes in the first test and the second test. This means that the mathematical proof process in the algebra courses supported by CT components affects the improvement of the mathematical proof skills of the prospective teachers.

Identification of the proof process in the Abstract Algebra course based on the CT stages

The proof process of the prospective teachers in abstract algebra was explored based on their written answers and interviews, with the latter being intended to clarify the former. This analysis was conducted based on the contexts of mathematical proof skills indicators according to the CT components. The worksheet the prospective teachers worked on contained four guidelines representing the contexts of the mathematical proof skills indicators, which served to affirm prospective teachers in the construction of the correct solution. Based on the initial analysis of each response, three representative responses were analyzed in more depth through semi-structured interviews. Figure 4 following is the answer of one of the prospective teachers (M1) on the topic of homomorphism. This student M1 illustrates the type of student who can construct answers correctly based on the CT stages. At each component, M1 wrote the proof rules well, which was reinforced by the interview process.


4. Diketahui C adalah himpunan bilangan kompleks, (C, +, x) adalah suatu ring. $\gamma: C \to C$ yang didefinisikan oleh $\gamma(a+bi) = a - bi$. Buktikan bahwa pemetaan γ merupakan suatu isomorfisma !

Langkah yang harus dilakukan:

a. Apa yang diketahui dan apa yang harus diselesaikan? Tuliskan !

Diketahui : C adalah himpunan bilangan Kompleks yang mana (C, +, x) adalah suatu ring
Pemetaan $\gamma: C \to C$ didefinisikan sebagai $\gamma(a+bi) = a - bi$
$C = \{x \mid x = a + bi ; a, b \in R\}$

Permasalahan : Membuktikan bahwa pemetaan γ adalah suatu isomorfisma, yang mana Isomorfisma adalah homomorfisma yang bersifat injektif dan bijektif

b. Konsep apa yang diperlukan untuk membuktikan masalah itu

Konsep yang digunakan :
- Pemetaan
- Pemetaan injektif dan surjektif
- Homomorfisma
- Isomorfisma

Translation

Let C be a set of complex numbers and (C, +, x) be a ring. γ: C → C is defined by γ (a + bi) = a - bi. Prove that γ is an isomorphism!

a. What is known and what needs to be done?

Known: C is a set of complex numbers, and (C, +, x) is a ring. γ: C → C is defined by γ (a + bi) = a - bi. C = {x I x = a + b; a, b € R}

Problem: To prove that γ mapping is an isomorphism, which means a homomorphism that is injective and bijective in nature

b. What concepts are needed to prove the problem?

The concepts used are:
1. mapping
2. injective and surjective mapping
3. homomorphism
4. isomorphism

Figure 4. M1's answer at the decomposition and pattern recognition component

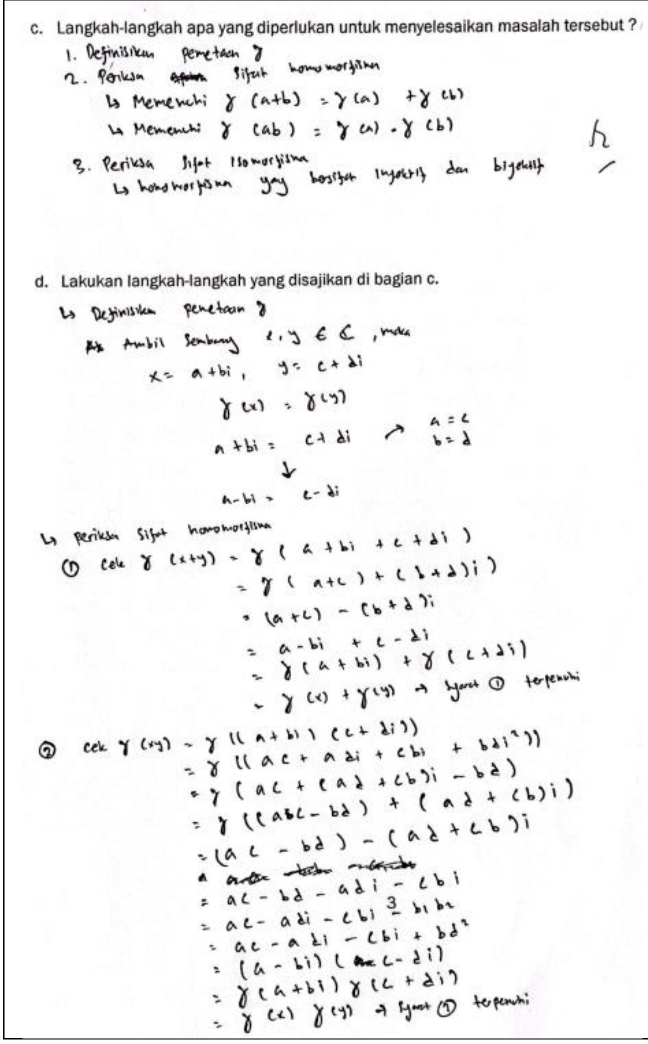
The solution proposed by M1 demonstrated that M1's initial assumption led them to the correct conclusion. By breaking down the proof into smaller stages, M1 redefined (C, +, x) as a ring in their language. This was evidenced by M1's ability to correctly write the set C in mathematical notations. Moreover, the indicator "creating more manageable parts" was demonstrated by M1 through the identification and arrangement of the components of the problem into appropriate categories, namely "known" and "problem."

