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Understanding mathematics prospective teachers' comprehension of function derivatives based on APOS theory: Insights from low mathematics anxiety levels

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Abstract

Understanding function derivatives shows global patterns of difficulty in comprehension and application. More research is needed to examine students' understanding of APOS theory. This research analyzes prospective mathematics teacher students' understanding of function derivatives based on mathematics anxiety. This study used a qualitative-exploratory design to describe the understanding of function derivatives of prospective mathematics teacher students with APOS theory, considering mathematics anxiety through assignments and interviews. A saturated sample of 26 students was studied. Instruments included math anxiety questionnaires, math ability tests, and function derivative tasks. Data was analyzed using triangulation, peer debriefing, member checking, data reduction, presentation, conclusion, and verification. The study of function derivatives, based on APOS Theory, integrates mental structures and mechanisms like encapsulation and coordination, showing proficiency in simple function derivatives and composition function derivatives but challenges with graphing function derivatives. This research emphasizes the need for teaching strategies that address math anxiety to improve conceptual understanding. It encourages further study of teaching interventions, emotional support, and the long-term impact of math anxiety.

Keywords:
APOS theory, Derivatives, Math anxiety, Understanding

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1. INTRODUCTION

Researchers in mathematics education have given much thought to the teaching and learning of differential calculus (Artigue, 2021), and the findings of this research have influenced how the topic is taught. Less emphasis has been placed on some basic issues, such as students' understanding of the relationships between ideas and the conceptual skills they need to be ready to move on to higher mathematics (Trigueros et al., 2024). A deeper understanding of the various calculus courses and the development of analytical skills for challenging mathematical and practical problems are prerequisites for advanced mathematics courses. Much attention has not been paid to research on how well students are prepared to handle the demands of advanced mathematics courses, particularly on the topic of derivatives.

Gaining a thorough understanding of derivatives at the university level is a challenging task. Several studies have shown that students struggle to understand calculus, especially when discussing the topic of derivatives of functions (Martínez-Planell et al., 2015). Students experience challenges when finding derivatives of functions that are displayed graphically and when solving derivative problems that require the use of the chain rule (Maharaj, 2013). Furthermore, students struggle with optimization problems that use the idea of derivatives of functions (Brijlall & Ndlovu, 2013). Findings from a study by Fuentealba et al. (2017) showed that it is difficult to thematize the idea of a system of understanding derivatives in students. The majority of students struggle with derivative problems both practically and conceptually, especially when discussing the mental structures required to determine derivatives at critical points and to determine velocities (Borji et al., 2018). According to findings from several studies, students struggle with concepts of derivatives of functions such as graphs and the chain rule as well as applications of derivatives of functions, such as solving speed and optimization problems. Therefore, it is important to conduct further research to understand how derivative concepts are formed in students' minds and improve teaching approaches to improve overall understanding of mathematics (Şefik & Dost, 2020).

One approach to the mathematics learning model that focuses on deriving functions in calculus is theoryAPOS (Action-Process-Object-Schema) (DeVries & Arnon, 2004; Dubinsky & Mcdonald, 2001; Fuentealba et al., 2019). A constructivist approach in mathematics education, based on Piaget's genetic psychology theory, is adopted in the APOS (Action-Process-Object-Scheme) theory to understand how students build mathematical knowledge. This theory was developed by Dubinsky and the Undergraduate Mathematics Education Research Community (RUMEC), and focuses on mental processes such as interiorization, encapsulation, coordination, and thematization that shape students' cognitive structures (Clark et al., 1997). APOS describes how students learn mathematical concepts through the transformation of mental objects, by connecting new ideas into existing schemas (Clark et al., 1997; Şefik & Dost, 2020). Research using APOS theory to understand derivatives of functions shows that students often face difficulties in applying the concept effectively. For example, Maharaj (2013), found that students had difficulty applying derived rules due to a lack of appropriate mental structures. Studies Clark et al. (1997) uses APOS theory to explore students' understanding of chain rules and function graphs and Font Moll et al. (2016), with a focus on developing Intra, Inter and Trans schemes.

There has been no comprehensive study of students' understanding of function derivatives using the complete APOS theory. Previous studies are often limited to certain aspects of the function derivative concept. For example, research by Clark et al. (1997) only studied the chain rule, while Asiala et al. (1997) just focus on the graph of the function. On the contrary, previous research provides deeper insight into students' understanding of differential and derivative calculus functions, by considering various aspects of scheme development (Fuentealba et al., 2019; Fuentealba et al., 2017).

Further research is needed to comprehensively examine students' understanding of function derivatives, considering mental structures and mental mechanisms in APOS theory in order to improve mathematics teaching and learning outcomes at the university level. In APOS theory, this study will explore the subject's understanding of function derivatives by integrating indicators of actions, processes, objects, and schemes by combining the concepts of simple function derivatives, composition function derivatives, and drawing function derivative graphs. This study introduces an in-depth approach in mapping the difficulties and mental abilities of subjects at each stage of mastering function derivatives. The mathematical scheme for derivative material emphasizes the urgency of a deep and gradual understanding of mathematical concepts, from actions to schemes (Ndlovu & Brijlall, 2019). At the action stage, students perform direct actions such as number operations and apply the concepts of exponents and roots, which serve as a basis for further understanding. At the process stage, students develop the ability to explain and apply substitution rules, and begin to connect different concepts such as drawing function graphs (Cooley et al., 2007). Objects involve the ability to see concepts as a more complex whole and connect these concepts through one-way mental mechanisms (Montiel et al., 2009). In the schema stage, students integrate various concepts and procedures, allowing for more flexible and comprehensive understanding, as well as the use of two-way mental mechanisms. APOS provides an important framework for building holistic mathematical skills, where each stage is interconnected and contributes to a broader understanding, but also indicates any difficulties or imperfections in some indicators that must be overcome. Furthermore, a person's understanding and performance when faced with mathematical concepts or when solving mathematical problems can also be influenced by other factors such as mathematics anxiety.

Mathematics anxiety refers to the negative emotional response experienced by some individuals when they work with numbers or are in situations related to mathematics (Diponegoro et al., 2024; Suárez-Pellicioni et al., 2016). In addition, mathematics anxiety also refers to the experience of fear and apprehension when engaging in mathematics-related activities (Haase et al., 2019; Mutodi & Ngirande, 2014; Stoehr, 2017; Wang et al., 2020). Based on the research results of Prahmana et al. (2019), one of the causes of students' mathematics anxiety is that classes discuss the traditional mathematics learning process and classroom culture, including the experience of learning mathematics in class and the friendships formed during the learning process. The results of research by Wu et al. (2012); Al Mutawah (2015); Hunt et al. (2017); and Pantaleon et al. (2018) showed a negative effect of mathematics anxiety on arithmetic skills and mathematics performance. However, this is contrary to research from Delima et al. (2024), which shows that there is no relationship between mathematics anxiety and students' academic achievement. From the results of this study, researchers suspect that there is a relationship between mathematics anxiety and students' understanding of mathematical concepts, especially the concept of derivative functions. The researcher's hypothesis is supported by Wahyuni et al. (2024) on how math anxiety affects geometry problem solving, where the study showed that a person's ability to solve geometry problems decreased as anxiety increased. In contrast to students with high math anxiety, those with low math anxiety appeared to be able to solve geometry problems accurately and successfully. So in this study, the level of mathematical anxiety was used as a review in selecting research subjects.

This study aims to describe the understanding of the concept of derivative functions of prospective mathematics teachers with low levels of mathematical anxiety, using the APOS theoretical framework. This study will analyze how students with low levels of mathematical anxiety understand the concept of derivative functions based on mental structures (Action, Process, Object, Schema) and mental mechanisms (interiorization, encapsulation, de-encapsulation, coordination, reversal, thematization). Thus, this study is expected to provide an in-depth picture of how mathematics anxiety affects the understanding of the concept of derivative functions and help in designing more effective teaching strategies.

This approach is important because in mathematics learning, understanding the concept of derivative functions does not only depend on the ability to perform symbolic operations or manipulations, but also involves complex mental structures and mental mechanisms such as interiorization, encapsulation, de-encapsulation, coordination, reversal, and thematization. The APOS framework provides important guidance in building holistic mathematics skills, where each stage is interrelated and contributes to a broader understanding. This study aims to map the difficulties and mental abilities of students at each stage of mastering derivative functions, as well as identifying obstacles they may face, with the aim of improving mathematics learning outcomes at the university level.

2. METHOD

2.1. Design

The research design uses a qualitative-exploratory model because it describes the understanding of derivative functions of prospective mathematics teacher students based on APOS theory (action, process, object and scheme) in terms of the level of mathematics anxiety, through assignments and interviews. This design was chosen because it lies in understanding how the level of mathematics anxiety influences the way prospective mathematics teacher students understand the concept of derivatives of algebraic functions (Baloğlu & Zelhart, 2007). Given the important role of prospective teachers in forming the foundations of future mathematical knowledge, understanding differences in this understanding, which are influenced by emotional factors such as anxiety, is crucial to improving teaching methods that are more effective and responsive to the psychological needs of students according to gender, namely male – men and women (Devine et al., 2012; Hembree, 1990; Hoffman, 2010). This also helps in designing educational strategies that can reduce mathematics anxiety and increase understanding of concepts, which in turn can improve the quality of mathematics education regarding derivatives of functions. APOS theory (Action, Process, Object, and Scheme) is used in mathematics learning to help students understand concepts in depth through four stages. First, in the Action stage, students carry out explicit mathematical procedures step by step. Then, in the Process stage, they begin to understand and process the procedure internally without having to physically carry out each step. Next, at the Object stage, students see concepts as overall entities that can be explained and used to build new concepts. Finally, in the Scheme stage, they integrate various mathematical objects and processes into coherent knowledge structures to solve more complex problems. By following these stages, students can develop a deeper and more flexible understanding of mathematical concepts (Abdelhaq et al., 2024; Erdem, 2017).

2.2. Research Subjects and Objects

Research subjects who had low levels of mathematics anxiety (LAS) showed a score of 24 on the mathematics anxiety questionnaire, and obtained a score of 78 on the mathematics ability test. This subject, even though he has low anxiety, still shows active participation and good communication skills in class. In contrast, subjects with high levels of mathematics anxiety (HAS) recorded a score of 44 on the anxiety questionnaire, but also obtained the same score, namely 78, on the mathematics ability test. Even though they have higher anxiety, HAS subjects remain active and communicative in the learning process. These two subjects, although they have different levels of anxiety, show that anxiety does not always hinder active participation and communication performance in the classroom (see Table 1). The mathematics anxiety questionnaire contains 15 items with a score of 1-4, a total score of 15-60. Students were categorized as having low anxiety ($15 \leq$ score ≤ 37) or high ($38 \leq$ score ≤ 60), to identify the influence of anxiety on understanding function derivatives according to APOS theory.

Table 1. Math Anxiety Levels

Math Anxiety Levels	Gender		Total
	Male	Female	
Low ($15 \leq$ score ≤ 37)	4	6	10
High ($38 \leq$ score ≤ 60)	4	12	16
Total	8	18	26

The level of mathematics anxiety is measured based on scores from the Mathematics Anxiety Questionnaire (MAQ) (Juniati & Budayasa, 2020), which is then divided into two categories: low and high. Based on the data, there were 26 research subjects consisting of 10 students with low anxiety (MAQ scores between 15 and 37) and 16 students with high anxiety (MAQ scores between 38 and 60). The gender distribution shows that low mathematics anxiety is experienced by 4 men and 6 women, while high anxiety is experienced by 4 men and 12 women. This indicates that mathematics anxiety tends to be higher among women than men in the population studied (see Table 2).

Table 2. Respondent pair code for MAT and MAQ

Respondent Spouse Code	MAT value	MAQ Score	Information
Rp-01; Rp-02	54; 54	36; 47	Low; High
Rp-06; Rp-21	63; 63	35; 45	Low; High
Rp-10; Rp-18	61; 61	31; 45	Low; High
Rp-15; Rp-07	78; 78	24; 44	Low; High
Rp-22; Rp-14	63; 66	37; 46	Low; High

The coding of pairs of respondents with equivalent Mathematics Ability Test (MAT) scores indicates variations in the level of mathematics anxiety between these subjects. For example, the pair Rp-15 and Rp-07 both obtained the same MAT score, namely 78, but their MAQ scores differed significantly, with Rp-15 having low anxiety (score 24) and Rp-07 having high anxiety (score 44). This data is important because it shows that even though their mathematical abilities are similar, differences in mathematics anxiety can influence their perception of mathematics and also their performance in certain situations while in the classroom in the process of learning mathematics material in the chapter on derivatives of functions *f(x)* (see Table 3).

Table 3. Mathematics anxiety level categories based on MAT and MAQ scores

Subject Code	MAT Score	MAQ Score	Mathematics Anxiety Level Categories
LAS	78	24	Low
HAS	78	44	High

The subject's understanding of function derivatives using APOS theory was obtained from written work and interviews, with the stages of mental structure identified. The data collection process was carried out twice to ensure credibility. Understanding of subjects with Low Mathematics Anxiety Levels (LAS) on Function Derivative Tasks (FDT). In this study, the subjects used were subjects with low mathematics anxiety with the LAS code who had a MAQ score of 24.

2.3. Operational Research Variables

This study uses APOS theory with the mental structure of action, process, object and scheme by describing the respective indicators shown in the Table 4.

Table 4. Operational research variables

Category	Indicator
Action	Determine the derivative of a simple function
	Determine the derivative of the composition function
	Draw graphs of derivatives of functions
Process	Explain the steps to determine the derivative of a simple function
	Explain the steps to determine the derivative of a composition function
	Explain the steps for drawing graphs of function derivatives
Object	Determine the derivative relationship of two functions where one function is a constant product of the other function.
	Determine the derivative relationship of two functions where one function is a power of the other function.
	Determine the derivative relationship of two functions based on the given graph
Scheme	Explains the nature of derivation of functions by connecting actions, processes, objects, and other schemas that may be used to complete the derivation task

2.4. Observations and Interviews

This research activity involves observation studies, interview studies, test studies for students using mathematics material in the chapter on derivatives of functions, documentation studies. In this instrument for test studies with the first function derivative model *f(x) (FDT-1)* and composition functions with assignments related to algebraic function derivative material *f'(x)* and *g'(x)*. Where the task is carried out by determining the derivative of the function in x and draw graphs of functions *f(x)* and *f'(x)*.

2.5. Research Instrument

This research instrument first used a valid and reliable mathematics anxiety questionnaire with 15 statement items, namely 4 items about anxiety in studying mathematics, 5 items about anxiety in taking mathematics lectures, and 6 items about anxiety in taking mathematics tests, to measure student anxiety, it was developed by Juniati and Budayasa (2020), for accurate identification of anxiety levels.

The mathematics test instrument consists of 10 essay questions to assess students' abilities, with an average validity criterion for validator assessments of 3.903. The validators of this instrument are 3 mathematics education lecturers from 3 different universities. This test determines subjects with a maximum score difference of 5 (see Table 5).

Table 5. Average assessment of the mathematics ability test (MAT) instrument

Validator 1	Validator 2	Validator 3	Average assessment of the Mathematics Ability Test (MAT) instrument
3.84	3.91	3.96	3.903

The Function Derivative Task, with 10 of questions, measures students' understanding of function derivatives based on APOS Theory. The questions are designed to identify understanding of the pada various stages of the theory (see Table 6).

Table 6. Average assessment of the function derivative task (FDT) instrument

Validator 1	Validator 2	Validator 3	The Function Derivative Task (FDT) instrument
3.83	3.97	3.97	3.92

2.6. Research Procedures

The research procedure uses model analysis (Miles & Huberman, 1994), includes four main stages: data reduction, data presentation, drawing conclusions, and verification, following the data reduction method carried out to filter and simplify relevant data, focusing on the subject's understanding of function derivatives based on APOS theory (Baker et al., 2000). Presentation of data organizes reduction results in the form of descriptions, charts and categories, making understanding and analysis easier. Drawing conclusions provides meaning to the data that has been presented, with a focus on comparing the understanding of students with high and low mathematics anxiety (Cooley et al., 2007). This process ensures sharper, specific, and organized data for in-depth verification and interpretation.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1. Subject's Understanding of Function Derivatives at the Action Stage

To explore the subject's understanding at the action stage, the following function derivative task is used.

(1) Determine the derivative of the function $f(x) = x^4 – 3x^3 + 16x$!

(2) Determine the derivative of the function $f(x) = (x^8 + 2x)^5$!

(3) Draw the function graphs f and f' of $f(x) = x^2$!

(4) Draw the function graphs f and f' of $f(x) = x^2+3$!

Based on this question, the subject's answer for number 1 and number 2 can be seen in Figure 1.

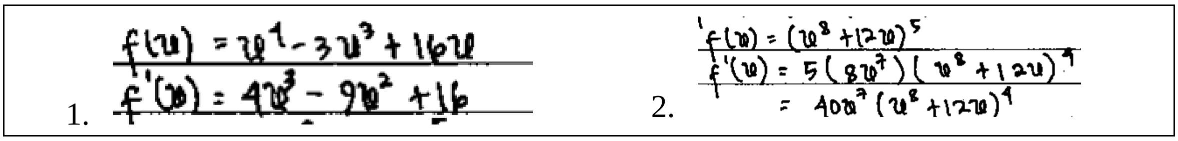


Figure 1. Subject's answer at action stage number 1 and 2

Next, interviews were conducted with the subjects based on their work results.

R : *How do you determine the derivative of the function number 1?*

S : *The first one is for x^4, so the exponent is multiplied by the coefficient of x^4, which is 1, then the exponent is reduced by 1 so that it becomes $4x^3$.*

R : *What formula does this use?*

S : *Use the formula, for ax^n, so the derivative is $anx^{(n-1)}$*

R : *What about the others?*

S : *It's the same, the formula used is the same as before*

R : *What about the next?*

S : *The power of 5 is multiplied by the derivative of x^8, which is $8x^7$, then multiplied by $(x^8+12x)^4$ because the power is reduced by 1*

R : *$8x^7$ is derived from which one?*

S : *The derivative in brackets is x^8+12x, oh yes, the correct one is $x^8+12x8x^7+12$*

R : *So, what is the correct answer?*

S : *$5(8x^7+12)(x^8+12x)^4$*

R : *What formula do you use?*

S : *If $a(f(x))^n$ then the derivative is $a.n.f'(x)(f(x))^{(n-1)}$*

R : *If that's the case, what is the derivative function called?*

S : *Composition function*

Based on the results of interviews conducted with subject, it is known that subject determines the derivative of a simple function in the model $f(x)=x^4 – 3x^3 + 16x$ by lowering each term one by one using the derivative formula if $f(x) = ax^n$ so $f'(x) = anx^{(n-1)}$, with the result $f'(x)= 4x^3 – 9x^2 + 16$. Next, subject determines the derivative of the composition function $f(x) = (x^8 + 12x)^5$ by using the chain rule, namely if the function is in the form $g(x)$

$= a(f(x))^n$ then the derivative is $g'(x) = a.n.f'(x)(f(x))^{(n-1)}$ so that the derivative obtained is $f'(x) = 5(8x^7+12)(x^8+12x)^4$.