To facilitate the appropriate course of action, M1 articulated a comprehensive and logical framework in Part B. The outcomes demonstrated that M1 was capable of identifying patterns within the proof to inform the solution process. These analysis results aligned with the findings from the interview with M1.

Interviewer	:	*When you wrote the known or unknown parts of the problem, did you understand what you were writing or did you just copy it?*
M1	:	*Understood the problem first, then explained it in my language*
Interviewer	:	*Did you do points a to "d" in order?*
M1	:	*Yes, because point b helped me do the next point. Point b must be done well to execute the steps correctly.*
Interviewer	:	*What were the advantages of working on point b for constructing the solution?*
M1	:	*Enabled more systematic construction of a pathway to arrive at the solution.*

After formulating the concepts, the steps that had to be taken were identified. The next assessment indicator focused on the fundamental aspects of the proof. M1 on homomorphism had correctly formulated the necessary steps to prove that the set C is an isomorphism. This was demonstrated by a step in which M1 showed that γ is a mapping and a homomorphism, which is a requirement for an isomorphism (injective and bijective). For more details, see Figure 4.

The final stage of solution construction was to develop a step-by-step logical sequence to efficiently solve each part of the proof. The correctness of the steps written by M1 in the previous stages facilitated the completion of this final stage. M1 correctly defined the mapping and performed the appropriate algorithm. Furthermore, the homomorphism conditions were checked with proper variable manipulation. The term "variable manipulation" is used to describe the process of operating or changing variables in an equation or mathematical expression to simplify, solve, or resolve a particular problem. This process may include a range of operations, such as addition, subtraction, multiplication, division, substitution, or other algebraic transformations, to achieve the desired form or solution as shown in Figure 5.



Translation

c. What steps are needed to solve the problem?
   1. Define the mapping of γ
   2. Check the homomorphism properties
      Meeting $\gamma(a + b) = \gamma(a) + \gamma(b)$
      Meeting $\gamma(ab) = \gamma(a) \cdot \gamma(b)$
   3. Check the isomorphism properties
      Homomorphisms that are injective and bijective

d. Carry out the steps presented in section c!
   - Define the mapping of γ

   any x, y ∈ C, hence x = a + bi and y = c + di … **(for complete answer look at the picture)**

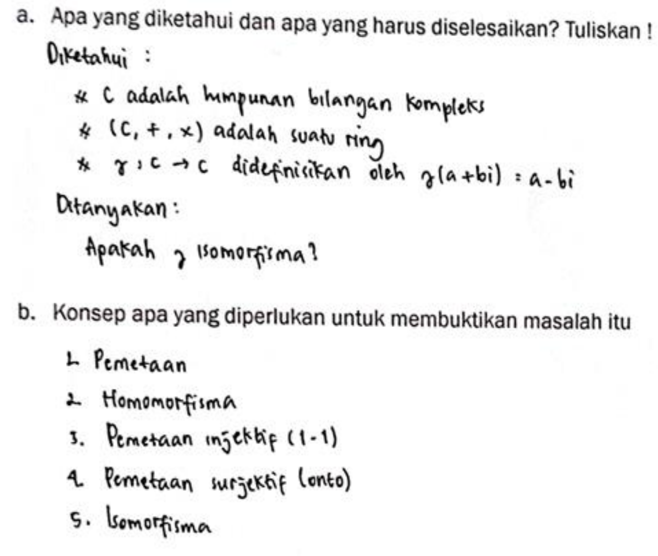
   - Check the isomorphism properties

   $\gamma(x + y) = \gamma(a + bi + c + di)$
   $= \gamma(a + c) + (b + d)i)$ …
   $\gamma(xy) = \gamma((a + bi)(c + di))$
   $= \gamma((ac + adi + cbi + bdi^2))$ …

(for complete answer look at the picture)