Next, for the task of drawing a function derivative graph using question number 3 and the subject's answer can be seen in Figure 2.

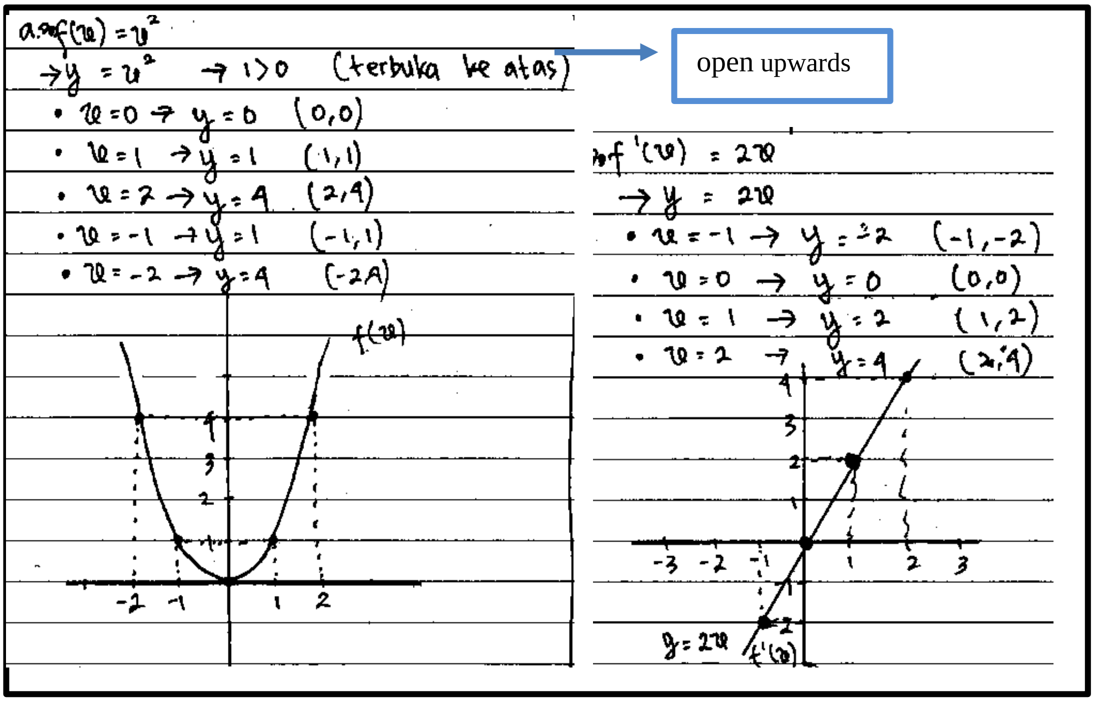


Figure 2. Subject's answer at action stage number 3

Then an interview was conducted based on the subject's answers in the Figure 2.

R : *Now try to explain the steps for drawing this graph.*

S : *For the equation $f(x)=x^2$, I change it first in form $y=x^2$, then I first check the coefficient x^2, namely 1, because 1 is more than 0, the graph opens upwards. Then I took $x=0$ the obtained $y=0$, then I also took the other points. From the points obtained earlier, I connected them to form a graph $f(x)=x^2$*

R : *How about drawing a derivative graph?*

S : *The derivative of x^2 is 2x, so I suppose that $y=2x$ then I take $x= -1$ so that I get = -2. Next, I took other points, then I connected these points to make a graph $f'(x)=2x$*

Next, for the task of drawing a function derivative graph using question number 4 and the subject's answer can be seen in Figure 3.

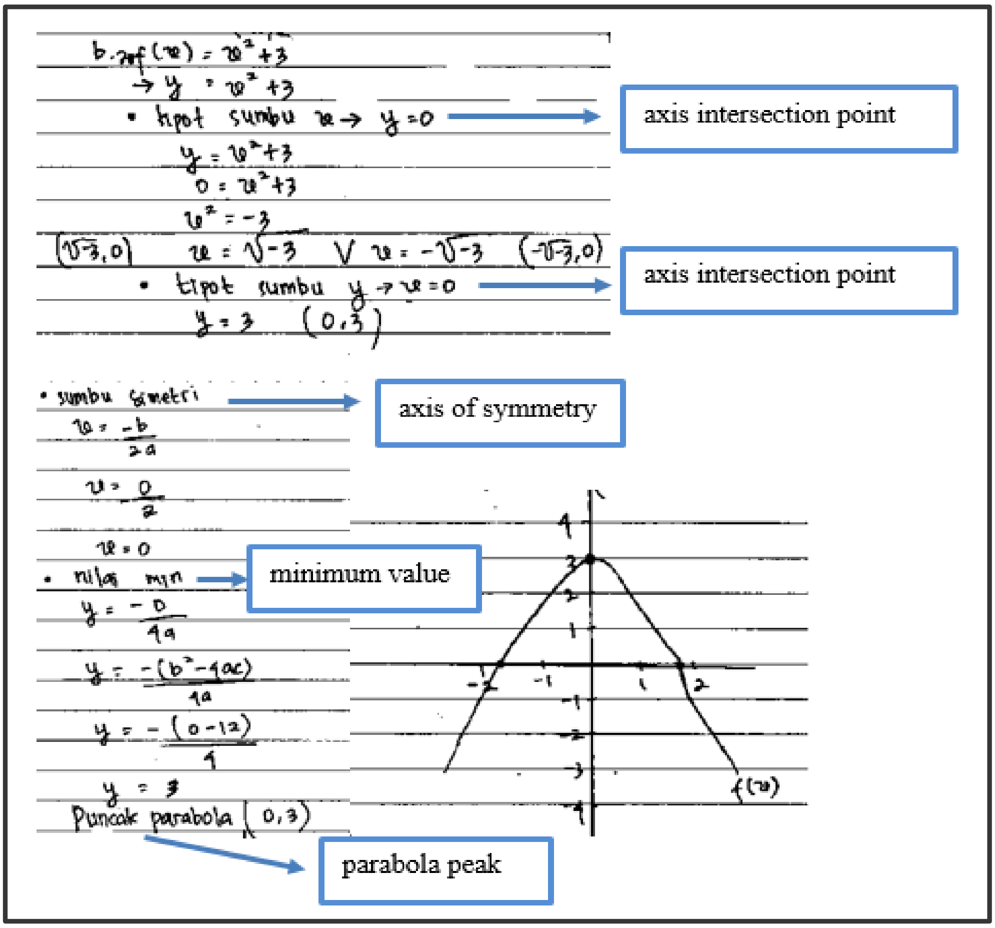


Figure 3. Subject's answer at action stage number 4

Then an interview was conducted based on the subject's answers in the Figure 3.

R : *Now try to explain the steps for drawing this graph.*

S : *The equation is* $f(x)=x^2+3$, *I changed it to* $y=x^2+3x$. *Then I looked for the point of intersection of the graph with the x-axis by taking* $y=0$ *then inserted into the equation so that* $x=\sqrt{-3}$ *or* $x=-\sqrt{-3}$ *was obtained. So the intersection points were found at points* $(\sqrt{-3},0)$ *and* $(-\sqrt{-3},0)$. *Then determine the point of intersection of the graph with the y-axis by taking* $x=0$ *and obtained* $y=3$, *so the intersection point is at (0,3). Next, determine the axis of symmetry using the formula and obtained* $x=0$. *Then I looked for the minimum value using the formula and obtained* $y=3$, *so the peak of the parabola is at (0,3). From the points obtained earlier, I drew the graph, Mam.*

R : *So what about the derivative graph?*

S : *The derivative is the same as number 3 Mam, so the method is the same*

Based on the results of interviews conducted with subject, it is known that subject subject in drawing graphs of function derivatives $f(x)=x^2$ by determining function derivatives $f'(x)=2x$. Subject uses the assumption that $y=2x$ and needs to determine the coordinates (x,y) that satisfy the equation of $y=2x$. From several points obtained, the subject draws a graph by connecting these points. Next, subject draw a function derivative graph $f(x)=x^2+3$ by determining the derivative in the model $f'(x)=2x$. The subject assumes that $y=2x$ and looks for several points (x,y) that satisfy the equation $y=2x$. From several points obtained, the subject draws a graph by connecting the points. Furthermore, in drawing a graph of the derivative of a function by determines the derivative of the function first.Then the subject draws a graph of the derivative of the function by finding several points that satisfy the equation and connecting the points obtained so that a graph of the derivative of the function is formed in the form of a straight line.