Figure 5. M1's answer at the abstraction and algorithmic thinking component

In other cases, there were groups of answers showing that prospective teachers managed to complete the component of decomposition, pattern recognition, and abstraction but failed to construct a good equation at the algorithmic thinking component. The provision of an accurate concept did not guarantee that prospective teachers would arrive at the optimal final solution, as illustrated in Figure 5. For example, M2 effectively identified the requisite concept for the solution. However, M2 encountered a difficulty in manipulating variables to demonstrate that the equation $\gamma(m + ni)$ is a function and bijective. This is because, at the stage of developing a step-by-step logical sequence to solve each part of the proof efficiently, it is necessary to demonstrate both shrewdness and accuracy in identifying opportunities to arrange the variables sought in optimal ways. M2's thought process at the decomposition and pattern recognition components is shown in Figure 6.


a. Apa yang diketahui dan apa yang harus diselesaikan? Tuliskan !

Diketahui :

* C adalah himpunan bilangan kompleks
* (C, +, x) adalah suatu ring
* $\gamma : C \to C$ didefinisikan oleh $\gamma(a+bi) = a-bi$

Ditanyakan:

Apakah γ isomorfisma?

b. Konsep apa yang diperlukan untuk membuktikan masalah itu

1. Pemetaan
2. Homomorfisma
3. Pemetaan injektif (1-1)
4. Pemetaan surjektif (onto)
5. Isomorfisma

Translation

a. What is known and what needs to be done?
Known:
C is a set of complex numbers
(C, +, x) is a ring
γ: C → C is defined by
$\gamma(a + bi) = a - bi$
Problem: Is γ an isomorphism?

b. What concepts are needed to prove the problem?

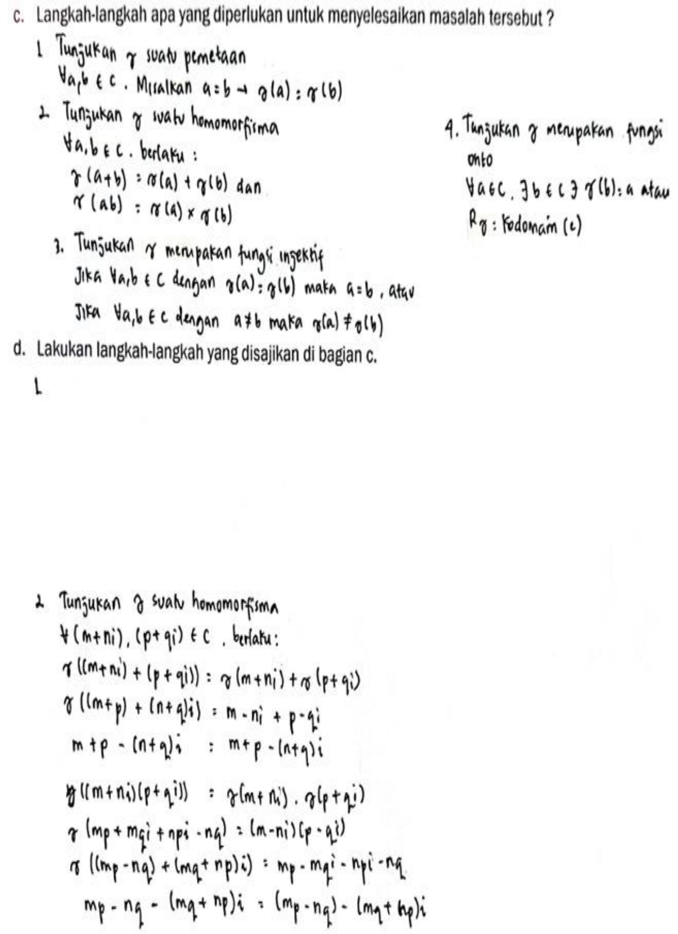
1. Mapping
2. Homomorphism
3. Injective (1-1)
4. Surjective (onto)
5. Isomorphism

Figure 6. M2's answer at the decomposition and pattern recognition components

At the decomposition stage, M2 was doing well in constructing the solution. M2 could "break the problem down into smaller problems," as characterized by the arrangement of important points derived from the complex problem. M2 systematically arranged the important points so that the overall problem could still be properly identified, which satisfied the indicator "creating more manageable parts." This made it easier for M2 to develop concepts to be used to arrive at the solution.

M2 was 'identifying patterns within the proof to guide the solution process' by stating the concepts needed to solve the problem in an organized and sequential way. To be able to prove that all equations are isomorphic, M2 needed to prove the mapping, homomorphism, injective, and surjective first. Figure 6 illustrates M2's thought process at the abstraction and algorithmic thinking components.

At the abstraction components, M2 was 'focusing on fundamental aspects of the proof' by emphasizing the important elements making up a valid mathematical proof. This was evidenced by clear statements concerning the definitions and theorems of mapping, homomorphism, and isomorphism. In addition, M2 also ensured the logical consistency of each step. This involved organizing the proof in a systematic format, starting with the premise and followed by the argument, as illustrated in M2's proof that γ is injective in Figure 7: "If $\forall$ a, b $\in$ c, $\gamma(a) = \gamma(b)$, then a = b, or if a ≠ b, then $\gamma(a) \neq \gamma(b)$."



Translation

c. What steps are needed to solve the problem?
   1. Proof that γ is a mapping
      ∀a, b ∈ c. ex: a = b → γ(a) = γ(b)
   2. Proof that γ is a homomorphism
      ∀a, b ∈ c, the following apply:
      γ(a + b) = γ(a) + γ(b)
      γ(ab) = γ(a) · γ(b)
   3. Proof that γ is injective
      If ∀a,b ∈ c with γ(a) = γ(b), then a = b, or if a ≠ b, then γ(a) ≠ γ(b)
   4. Proof that γ is onto
      ∀a, b ∈ c, ∃ b ∈ c → γ(b) = a or Rγ: codomain (c)

d. Carry out the steps presented in section c!
   1. …
   2. Proof that γ is a homomorphism
      ∀(m + ni), (p + qi) ∈ c, the following apply:
      γ((m + ni) + (p + qi)) = γ(m + ni) + γ(p + qi)
      γ((m + ni) + (p + qi)) = m - ni + p - qi …

(for complete answer look at the picture)

Figure 7. M2's answer at the abstraction and algorithmic thinking components

In addition to systematic organization, proof requires the selection of appropriate techniques, such as direct proof, indirect proof, or contradiction proof, according to the context of the problem. Errors in the choice of proof techniques can hinder the preparation of arguments. In addition, it is important to be clever in the manipulation of variables. The correct manipulation of variables is key to mathematical proof, especially in the process of simplifying expressions and solving equations. M2 could not prove that γ is a homomorphism because they did not finish manipulating the variables. M2 failed to recognize the important relationship between the variables, which meant that the technique used did not lead to the final solution.

The data derived from the analysis of prospective teachers' written test responses were supported by interview data. The following interview excerpt provides an illustrative example.

Interviewer	:	*Were points a to d useful for solving the problem?*
M2	:	*They helped you work through step d more easily.*
Interviewer	:	*What stage was hardest to work on?*

M2 : *Point d, because it required the ability to manipulate numbers. It takes a lot of practice with problems of this sort.*

Based on the results of the interview obtained information that students have difficulty in manipulating numbers due to limited ideas, so what is needed in opening their horizons is to do more problems with different types.

3.2. Discussion

Computational thinking (CT) can help prospective teachers overcome difficulties in constructing mathematical proofs on abstract algebra concepts. CT can contribute to mental processes in not only instrumental but also more important and conceptual ways, which affects the way of working and thinking. This notion is supported by research (Lee et al., 2024; Masfingatin & Maharani, 2019) which states that work steps arranged based on CT components can improve students' mathematical reasoning, problem-solving, and thinking systems.

Computational thinking (CT) approaches, constructed in the form of questions, can guide prospective teacher to envisage novel problem-solving strategies and evaluate new solutions. This is achieved by applying algorithms, decomposition, abstraction, and logic to solve complex problems (Masfingatin & Maharani, 2019; Wu et al., 2024). Decomposition is a fundamental concept in CT, whereby the essential features of a problem are identified and irrelevant details are disregarded (Coşkun et al., 2024). The ability to define the problem is required in the investigation of complex systems (Salwadila & Hapizah, 2024). Meanwhile, pattern recognition is defined as the process of establishing relationships and identifying patterns. The ability to identify patterns is a valuable skill in several contexts, particularly in one where prospective teachers are required to discern regularities and establish rules. In the field of abstract algebra, for instance, patterns can be identified through the examination of numerical regularities (Calado et al., 2024). The capacity to apply modeling concepts is also crucial for the development of suitable solutions.

The next component is abstraction, which is the capacity to conceptualize and represent an idea or process in more general terms by emphasizing the salient aspects of the idea. It is defined as the process of developing descriptive and representative models (Csizmadia et al., 2015). Therefore, abstraction represents a principal means of applying computational power to mathematical and scientific problems. The final stage, the ability to create appropriate algorithmic systems to build models, is of great importance in mathematics, as there is often more than one possible course of action to construct a solution (Curzon & McOwan, 2017). Even if two different methods produce the same correct result, other aspects must be considered when using these methods. Based on these four stages, CT analysis focuses on an inclusive examination of how the system and its constituent parts interact and relate to each other as a whole (Assaraf & Orion, 2005).