3.1.2. Subject’s Understanding of Function Derivatives at the Process Stage

To explore the subject's understanding at the process stage, the following function derivative task is used.

(1) Given function *f* and *g*, if $g(x)=3f(x)$ explain the determining steps $g'(x)$!

(2) Given function *f* and *g*, if $g(x)=[f(x)]^4$ explain the determining steps $g'(x)$!

(3) Explain how to draw a graph of f' from the graph of f ! (see Figure 4)

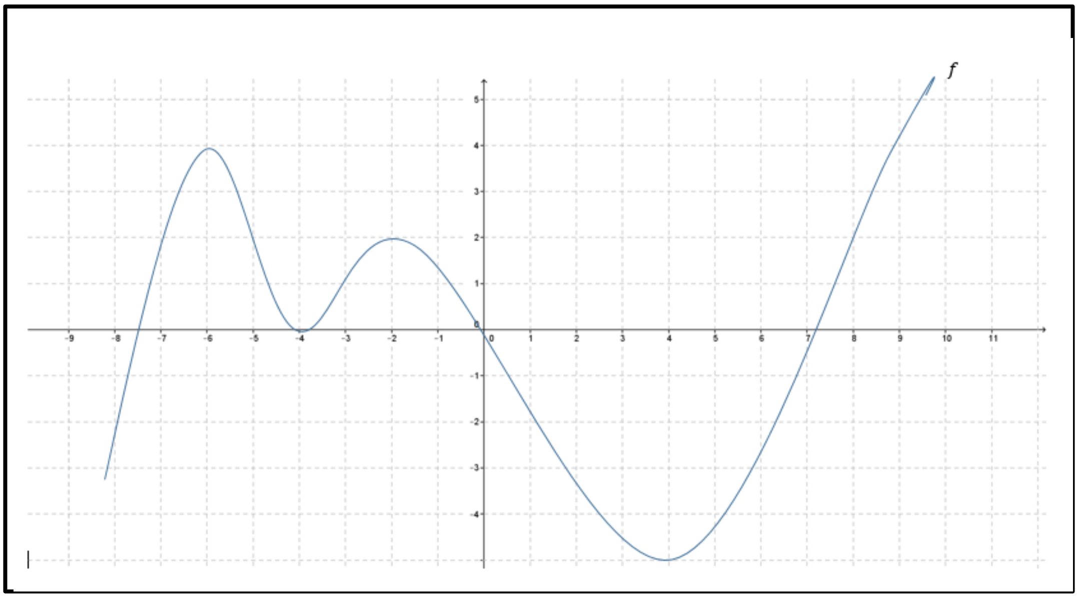
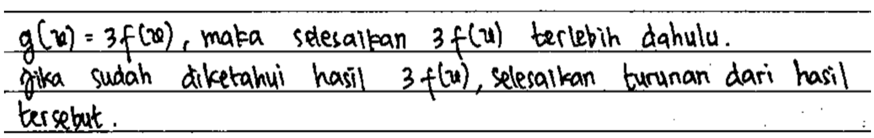


Figure 4. Task of drawing a graph of the derivative function

Based on this question, the subject's answer for number 1 can be seen in Figure 5.



$g(x) = 3f(x)$ solve $3f(x)$ first. If the result is $3f(x)$, solve the derivative of the result.

Figure 5. Subject's answer at process stage number 1

Then an interview was conducted based on the subject's answers in the Figure 5.

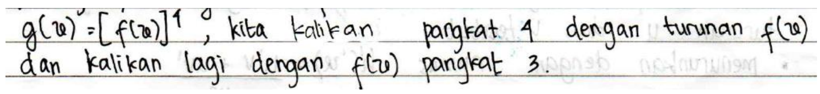
R : *Try to explain how you determine the derivative g(x) if g(x)=3f(x)*

S : *I'll finish 3f(x) first*

R : *What do you mean by finishing 3f(x) first?*
S : *3 times with f(x)*
R : *Then after that how?*
S : *The results are derived*
R : *Is there another way to determine the derivative of g(x)?*
S : *Yes ma'am, look for f'(x) first, then multiply by 3*
R : *Try writing down what the results look like*
S : *This is ma'am g'(x)=3f'(x)*

Based on the results of interviews conducted with subject, it is known that subject explains the steps to determine the derivative of a simple function *g(x)=3f(x)* in two ways. The first way is that the subject multiplies *f(x)* by *3* and then the result is derived. The second method is that the subject determines *f'(x)* first and then the result is multiplied by *3* to obtain *g'(x)=3f'(x)*. Based on this, it can be said that subject explains the steps to determine the derivative of a simple function in two ways. The first way determines the result of multiplying the constant a by *f(x)*, then subject determines the derivative of the result of the operation of the function. While for the second way, subject reduces the function *f(x)* one by one to *f'(x)* then determines the result of the operation of each function sought so that *g'(x)=af'(x)* is obtained. Based on this, it can be concluded that there is a mental mechanism of interiorization in determining the derivative of the function of multiplying a constant by a function that is carried out repeatedly and reflects the action in his mind so that subject can explain how to determine the derivative of a simple function using two different methods.

Next, the subject's answer for number 2 can be seen in Figure 6.


g(x) = [f(x)]⁴ ; kita kalikan pangkat 4 dengan turunan f(x) dan kalikan lagi dengan f(x) pangkat 3.

$g(x) = [f(x)]^4$, we multiply the power of 4 by the derivative of $f(x)$ and multiply it again by $f(x)$ to the power of 3

Figure 6. Subject's answer at process stage number 2

Then an interview was conducted based on the subject's answers in the Figure 6.

R : *Try to explain how you determine the derivative g(x)*
S : *For this number, because the problem is g(x)=[f(x)]^4 so to find the derivative of g(x), I multiply the power of 4 by the derivative of f(x) then multiply it again by f(x) where the power is reduced by 1*
R : *Why do you use such methods?*
S : *That is a composition function, so I use the derivative formula If g(x)=a(f(x))^n then the derivative is g'(x)=a.n.f'(x)(f(x))$^{(n-1)}$*

Based on the results of interviews conducted with subject, it is known that subject explains the steps to determine the derivative of the composition function *g(x)=[f(x)]*^{4} by multiplying to the power of *4* with the derivative of *f(x)* then multiplied again by *f(x)* where the power is *4* minus *1*. The subject uses the derivative formula for the composition function, namely if the derivative is $g(x)=a(f(x))^n$ so that the derivative is $g'(x)=a.n.f'(x)(f(x))^{(n-1)}$ obtained $g'(x)=4.f'(x)[f(x)]^3$. Based on this, it can be said that subject explains the steps to determine the derivative of the composition function by using the chain rule. It can be concluded that there is a mental mechanism of interiorization to determine the derivative of the composition function which is carried out repeatedly and reflects the action in his mind so that subject can explain how to determine the derivative of the composition function using the chain rule.

However, subject could not explain the steps to draw a function derivative graph from a given function graph. Based on this, it can be concluded that there is no mental mechanism of interiorization in explaining the steps to draw a function derivative graph from a given function graph which is indicated by subject not being able to explain the steps to draw a function derivative graph from a given function graph.

3.1.3. Subject's Understanding of Function Derivatives at the Object Stage

To explore the subject's understanding at the object stage, the following function derivative task is used.

(1) Given the functions *f* and *g*, If $g(x)=3f(x)$ so, what is the relationship between $f'(x)$ and $g'(x)$?

(2) Given the functions *f* and *g*, If $g(x)=[f(x)]^4$ so, what is the relationship between $f'(x)$ and $g'(x)$?

(3) The following image shows a graphic image of the four functions shown by the graph *a*, *b*, *c*, *d* (see Figure 7). Based on the graph, what is the relationship between: (a) functions *a* and *b* ?; (b) functions *c* and *d* ?

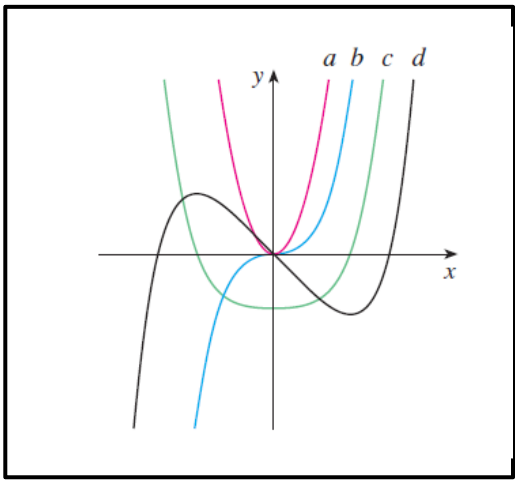


Figure 7. Function derivative graph task

Based on the task given, the subject then answers the questions. Next, an interview is conducted based on the subject's answers.