CT can assist prospective teachers in developing critical thinking skills and new ways of thinking solve complex problems even on difficult topics such as abstract algebra course. The integration of CT in learning, particularly in mathematics, has been a subject of extensive research and has been demonstrated to enhance students' mathematical abilities and problem-solving skills (Chevalier et al., 2020; Gabriele et al., 2019; Masfingatin & Maharani, 2019; Rodríguez-Martínez et al., 2020). The CT model demonstrates ideas that lead students to a deeper understanding of mathematical concepts. Therefore, the application of CT in the learning of abstract algebra can help students develop problem-solving skills and overcome difficulties in constructing mathematical proofs (Cetin & Dubinsky, 2017; Kilhamn et al., 2022; Rodríguez-Martínez et al., 2020). Another advantage of employing computational thinking in the teaching and learning process is that the abstraction and reasoning skills that are involved will enhance students' intellectual abilities (Wing, 2006). The integration of CT components into learning, for instance in the form of worksheets which are structured with attention to the stages of CT thinking, can facilitate students' performance on tasks.

The use of worksheets based on CT components enables students to develop the capacity to think systematically. The CT components facilitate the simplification of problems and the systematic construction of solutions. CT empowers all students to conceptualize, analyze, and solve complex problems more effectively by selecting and applying appropriate strategies and tools (Wilkerson & Fenwick, 2016). Furthermore, the questions presented at each stage of the worksheet minimize the likelihood of errors in the construction of solutions. This is because the components of CT thinking allow students to cross-check the concepts used, thus preventing any errors in the construction of solutions. The worksheet guides students to carry out proofs based on the components of computational thinking, which in turn familiarizes students with the construction of precise and systematic proofs.

Nevertheless, difficulties to prove persist at each component of CT (Doruk & Kaplan, 2015; Selden & Selden, 2008). In decomposition, the primary challenge is the inability to comprehend the problem. Indeed, the capacity to grasp the problem represents the initial step in conceptualizing and formulating solutions. Ultimately, some students merely reiterate the salient details of the problem. However, rewriting these crucial elements can also facilitate the identification of the problem. At the very least, the student's attention is directed toward the significant information that has been documented.

Furthermore, at the components of pattern recognition, the difficulty that arises is to determine the concept that is in accordance with the pattern formed. A lack of understanding of the theoretical concepts that have been learned becomes an obstacle in the process of proving (Belay et al., 2024). As a result, students experience a deadlock when faced with problems. Finally, difficulties at the algorithmic thinking stage occur due to students' inexperience in manipulating variables. Variable manipulation is closely related to algebraic operations, and therefore the possibility of this occurrence is due to students' lack of practice in working on mathematical problems. Algebraic manipulation itself is a challenging activity as it requires problem-solving skills on non-routine problems.

4. CONCLUSION

The use of worksheets integrated with CT enables students to modify the process of proving. This is because these worksheets facilitate the acquisition of the skills required to construct proofs by encouraging students to compile and elaborate on CT components. The

CT component is designed in the form of questions that can construct students' thinking processes in proving algebraic problems. The improvement in the ability to prove in this study was observed based on the first and second test scores after the learning process. This means that the mathematical proof process in algebra courses can be supported by integrating the CT component into the worksheet used. The prospective teacher's thought process of proving algebraic problems with the help of decomposition questions on CT, students can translate and simplify the problem well. The second direction of pattern recognition based on CT components can help students find patterns based on the statements made. Furthermore, the abstraction component helps students to use appropriate concepts to solve proof problems. Finally, the algorithmic thinking component helps students construct a systematic and orderly solution algorithm without missing any steps. However, some of the difficulties that arise at this stage are difficulties in manipulating variables to reach a solution. The results of this study can be used to develop strategies to help students construct and carry out proofs, particularly in algebraic structure courses.

Acknowledgments

The authors would like to thank the LPPM of Universitas Pendidikan Indonesia for providing research financing through the research center system (contract number 535/UN40/PT.01.02/2023).

Declarations

Author Contribution	: EN: Conceptualization, Writing - Original Draft, Formal analysis, and Methodology; AP: Writing - Original Draft, Formal analysis, and Methodology; MT: Writing - Review & Editing; JAD: Editing and Visualization; DU: Validation and Supervision.
Funding Statement	: This research was funded by the research center system (contract number 535/UN40/PT.01.02/2023).
Conflict of Interest	: The authors declare no conflict of interest.
Additional Information	: Additional information is available for this paper.

REFERENCES

A'idah, Z. (2022). *Analis kemampuan argumentasi matematis siswa dalam pembuktian matematika ditinjau dari kecerdasan linguistik*. FITK UIN Syarif Hidayatullah Jakarta. https://repository.uinjkt.ac.id/dspace/handle/123456789/58913

Albert, A. A. (2018). *Modern higher algebra*. Courier Dover Publications.

Ambarwati, S., Fauzan, A., & Fauzi, A. (2017). Improving communication ability and problem solving mathematical by using PACE model learning reviewed from difference learning style and self efficacy In International Conference on Mathematics and Mathematics Education (ICM2E 2017), Universitas Negeri Padang, West Sumatera, Indonesia (pp. 335-361).

Arbaugh, F., Smith, M., Boyle, J., Stylianides, G. J., & Steele, M. (2018). *We reason & we prove for all mathematics: Building students' critical thinking, grades 6–12*. Corwin Press. https://doi.org/10.4135/9781071800812

Arnawa, I. M., Yerizon, Y., & Nita, S. (2020). Improvement students' achievement in elementary linear algebra through APOS theory approach. *Journal of Physics: Conference Series*, *1567*(2), 022080. https://doi.org/10.1088/1742-6596/1567/2/022080

Assaraf, O. B. Z., & Orion, N. (2005). Development of system thinking skills in the context of earth system education. *Journal of Research in Science Teaching*, *42*(5), 518-560. https://doi.org/10.1002/tea.20061

Aydeniz, M. (2018). Integrating computational thinking in school curriculum. In M. S. Khine (Ed.), *Computational Thinking in the STEM Disciplines: Foundations and Research Highlights* (pp. 253-277). Springer International Publishing. https://doi.org/10.1007/978-3-319-93566-9_13

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 27-44). National Council of Teachers of Mathematics.

Belay, A. M., Machaba, F., & Makgakga, T. P. (2024). Introducing a supportive framework to address students' misconceptions and difficulties in the learning mathematical proof techniques: A case of debark university in Ethiopia. *Research in Social Sciences and Technology*, *9*(1), 63-84. https://doi.org/10.46303/ressat.2024.4

Bergwall, A. (2019). Proof-related reasoning in upper secondary school: characteristics of Swedish and Finnish textbooks. *International Journal of Mathematical Education in Science and Technology*, *52*(5), 731-751. https://doi.org/10.1080/0020739X.2019.1704085

Bruner, J. (1974). *Toward a theory of instruction*. Harvard university press.

Calado, A., Roselli, P., Gruppioni, E., Marinelli, A., Bellingegni, A. D., Boccardo, N., & Saggio, G. (2024). A geometric algebra-based approach for myoelectric pattern recognition control and faster prosthesis recalibration. *Expert Systems with Applications*, *254*, 124373. https://doi.org/10.1016/j.eswa.2024.124373

Cansu, F. K., & Cansu, S. K. (2019). An overview of computational thinking. *International Journal of Computer Science Education in Schools*, *3*(1), 17-30. https://doi.org/10.21585/ijcses.v3i1.53

Cetin, I., & Dubinsky, E. (2017). Reflective abstraction in computational thinking. *Journal of Mathematical Behavior*, *47*, 70-80. https://doi.org/10.1016/j.jmathb.2017.06.004

Chevalier, M., Giang, C., Piatti, A., & Mondada, F. (2020). Fostering computational thinking through educational robotics: a model for creative computational problem solving. *International journal of STEM education*, *7*(1), 39. https://doi.org/10.1186/s40594-020-00238-z

Coşkun, I., Özen Ünal, D., Eryalçin, S., & Yazici, E. (2024). Investigation of pre-service middle school mathematics teachers habits of mind in pattern generalization problems. *Osmangazi Journal of Educational Research*, *10*(2), 52-75. https://doi.org/10.59409/ojer.1359752

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.

Csizmadia, A., Curzon, P., Dorling, M., Humphreys, S., Ng, T., Selby, C., & Woollard, J. (2015). *Computational thinking-A guide for teachers*. University of Southampton. https://eprints.soton.ac.uk/424545/

Curzon, P., & McOwan, P. W. (2017). *The power of computational thinking: Games, magic and puzzles to help you become a computational thinker*. World Scientific. https://doi.org/10.1142/q0054

D'Alessandro, W. (2019). Explanation in mathematics: Proofs and practice. *Philosophy Compass*, *14*, e12629. https://doi.org/10.1111/phc3.12629

Davison, R. C. R., & Smith, P. M. (2018). Quantitative data analyses. In S. R. Bird (Ed.), *Research Methods in Physical Activity and Health* (pp. 168-183). Routledge. https://doi.org/10.4324/9781315158501-17

Dong, Y., Cateté, V., Jocius, R., Lytle, N., Barnes, T., Albert, J., Joshi, D., Robinson, R., & Andrews, A. (2019). Prada: A practical model for integrating computational thinking in K-12 education In SIGCSE '19: The 50th ACM Technical Symposium on Computer Science Education, New York (pp. 906-912). https://doi.org/10.1145/3287324.3287431

Doruk, M., & Kaplan, A. (2015). Prospective mathematics teachers' difficulties in doing proofs and causes of their struggle with proofs. *Bayburt Eğitim Fakültesi Dergisi*, *10*(2), 315-328.