R : *Try to explain what the relationship is between f'(x) and g'(x)*

S : *The relationship g'(x)=3f'(x)*

R : *Why is the relationship like that?*

S : *I was just trying it out, Ma'am. I tried to take any function f(x) and g(x) and then I derived it, it turns out g'(x)=3f'(x)*
R : *Ok, now for the next question, what is the relationship between f'(x) and g'(x)?*
S : *I don't really understand the relationship between f'(x) and g'(x). I tried but I still couldn't do it.*
R : *Give it a try, try writing it again!*
S : *I tried to lower g(x) to g'(x) equals 4 times f'(x) then multiplied again by f(x) raised to the power of 4 minus 1 so the result is 3. Could* $g'(x)=4(f'(x))[f(x)]^3$ *be the relationship between f '(x) and g'(x).*
R : *Try to look at your work again and think again about relationship between f'(x) and g'(x).*
S : *I think,* $g'(x)=4(f'(x))[f(x)]^3$ *is the relationship between f'(x) and g'(x)*
R : *How do you determine the relationship between f'(x) and g'(x)?*
S : *I use the derivative formula for the composition function*
R : *OK, now let's move on to the next number. By looking at this graph, what is the relationship between functions a and b?*
S : *If I look at the graph, function a is a derivative of b*
R : *Why do you say that?*
S : *I see from previous question, the graph is similar, so I see from the picture that a is a derivative of b*
R : *What is the relationship between functions c and d?*
S : *c is a derivative of d, because I see the graph is also almost the same as previous question.*

Based on the results of interviews conducted with subject, it is known that subject determines the relationship between *f'(x)* and *g'(x)* if *g(x)=3f(x)* by trying to take any function *f(x)* and *g(x)* then determining *f'(x)* and *g'(x)* so that the relationship between *f'(x)* and *g'(x)* is obtained, namely *g'(x)=3f'(x)*. Based on this, it can be said that subject determines the relationship of the derivatives of two functions where one function is a constant multiplication of another function by trying to take any function then determining the result of the function operation, finding the derivative of the result of the function operation and determining the relationship of the derivatives of the two functions so that the relationship of the derivative of the second function is obtained equal to the constant multiplied by the derivative of the first function.

Thus, a mental encapsulation mechanism occurs when transforming the process of determining the derivative of a function into an object so that subject can determine the relationship of the derivatives of two functions where one function is a constant multiplication of another function so that the relationship of the derivative of the second function is obtained equal to the constant multiplied by the derivative of the first function. The mental coordination mechanism occurs when subject combines two processes, namely determining the derivative of a function in the form of a constant multiplication with a multiplication function.

Next, subject determines the derivative relationship of two functions $f'(x)$ and $g'(x)$ if $g(x)=[f(x)]^4$ by using the derivative formula for the composition function, namely finding $g'(x)$ equals *4* times $f'(x)$ then multiplying it again by $f(x)$ to the power of *4* minus *1*, namely *3*, so that the relationship between $f'(x)$ and $g'(x)$ is obtained, namely $g'(x)=4f'(x)[f(x)]^3$. Based on this, it can be said that the subject determines the relationship of the derivatives of two functions where one function is the power of the other function by changing the function into the form of a power function and then using the derivative rule for the composition function so that the relationship of the derivative of the second function is obtained as the power of the first function multiplied by the derivative of the first function multiplied by the first function raised to the power of the first function minus one. Thus, a mental encapsulation mechanism occurs in the form of transforming the process of determining the derivative of a function into an object so that subject can determine the relationship of the derivatives of two functions where one function is the power of the other function.

Furthermore, subject determines the relationship of the derivatives of two functions based on the given graph, namely determining the relationship between functions *a* and *b* and functions *c* and *d* based on the given graph by looking at the similarity of the graph with the previous question so that the relationship *a* is derivative of *b* and *c* is the derivative of *d*. Based on this, it can be said that the subject determines the relationship of the derivatives of two functions based on the given graph by using previous knowledge, namely seeing the similarity of the graph and function from the previous question so that subject determines the relationship of the function with a parabolic graph is a derivative of the function with a graph that has two peaks.

Thus, an encapsulation mental mechanism occurs in the form of transforming the process of drawing a function derivative graph into an object marked by subject being able to determine the relationship of the derivatives of two functions based on the given graph, namely a function with a parabolic graph is a derivative of a function with a graph that has two peaks. The coordination mental mechanism occurs when subject combines two processes, namely the function graph, the function derivative graph and the corresponding function. Furthermore, the reversal mental mechanism occurs when subject retraces the function graph that corresponds to the original function by looking at the similarity of the function graph and the function in previous knowledge. The reversal mental mechanism also occurs when the subject retraces the function derivative graph that matches the original function by seeing the similarity of the function derivative graph and the function derivative in previous knowledge. The mental mechanism of De-Encapsulation occurs when the subject breaks down the relationship between the derivatives of two functions into two mental structures. The process is determining the function graph of a given function and determining the function derivative graph of a given function.

3.1.4. Subject’s Understanding of Function Derivatives at the Scheme Stage

To explore the subject's understanding at the schema stage, interviews were conducted based on the subject's answers. The following presents data from interviews with subjects based on the subject's answers at the scheme stage.

R : *Try to pay attention to your answer again. How do you determine the derivative of the function?*
S : *The first is for x^4, so the exponent is multiplied by the coefficient of x^4, which is 1, then the exponent is reduced by 1 so that it becomes $4x^3$*
R : *What formula does this use?*
S : *Use the formula, if ax^n, so the derivative is $anx^{(n-1)}$*
R : *What about the others?*
S : *It's the same, ma'am, the formula used is the same as before*
R : *What about the next?*
S : *The power of 5 is multiplied by the derivative of x^8, which is $8x^7$, then multiplied by $(x^8+12x)^4$ because the power is reduced by 1*
R : *$8x^7$ is derived from which one?*
S : *The derivative in brackets is x^8+12x, oh yes, the correct one is $x^8+12x8x^7+12$*
R : *So, what is the correct answer?*
S : *$5(8x^7+12)(x^8+12x)^4$*
R : *What formula do you use?*
S : *If $a(f(x)^n$ then the derivative is $a.n.f'(x)(f(x))^{(n-1)}$*
R : *If that's the case, what is the function derivative called?*
S : *Composition function*
R : *Okay for this one, how do you draw the derivative graph?*
S : *The derivative of $x^2$is 2x, so I assume y=2x then I take x=-1 so that I get y=-2. Next I take other points then I connect these points so that they become a graph of f'(x)=2x.*
R : *Okay, so what about the derivative graph for this question?*
S : *For the derivative, it is the same as the previous number, so the method is the same*

Based on the results of interviews conducted with subject, it is known that subject explains the nature of function derivatives by connecting actions, processes, objects, and other schemes that allow them to be used in solving simple function derivative tasks $f(x)=x^4-3x^3+16x$ by deriving each term using the derivative formula if $f(x)=ax^n$ then the derivative $f'(x)=anx^{(n-1)}$. Then, subject explains the nature of function derivation by linking actions, processes, objects, and other possible schemes to be used in solving the task of deriving function composition $f(x)= (x^8+12x)^5$ by using the chain rule, namely if $a(f(x))^n$ the function is of the form then the derivative is $a.n.f'(x)(f(x))^{(n-1)}$.

In addition, subject explains the nature of the derivative of a function by connecting actions, processes, objects, and other schemes that can be used to complete the derivative task of drawing a graph of the derivative of a function $f(x)=x^2$ starting with determining the derivative of $f(x)=x^2$, namely $f'(x)=2x$. Next, the subject assumes $y=2x$ and looks for several points (x,y) that satisfy the equation $y=2x$. From several points obtained, the subject draws a graph by connecting the points. Next, subejct draws a graph of the derivative of a function $f(x)=x^2+3$ starting with determining the derivative of $f(x)=x^2+3$, namely $f'(x)=2x$. Next, the subject assumes $y=2x$ and looks for several points (x,y) that satisfy the equation $y=2x$. From several points obtained, the subject draws a graph by connecting the points.

Based on this, it can be said that subject explains the nature of function derivatives by connecting actions, processes, objects, and other schemes that can be used to complete derivative tasks by considering the suitability between the given function and the appropriate derivative rules. For simple functions, subject uses the derivative rules of constant functions, the derivative rules of power functions, the derivative rules of addition functions, the derivative rules of subtraction functions, and for derivatives of composition functions, subject uses the chain rule. When determining the derivative of a function, subject also involves other rules such as the concept of number operations. Then in drawing a function derivative graph, subject uses the function derivative rules and the steps for drawing a function graph, namely determining the point of intersection of the graph with the coordinate axis, and determining other points on the graph. The mental mechanism of thematization occurs when subejct identifies the suitability of the given function with the derivative rules used and when subject identifies the suitability of the function derivative obtained with the steps used to draw a function derivative graph. Based on the results of research and data analysis related to subject's understanding of function derivatives based on APOS theory, it is summarized and presented in Table 7.