Emara, M., Hutchins, N. M., Grover, S., Snyder, C., & Biswas, G. (2021). Examining student regulation of collaborative, computational, problem-solving processes in openended learning environments. *Journal of Learning Analytics*, *8*(1), 49-74. https://doi.org/10.18608/JLA.2021.7230

Etikan, I., Musa, S. A., & Alkassim, R. S. (2015). Comparison of convenience sampling and purposive sampling. *American journal of theoretical and applied statistics*, *5*(1), 1-4. https://doi.org/10.11648/j.ajtas.20160501.11

Feudel, F., & Unger, A. (2024). Development and implementation of concept-test questions in abstract algebra. *International Journal of Mathematical Education in Science and Technology*, 1-30. https://doi.org/10.1080/0020739X.2023.2297002

Fraenkel, J., Wallen, N., & Hyun, H. (1993). *How to design and evaluate research in education* (10th ed.). McGraw-Hill Education.

Gabriel, F., Buckley, S., & Barthakur, A. (2020). The impact of mathematics anxiety on self-regulated learning and mathematical literacy. *Australian Journal of Education*, *64*(3), 227-242. https://doi.org/10.1177/0004944120947881

Gabriele, L., Bertacchini, F., Tavernise, A., Vaca-Cárdenas, L., Pantano, P., & Bilotta, E. (2019). Lesson planning by computational thinking skills in Italian pre-service teachers. *Informatics in Education*, *18*(1), 69-104. https://doi.org/10.15388/infedu.2019.04

Gouvêa, F. Q. (2012). *A guide to groups, rings, and fields*. MAA. https://doi.org/10.5948/upo9781614442110

Grover, S., & Pea, R. (2023). Computational thinking: A competency whose time has come. In S. Sentance, E. Barendsen, N. R. Howard, & C. Schulte (Eds.), *Computer Science*

Education (pp. 51-68). Bloomsbury Academic. https://doi.org/10.5040/9781350296947.ch-005

Halbeisen, L., Hamilton, M., & Růžička, P. (2007). Minimal generating sets of groups, rings, and fields. *Quaestiones Mathematicae, 30*(3), 355-363. https://doi.org/10.2989/16073600709486205

Hanna, G., & Barbeau, E. (2002). *Proof in mathematics*. University of Toronto.

Hanna, G., & Barbeau, E. (2010). Proofs as bearers of mathematical knowledge. In G. Hanna, H. N. Jahnke, & H. Pulte (Eds.), *Explanation and Proof in Mathematics: Philosophical and Educational Perspectives* (pp. 85-100). Springer US. https://doi.org/10.1007/978-1-4419-0576-5_7

Hanna, G., & de Villiers, M. (2008). ICMI study 19: Proof and proving in mathematics education. *Zdm*, *40*, 329-336. https://doi.org/10.1007/s11858-008-0073-4

Intan, R., Rejeki, R., Panggabean, R. R., Sari, I. P., & Sari, S. Y. (2022). Application of the M-APOS learning model (modification-action, process, object, scheme) to improve mathematics outcomes in class IV state elementary school 011 Rambah Samo. *Indonesian Journal of Basic Education*, *5*(3), 236-249. https://doi.org/10.37728/ijobe.v5i3.631

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, *96*, 1-16. https://doi.org/10.1007/s10649-017-9761-8

Kallia, M., van Borkulo, S. P., Drijvers, P., Barendsen, E., & Tolboom, J. (2021). Characterising computational thinking in mathematics education: A literature-informed Delphi study. *Research in Mathematics Education*, *23*(2), 159-187. https://doi.org/10.1080/14794802.2020.1852104

Kieran, C. (2018). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research Issues in the Learning and Teaching of Algebra* (pp. 33-56). Routledge. https://doi.org/10.4324/9781315044378-4

Kilhamn, C., Bråting, K., Helenius, O., & Mason, J. (2022). Variables in early algebra: exploring didactic potentials in programming activities. *ZDM - Mathematics Education*, *54*(6), 1273-1288. https://doi.org/10.1007/s11858-022-01384-0

Kong, S.-C. (2016). A framework of curriculum design for computational thinking development in K-12 education. *Journal of Computers in Education*, *3*, 377-394. https://doi.org/10.1007/s40692-016-0076-z

Kuo, W.-C., & Hsu, T.-C. (2020). Learning computational thinking without a computer: How computational participation happens in a computational thinking board game. *Asia-Pacific Education Researcher*, *29*, 67-83. https://doi.org/10.1007/s40299-019-00479-9

Lee, H.-Y., Wu, T.-T., Lin, C.-J., Wang, W.-S., & Huang, Y.-M. (2024). Integrating computational thinking into scaffolding learning: An innovative approach to enhance science, technology, engineering, and mathematics hands-on learning. *Journal of Educational Computing Research*, *62*(2), 431-467. https://doi.org/10.1177/07356331231211916

Masfingatin, T., & Maharani, S. (2019). Computational thinking: Students on proving geometry theorem. *International Journal of Scientific and Technology Research*, *8*(9), 2216-2223.

McCarthy, T. (2021). Induction, constructivity, and grounding. *Notre Dame Journal of Formal Logic*, *62*(1), 83-105. https://doi.org/10.1215/00294527-2021-0004

Orban, C. M., & Teeling-Smith, R. M. (2020). Computational thinking in introductory physics. *The Physics Teacher*, *58*(4), 247-251. https://doi.org/10.1119/1.5145470

Palts, T., & Pedaste, M. (2020). A model for developing computational thinking skills. *Informatics in Education*, *19*(1), 113-128. https://doi.org/10.15388/INFEDU.2020.06

Putra, Z. H., Afrillia, Y. M., Dahnilsyah, D., & Tjoe, H. (2023). Prospective elementary teachers' informal mathematical proof using GeoGebra: The case of 3D shapes. *Journal on Mathematics Education*, *14*(3), 449-468. https://doi.org/10.22342/jme.v14i3.pp449-468

Rodríguez-Martínez, J. A., González-Calero, J. A., & Sáez-López, J. M. (2020). Computational thinking and mathematics using Scratch: an experiment with sixth-grade students. *Interactive Learning Environments*, *28*(3), 316-327. https://doi.org/10.1080/10494820.2019.1612448

Rowen, L. (2018). *Algebra: groups, rings, and fields*. A K Peters/CRC Press. https://doi.org/10.1201/9781315275598

Salwadila, T., & Hapizah. (2024). Computational thinking ability in mathematics learning of exponents in grade IX. *Infinity Journal*, *13*(2), 441-456. https://doi.org/10.22460/infinity.v13i2.p441-456

Selden, A., & Selden, J. (2008). Overcoming students' difficulties in learning to understand and construct proofs. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 95-110). The Mathematical Association of America. https://doi.org/10.5948/UPO9780883859759.009

Stylianides, A. J. (2016). *Proving in the elementary mathematics classroom*. Oxford Academic. https://doi.org/10.1093/acprof:oso/9780198723066.001.0001

Stylianides, A. L. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education (JRME)*, *38*(3), 289-321. https://doi.org/10.2307/30034869

Tabach, M., & Nachlieli, T. (2016). Communicational perspectives on learning and teaching mathematics: Prologue. *Educational Studies in Mathematics*, *91*(3), 299-306. https://doi.org/10.1007/s10649-015-9638-7

Tang, X., Yin, Y., Lin, Q., Hadad, R., & Zhai, X. (2020). Assessing computational thinking: A systematic review of empirical studies. *Computers & Education*, *148*, 103798. https://doi.org/10.1016/j.compedu.2019.103798

Valenta, A., & Enge, O. (2022). Teaching practices promoting meta-level learning in work on exploration-requiring proving tasks. *The Journal of Mathematical Behavior*, *67*, 100997. https://doi.org/10.1016/j.jmathb.2022.100997

Wagner, S., & Parker, S. (1993). *Advancing algebra. Research ideas for the classroom: high school mathematics*. National Council of Teachers of Mathematics.

Waterman, K. P., Goldsmith, L., & Pasquale, M. (2020). Integrating computational thinking into elementary science curriculum: An examination of activities that support students' computational thinking in the service of disciplinary learning. *Journal of Science Education and Technology*, *29*(1), 53-64. https://doi.org/10.1007/s10956-019-09801-y

Wilkerson, M. H., & Fenwick, M. (2016). The practice of using mathematics and computational thinking. In C. V. Schwarz, C. Passmore, & B. J. Reiser (Eds.), *Helping Students Make Sense of the World Using Next Generation Science and Engineering Practices* (pp. 181-204). National Science Teachers' Association Press.

Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, *49*(3), 33–35. https://doi.org/10.1145/1118178.1118215

Wu, T.-T., Asmara, A., Huang, Y.-M., & Hapsari, I. P. (2024). Identification of problem-solving techniques in computational thinking studies: Systematic literature review. *SAGE Open*, *14*(2), 1-20. https://doi.org/10.1177/21582440241249897