Table 7. Subject 's understanding of function derivations based on APOS theory

Mental Structure/Mental Mechanism	**Indicator**	**Description Understanding**
Action	Determine the derivative of a simple function	Determine the derivative of a simple function by lowering each term one by one using the derivative formula for a power function, namely if $f(x) = ax^n$ then the derivative is $f'(x) = anx^{n-1}$
	Determine the derivative of the composition function	Determine the derivative of a composition function using the chain rule, if $g(x) = a(f(x))^n$, so derivatives $g'(x) = a.n.f'(x)(f(x))^{n-1}$
	Draw graphs of derivatives of functions	Draw a function derivative graph by determining the function derivative first. Then find several points that satisfy the equation and connect the points obtained so that a function derivative graph is formed in the form of a straight line.
Interiorization		The mental mechanism of interiorization determines the derivative of a simple function that is done repeatedly and reflects the action in his mind so that subject can explain how to determine the derivative of a simple function using two different methods, the first method determines the result of the operation of a given function and then determines the derivative of the result of the operation of that function. While the second method is to

Mental Structure/Mental Mechanism	**Indicator**	**Description Understanding**
		determine the derivative of the given function one by one and then determine the result of the operation of the derivative of the function in question.
		The mental mechanism of interiorization determines the derivative of the composition function which is carried out repeatedly and reflects the action in his mind so that subject can explain how to determine the derivative of the composition function by using the chain rule.
		There is no mental mechanism of interiorization in explaining the steps for drawing a function derivative graph from a given function graph, which is indicated by subject not being able to explain the steps for drawing a function derivative graph from a given function graph.
Process	Explain the steps to determine the derivative of a simple function	Subject explains the steps to determine the derivative of a simple function in two ways. The first way subject determines the result of the operation of the given function, namely determining the result of multiplying the constant a by $f(x)$, then subject determines the derivative of the result of the operation of the function. While for the second way subject reduces one by one the functions(x) and $g(x)$ become $f'(x)$ and $g'(x)$ then determines the result of the operation of each function sought so that $g'(x) = af'(x)$ is obtained.
	Explain the steps to determine the derivative of a composition function	Subject explains the steps to determine the derivative of a composition function, by using the chain rule, if the function is in the form $g(x) = a[f(x)]^n$ then the subject uses the derivative rule $g'(x) = a.n.f'(x)(f(x))^{n-1}$.
	Explain the steps for drawing graphs of function derivatives	Subject cannot explain the steps to draw a function derivative graph from a given function graph
Encapsulation		The mental mechanism of encapsulation when transforming the process of determining the derivative of a function into an object so that subject can determine the relationship of the derivative of two functions where one function

Mental Structure/Mental Mechanism	**Indicator**	**Description Understanding**
		is a constant multiplication of another function so that the relationship obtained is that the derivative of the second function is the same as the constant multiplied by the derivative of the first function. The mental mechanism of encapsulation is in the form of transforming the process of determining the derivative of a function into an object so that subject can determine the relationship of the derivative of two functions where one function is a power of the other function. The mental mechanism of encapsulation in the form of transforming the process of drawing a function derivative graph into an object marked with subject can determine the relationship between the derivatives of two functions based on the given graph, namely a function with a parabolic graph is a derivative of a function with a graph that has two peak points.
Object	Determine the derivative relationship of two functions where one function is a constant product of the other function.	Subject determines the relationship of the derivatives of two functions where one function is a constant multiplication of another function in two ways. The first way is to take any function then determine the result of the function operation, find the derivative of the result of the function operation and determine the relationship of the derivatives of the two functions. The second way is to determine the derivative of the first function, then substitute the first function into the second function, then determine the derivative of the second function so that the relationship of the derivative of the second function is obtained equal to a constant multiplied by the derivative of the first function..
	Determine the derivative relationship of two functions where one function is a power of the other function.	Subject determines the relationship between the derivatives of two functions where one function is a power of the other function by changing the function into a power function form and then using the derivative rule for the composition function so that the relationship

Mental Structure/Mental Mechanism	Indicator	Description Understanding
		between the derivatives of the two functions is obtained
	Determine the derivative relationship of two functions based on the given graph	Subject determines the relationship between the derivatives of two functions based on the given graphs by using previous knowledge, seeing the similarity of the graphs and functions from the previous question so that subject determines the relationship between functions with parabolic graphs which are derivatives of functions with graphs that have two peak points..
Coordination		The mental coordination mechanism occurs when subject combines two processes, namely determining the derivative of a function in the form of a constant multiplication by a multiplication function.
		The mental coordination mechanism occurs when the subject combines two processes, namely the function graph, the function derivative graph and the corresponding function.
Reversal		The mental reversal mechanism occurs when the subject retraces the function graph that corresponds to the original function by seeing similarities between the function graph and the function in previous knowledge.
		The mental reversal mechanism also occurs when Subject retraces the function derivative graph that corresponds to the original function by seeing the similarity of the function derivative graph and the function derivative in previous knowledge.
De-Encapsulation		De-Encapsulation mental mechanism occurs when the subject decomposes the relationship between the derivatives of two functions into two mental structures. The process is determining the function graph of the given function and determining the function derivative graph of the given function
Scheme	Explains the nature of derivation of functions by	Subject explains the nature of function derivatives by connecting actions, processes, objects, and other schemes that allow them to be used in solving derivative tasks by

Mental Structure/Mental Mechanism	**Indicator**	**Description Understanding**
	connecting actions, processes, objects, and other schemas that may be used to complete the derivation task	considering the suitability between the given function and the appropriate derivative rules. For simple functions, Subject uses several rules, namely the derivative rule of constant functions, the derivative rule of power functions, the derivative rule of addition functions, the derivative rule of subtraction functions, and for derivatives of composition functions, Subject uses the chain rule. When determining the derivative of a function, Subject also involves other rules such as the concept of number operations and substitution rules. Then in drawing a function derivative graph, subject uses the function derivative rule and the steps for drawing a function graph, namely determining the point of intersection of the graph with the coordinate axis, and determining other points on the graph.
Thematization		The mental mechanism of thematization occurs when subject identifies the suitability of a given function with the derivative rule used and when subject identifies the suitability of the derivative of the function obtained with the steps used to draw the function derivative graph.

Based on the summary of subject's explanation in Table 7, it can be described how the understanding of the function derivative is built in subject's mind. Figure 8 is a description of the subject scheme for the function derivative based on the APOS theory.

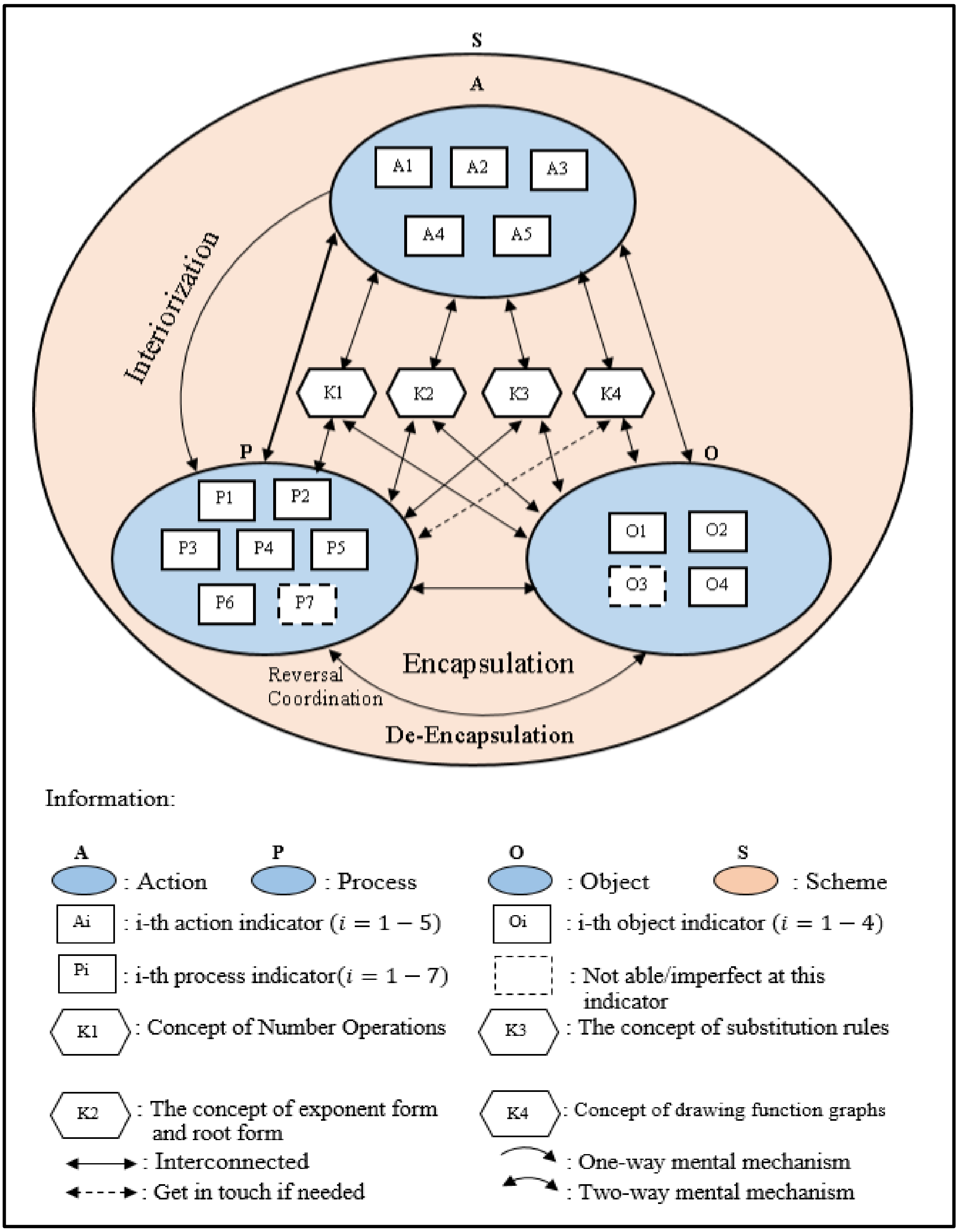


Figure 8. Subject' understanding scheme for function derivations based on APOS theory

Based on Figure 8, it can be explained that the subject understanding scheme for function derivatives based on APOS Theory is a collection of mental structures of actions, processes, and objects with mental mechanisms of interiorization, encapsulation, de-encapsulation, coordination, reversal and thematization using other concepts, namely the concept of number operations, the concept of exponents and root forms, the concept of substitution rules, and the concept of drawing function graphs. The subject scheme shows that the mental structure of action meets all indicators of understanding. Furthermore, a mental mechanism of interiorization occurs when transforming the mental structure of action into a mental structure of process. However, the subject scheme shows that there are indicators of a process that is unable/imperfectly carried out by subject, namely explaining the steps for drawing a function derivative graph which is caused because subject cannot draw a function derivative graph from a given function graph image, thus the concept of drawing a function graph is not related to the mental structure of the process. Furthermore, a mental encapsulation mechanism occurs when transforming the mental structure of the process into a mental structure of the object. In addition, a mental mechanism of coordination occurs when subject combines two processes into an object. There is also a mental reversal mechanism when subject retraces the function graph that matches the original function by seeing the similarity of the function graph and the function in previous knowledge and when subject retraces the function derivative graph that matches the original function by seeing the similarity of the function derivative graph and the function derivative in previous knowledge. The mental mechanism of De-Encapsulation occurs when the subject breaks down the relationship between the derivatives of two functions into two mental structures. The process is determining the function graph of a given function and determining the function derivative graph of a given function.

3.2. Discussion

The subject's understanding scheme of function derivatives based on APOS Theory includes the mental structure of actions, processes, and objects with mental mechanisms of interiorization, encapsulation, de-encapsulation, coordination, reversal, and thematization adopted from the framework of the APOS mental structure and mechanism theory (Arnon et al., 2014; Dubinsky & Mcdonald, 2001). The subject showed good understanding when determining simple function derivatives, determining composition function derivatives and drawing function derivative graphs on the mental structure of action. This is in line with research from Langi et al. (2023) which shows that there is good understanding at the action stage when the subject works on integral tasks. At the action stage, the subject completes the function derivative task comprehensively and gradually according to the procedural instructions. This is in accordance with Kazunga and Bansilal (2020) which states that in APOS theory, action is a reaction to external stimuli felt by the individual. Supported by the statement Parraguez and Oktaç (2010) which explains that at the action stage, individuals can perform calculations and transformations of mathematical objects due to external stimuli in which the previous step triggers each subsequent step. In this study, the external stimuli received by the subject were in the form of a function derivative problem that was given while the response was in the form of a complete solution with a structured process according to the rules of function derivatives.

In the mental structure of the process, the subject had difficulty in explaining the steps of drawing a function derivative graph from a given function graph, this is because the subject has a lack of understanding of the function derivative graph. This is in line with research Listiawati et al. (2023) which shows that students have difficulty when faced with tasks related to function graphs, namely determining the gradient of a graph at a certain point as the first derivative of the function. This shows that the concept of understanding the function derivative of students is not complete and the APOS level is far below the schema level. In addition, research Listiawati and Juniati (2021) also shows that subjects have difficulty drawing quadratic function graphs. This is because the subject is confused in translating graphs into mathematical symbols in the form of function formulas. From several previous studies, it strongly supports the findings of this study where the greatest difficulty experienced by the subject is related to function graphs.

At the object stage, the subject determines the relationship between the derivatives of two functions based on the given graph using previous knowledge, seeing the similarity of the graph and function from the previous question so that the subject determines the relationship between the function and the parabola graph which is a derivative of the function with a graph that has two peak points. A encapsulation mental mechanism occurs in the form of a transformation of the process of depicting the function derivative graph into an object marked by the subject being able to determine the relationship between the derivatives of two functions based on the given graph. The coordination mental mechanism occurs when the subject combines two processes, namely the function graph, the function derivative graph and the corresponding function. Then the de-encapsulation mental mechanism occurs when the subject describes the relationship between the derivatives of two functions into two mental structures. This is in line with Firdaus et al. (2023) that students decapsulate by verifying the depiction of the patterns found, especially finding the first term, the number of terms in the row, and the difference between each term in the row, to ensure that there are no errors or miscalculations during the process. In accordance with the view of Dubinsky and Mcdonald (2001) that a new process is generated by accommodating an existing process, when the current process grows into a technique that can be modified by an action, then the process is encapsulated into an object.

At the schema stage, the subject connects actions, processes, objects, and other possible schemes to be used in completing the derivative task by considering the suitability between the given function and the appropriate derivative rule. For simple functions, the subject uses several rules, namely the constant function derivative rule, the power function derivative rule, the addition function derivative rule, the subtraction function derivative rule, and for the composition function derivative, the subject uses the chain rule. In determining the derivative of a function, the subject also involves other rules such as the concept of number operations and substitution rules. Then in drawing the function derivative graph, the subject uses the function derivative rule and the steps for drawing the function graph, namely determining the point of intersection of the graph with the coordinate axis, and determining other points on the graph. The mental mechanism of thematization occurs when the subject identifies the suitability of a given function with the derivative rule used and when the subject identifies the suitability of the function derivative obtained with the steps used to draw the function derivative graph. In line with Firdaus et al. (2020) that thematization of number series involves a special relationship between the number series and the concept of function. A student has been able to mathematize a number series as a scheme if he can show the relationship of the series by linking it to the concept of function. This is in accordance with Dubinsky and Mcdonald (2001) statement that thematization is a construction that connects separate actions, processes and objects to a particular object and then produces a scheme.

4. CONCLUSION

This study found that students with low math anxiety showed a good understanding of the mental structure of actions, but had difficulty explaining the steps in drawing function derivative graphs on the mental structure of processes and objects. Students with high math anxiety showed variation in understanding, with significant differences in math anxiety scores and conceptual understanding. This research also states that math anxiety can affect the ability to understand and draw graphs of functions. By using APOS theory, this study aims to provide an in-depth picture of how the level of mathematics anxiety influences the understanding of derivatives of functions and helps in designing more effective teaching strategies.

This study shows the importance of considering the level of mathematics anxiety in designing mathematics teaching strategies, especially in understanding the concept of derivatives of functions. Teaching tailored to students' needs based on their anxiety levels can improve their understanding and skills in this topic. Therefore, instructors need to develop a more personalized and supportive approach to overcome the difficulties faced by students with high mathematics anxiety.

This study calls for further exploring how various teaching and emotional support interventions may influence overall understanding of mathematics concepts, including conducting longitudinal studies to assess the long-term effects of mathematics anxiety on academic achievement. Additionally, research could expand the focus on other subjects in mathematics and explore how various teaching strategies can be adapted to meet the needs of different individuals.

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REFERENCES

Abdelhaq, L., Syam, S. M., & Syam, M. I. (2024). An efficient numerical method for two-dimensional fractional integro-differential equations with modified Atangana–Baleanu fractional derivative using operational matrix approach. *Partial Differential Equations in Applied Mathematics*, *11*, 100824-100824. https://doi.org/10.1016/j.padiff.2024.100824

Al Mutawah, M. (2015). The influence of mathematics anxiety in middle and high school students math achievement. *International Education Studies*, *8*(11), 239-252. https://doi.org/10.5539/ies.v8n11p239

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S. R., Trigueros, M., & Weller, K. (2014). The APOS paradigm for research and curriculum development. In *APOS theory: A framework for research and curriculum development in mathematics education* (pp. 93-108). Springer New York. https://doi.org/10.1007/978-1-4614-7966-6_6

Artigue, M. (2021). Mathematics education research at university level: 3Achievements and challenges. In V. Durand-Guerrier, R. Hochmuth, E. Nardi, & C. Winsløw (Eds.), *Research and development in university mathematics education* (pp. 2-21). Routledge.

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. E. (1997). The development of students' graphical understanding of the derivative. *The Journal of Mathematical Behavior*, *16*(4), 399-431. https://doi.org/10.1016/S0732-3123(97)90015-8

Baker, B., Cooley, L., & Trigueros, M. (2000). A calculus graphing schema. *Journal for Research in Mathematics Education JRME*, *31*(5), 557-578. https://doi.org/10.2307/749887

Baloğlu, M., & Zelhart, P. F. (2007). Psychometric properties of the revised mathematics anxiety rating scale. *The Psychological Record*, *57*(4), 593-611. https://doi.org/10.1007/BF03395597

Borji, V., Font, V., Alamolhodaei, H., & Sánchez, A. (2018). Application of the complementarities of two theories, APOA and OSA, for the analysis of the university students' understanding on the graph of the function and its derivative. *Eurasia Journal of Mathematics Science and Technology Education*, *14*(6), 2301-2315. https://doi.org/10.29333/ejmste/89514

Brijlall, D., & Ndlovu, Z. (2013). High school learners' mental construction during solving optimisation problems in Calculus : a South African case study. *South African Journal of Education*, *33*(2), 1-18. https://journals.co.za/doi/abs/10.10520/EJC134992

Clark, J. M., Cordero, F., Cottrill, J., Czarnocha, B., DeVries, D. J., St. John, D., Tolias, G., & Vidakovic, D. (1997). Constructing a schema: The case of the chain rule? *The Journal of Mathematical Behavior*, *16*(4), 345-364. https://doi.org/10.1016/S0732-3123(97)90012-2

Cooley, L., Trigueros, M., & Baker, B. (2007). Schema thematization: A framework and an example. *Journal for Research in Mathematics Education JRME*, *38*(4), 370-392. https://doi.org/10.2307/30034879

Delima, N., Kusuma, D. A., & Paulus, E. (2024). The students' mathematics self-regulated learning and mathematics anxiety based on the use of chat GPT, music, study program, and academic achievement. *Infinity Journal*, *13*(2), 349-362. https://doi.org/10.22460/infinity.v13i2.p349-362

Devine, A., Fawcett, K., Szűcs, D., & Dowker, A. (2012). Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety. *Behavioral and Brain Functions*, *8*(1), 33-33. https://doi.org/10.1186/1744-9081-8-33

DeVries, D., & Arnon, I. (2004). Solution--What does it mean? Helping linear algebra students develop the concept while improving research tools. In *International Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 55-62).

Diponegoro, A. M., Khalil, I. A., & Prahmana, R. C. I. (2024). When religion meets mathematics: From mathematical anxiety to mathematical well-being for minority group student. *Infinity Journal*, *13*(2), 413-440. https://doi.org/10.22460/infinity.v13i2.p413-440

Dubinsky, E., & Mcdonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 275-282). Springer Netherlands. https://doi.org/10.1007/0-306-47231-7_25

Erdem, E. (2017). A current study on grade/age and gender-related change in math anxiety. *European Journal of Education Studies*, *3*(6), 396-413.

Firdaus, A. M., Junıatı, D., & Wijayanti, P. (2020). Number pattern generalization process by provincial mathematics olympiad winner students. *Journal for the Education of Gifted Young Scientists*, *8*(3), 991-1003. https://doi.org/10.17478/jegys.704984

Firdaus, A. M., Lestari, N. D. S., Murtafiah, W., Ernawati, T., Lukitasari, M., & Widodo, S. A. (2023). Generalization of patterns drawing of high-performance students based on action, process, object, and schema theory. *European Journal of Educational Research*, *12*(1), 421-433. https://doi.org/10.12973/eu-jer.12.1.421

Font Moll, V., Trigueros, M., Badillo, E., & Rubio, N. (2016). Mathematical objects through the lens of two different theoretical perspectives: APOS and OSA. *Educational Studies in Mathematics*, *91*(1), 107-122. https://doi.org/10.1007/s10649-015-9639-6

Fuentealba, C., Badillo, E., Sánchez-Matamoros, G., & Cárcamo, A. (2019). The understanding of the derivative concept in higher education. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(2), em1662. https://doi.org/10.29333/ejmste/100640

Fuentealba, C., Sánchez-Matamoros, G., Badillo, E., & Trigueros, M. (2017). Thematization of derivative schema in university students: nuances in constructing relations between a function's successive derivatives. *International Journal of Mathematical Education in Science and Technology*, *48*(3), 374-392. https://doi.org/10.1080/0020739X.2016.1248508

Haase, V. G., Guimarães, A. P. L., & Wood, G. (2019). Mathematics and emotions: The case of math anxiety. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the* *classroom* (pp. 469-503). Springer International Publishing. https://doi.org/10.1007/978-3-319-97148-3_29

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education, 21*(1), 33-46. https://doi.org/10.5951/jresematheduc.21.1.0033

Hoffman, B. (2010). "I think I can, but I'm afraid to try": The role of self-efficacy beliefs and mathematics anxiety in mathematics problem-solving efficiency. *Learning and Individual Differences*, *20*(3), 276-283. https://doi.org/10.1016/j.lindif.2010.02.001

Hunt, T. E., Bhardwa, J., & Sheffield, D. (2017). Mental arithmetic performance, physiological reactivity and mathematics anxiety amongst U.K. primary school children. *Learning and Individual Differences, 57*, 129-132. https://doi.org/10.1016/j.lindif.2017.03.016

Juniati, D., & Budayasa, I. K. (2020). Working memory capacity and mathematics anxiety of mathematics undergraduate students and its effect on mathematics achievement. *Journal for the Education of Gifted Young Scientists*, *8*(1), 271-290. https://doi.org/10.17478/jegys.653518

Kazunga, C., & Bansilal, S. (2020). An APOS analysis of solving systems of equations using the inverse matrix method. *Educational Studies in Mathematics*, *103*(3), 339-358. https://doi.org/10.1007/s10649-020-09935-6

Langi, E. L., Juniati, D., & Abadi, A. (2023). Students as prospective teachers' understanding of integral based on the APOS theory in terms of gender difference. *Journal of Higher Education Theory and Practice*, *23*(4), 169-185.

Listiawati, E., & Juniati, D. (2021). An APOS analysis of student's understanding of quadratic function graph. *Journal of Physics: Conference Series*, *1747*(1), 012028. https://doi.org/10.1088/1742-6596/1747/1/012028

Listiawati, E., Juniati, D., & Ekawati, R. (2023). Mathematics pre-service teachers' understanding of derivative concepts using APOS theory related to mathematic anxiety: A case study. *AIP Conference Proceedings*, *2733*(1), 030015. https://doi.org/10.1063/5.0140155

Maharaj, A. (2013). An APOS analysis of natural science students' understanding of derivatives. *South African Journal of Education*, *33*(1), 1-19. https://doi.org/10.10520/EJC130323

Martínez-Planell, R., Gaisman, M. T., & McGee, D. (2015). On students' understanding of the differential calculus of functions of two variables. *The Journal of Mathematical Behavior*, *38*, 57-86. https://doi.org/10.1016/j.jmathb.2015.03.003

Miles, B. M., & Huberman, M. A. (1994). *An expanded sourcebook: Qualitative data analysis*. Sage publications.

Montiel, M., Wilhelmi, M. R., Vidakovic, D., & Elstak, I. (2009). Using the onto-semiotic approach to identify and analyze mathematical meaning when transiting between different coordinate systems in a multivariate context. *Educational Studies in Mathematics*, *72*(2), 139-160. https://doi.org/10.1007/s10649-009-9184-2

Mutodi, P., & Ngirande, H. (2014). Exploring mathematics anxiety: Mathematics students' experiences. *Mediterranean Journal of Social Sciences*, *5*(1), 283-294. https://doi.org/10.5901/mjss.2014.v5n1p283

Ndlovu, Z., & Brijlall, D. (2019). Pre-service mathematics teachers' mental constructions when using Cramer's rule. *South African Journal of Education*, *39*(1), 1-13. https://doi.org/10.15700/saje.v39n1a1550

Pantaleon, K. V., Juniati, D., & Lukito, A. (2018). The proving skill profile of prospective math teacher with high math ability and high math anxiety. *Journal of Physics: Conference Series*, *1097*(1), 012154. https://doi.org/10.1088/1742-6596/1097/1/012154

Parraguez, M., & Oktaç, A. (2010). Construction of the vector space concept from the viewpoint of APOS theory. *Linear Algebra and its Applications*, *432*(8), 2112-2124. https://doi.org/10.1016/j.laa.2009.06.034

Prahmana, R. C. I., Sutanti, T., Wibawa, A. P., & Diponegoro, A. M. (2019). Mathematical anxiety among engineering students. *Infinity Journal*, *8*(2), 179-188. https://doi.org/10.22460/infinity.v8i2.p179-188

Şefik, Ö., & Dost, Ş. (2020). The analysis of the understanding of the three-dimensional (Euclidian) space and the two-variable function concept by university students. *The Journal of Mathematical Behavior*, *57*, 100697. https://doi.org/https://doi.org/10.1016/j.jmathb.2019.03.004

Stoehr, K. J. (2017). Mathematics anxiety: One size does not fit all. *Journal of Teacher Education*, *68*(1), 69-84. https://doi.org/10.1177/0022487116676316

Suárez-Pellicioni, M., Núñez-Peña, M. I., & Colomé, À. (2016). Math anxiety: A review of its cognitive consequences, psychophysiological correlates, and brain bases. *Cognitive, Affective, & Behavioral Neuroscience*, *16*(1), 3-22. https://doi.org/10.3758/s13415-015-0370-7

Trigueros, M., Badillo, E., Sánchez-Matamoros, G., & Hernández-Rebollar, L. A. (2024). Contributions to the characterization of the Schema using APOS theory: Graphing with derivative. *ZDM – Mathematics Education*, *56*(6), 1093-1108. https://doi.org/10.1007/s11858-024-01615-6

Wahyuni, R., Juniati, D., & Wijayanti, P. (2024). Mathematics anxiety of junior high school students in solving geometry problems. *Perspektivy Nauki i Obrazovania*, *71*(5), 452-463. https://doi.org/10.32744/pse.2024.5.26

Wang, Z., Rimfeld, K., Shakeshaft, N., Schofield, K., & Malanchini, M. (2020). The longitudinal role of mathematics anxiety in mathematics development: Issues of gender differences and domain-specificity. *Journal of Adolescence*, *80*(1), 220-232. https://doi.org/10.1016/j.adolescence.2020.03.003

Wu, S. S., Barth, M., Amin, H., Malcarne, V., & Menon, V. (2012). Math anxiety in second and third graders and its relation to mathematics achievement. *Frontiers in psychology*, *3*, 162. https://doi.org/10.3389/fpsyg.2012.00162