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The role of scaffolding in shaping reflective mathematical thinking of dependent field students in numeracy problems

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Abstract

In learning mathematics, reflective thinking is often overlooked due to an excessive emphasis on final results, which causes students to struggle in evaluating and reconstructing their problem-solving processes. Reflective thinking skills are necessary for students to solve problems, including numeracy. This study adopts a qualitative approach, focusing on the problem-solving process of two seventh-grade students with a Dependent Field (DF) cognitive style and similar initial mathematical abilities. Data were collected through the Group Embedded Figures Test, in-depth interviews, and initial mathematical and reflective thinking ability tests. Based on the research results, DF students couldn't analyze arguments from various perspectives and see if there were deeper implications. This finding reflects the characteristics of DF, who don't perform the 'result in context' process, leading to a lack of ability to understand, interpret, and use numerical results in concrete/situational contexts. This also includes the ability to relate numbers to real-world situations, make appropriate interpretations, and take suitable actions based on those numerical results. The results of this study can serve as a foundation for designing differentiated instruction that emphasizes the development of reflective thinking skills, particularly in numeracy, through approaches involving technology, models, pedagogy, or other learning strategies.

Keywords:
Dependent field, Junior high schools, Number, Numeracy, Reflective thinking process

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1. INTRODUCTION

When solving problems, students engage in various cognitive actions, such as understanding the problem statement, selecting the necessary data for the solution, applying concepts and operations to the solution, solving the problem, and determining whether the solution is correct (Sümen, 2023). In this context, problem-solving ability is considered a vital skill for individuals, and reflective thinking is regarded as a key factor in supporting the problem-solving process (Toraman et al., 2020). Furthermore, Whalen and Paez (2019) emphasize the importance of mathematical reflective thinking skills for students, as it enables them to evaluate, analyze, and reflect on their own thought processes in solving mathematical problems. This indicates that by mastering reflective thinking skills, students can identify errors or mistakes, understand various approaches to solving problems, and improve the strategies used.

Reflective thinking skills also assist students in connecting previously learned mathematical concepts to new problems, enabling them to develop a deeper understanding and more flexible problem-solving (Toraman et al., 2020). Additionally, reflective thinking encourages students to become independent and critical learners (Antonio, 2020), as they learn not only to accept the final answer but also to understand the process and reasoning behind it (Mohamad & Tasir, 2023). Therefore, reflective thinking skills play a crucial role in building a mindset that values not only outcomes but also the processes involved.

Several researchers have documented their findings regarding the role of reflective thinking skills in middle school students' mathematical problem-solving abilities. Reflective thinking is an essential advanced skill for students to nurture in the globalization era, as the complexity of problems in different areas of modern life becomes increasingly demanding (Hendriana et al., 2019). Reflective thinking skills significantly determine students' success in understanding and solving complex mathematical problems (Milner & Wolfer, 2023; Zerdali & Eğmir, 2025). Furthermore, Kholid et al. (2024) reveal that without adequate reflective thinking skills, students often fail to thoroughly evaluate their problem-solving steps. This shows that weak reflective thinking abilities hinder students from effectively connecting the necessary mathematical concepts to solve problems accurately (Ceylan, 2024; Katrancı & Şengül, 2020). Reflective thinking tends to emerge when students experience confusion, often occurring while working on challenging problems (Sa'dijah et al., 2020). The low performance of Indonesian students in PISA assessments indicates their struggle to solve PISA-level problems (Akbar et al., 2022), which aligns with the findings of the researcher's preliminary study. Based on the results of a preliminary research involving 42 eighth-grade students from SMPN 2 and SMPN 4 Cirebon, it was found that students' reflective thinking processes were still low when solving numeracy problems (Setiyani, Waluya, Sukestiyarno, Cahyono, et al., 2024).

To equip students with reflective thinking skills, it is important to consider how to develop these skills. The thinking process is a sequence of mental processes that occur naturally or in a planned and systematic manner, in the context of space, time, and the medium used, resulting in changes to the affected object. Dewey divides the reflective thinking process into three levels: pre-reflective, reflective, and post-reflective situations (Tan, 2014). The pre-reflective situation is when students show confusion or doubt about a problem. The post-reflective situation occurs when that confusion or doubt is resolved. Meanwhile, the reflective phase is a transition from pre-reflective to post-reflective, where a change in student behavior is observed as they engage in investigative thinking. According to Surbeck, the mathematical reflective thinking process consists of three phases: reacting, elaborating, and reflecting (Lee & Park, 2014). Reacting means responding to the situation or problem at hand with opinions based on theories or concepts, such as the ability to use mathematics in other fields of study or daily life and the ability to identify mathematical concepts or formulas involved in the problem. Elaboration is defined as the ability to compare one situation with another based on experience or by referring to general principles. Reflecting involves deep thinking to solve problems and re-evaluating the solutions that have been used. The reflective thinking process, according to Mezirow and Kember, consists of four stages: habitual action, understanding, reflection, and critical reflection (Ghanizadeh & Jahedizadeh, 2017). Lee further states that there are six processes of reflective thinking: problem identification, defining the problem, seeking possible solutions, trying one of the possible solutions or the best solution, evaluation, and acceptance or rejection (Lee, 2005). Baron (1981) mentions five stages in the reflective thinking process: problem recognition, constructing various possibilities, reasoning, revision, and evaluation. Meanwhile, Zehavi and Mann (2005) mention four processes of reflective thinking: technique selection, solution process monitoring, insight, and conceptualization. The reflective thinking process constructed by the researchers in this study is based on modifications from previous studies (Setiyani et al., 2022; Setiyani, Waluya, Sukestiyarno, & Cahyono, 2024) with the following stages: reacting, seeking possible solutions, elaboration, and critical reflection.

The process of mathematical reflective thinking in solving problems is greatly influenced by each student's cognitive style (Blackburn et al., 2015; Verawati et al., 2020) as the way students understand, process, and evaluate information determines how they reflect on their learning experiences (Ekawati & Asih, 2019). Numerous literatures have identified cognitive styles, including Witkin’s introduction in 1977 of two cognitive styles, namely Independent Field and Dependent Field (Witkin et al., 1977). Students with an independent cognitive style are able to break down the components of a problem, answer using various strategies (Syamsuddin et al., 2020), connect the concept of integer and rational number operations with other mathematical concepts, and respond well to numerical problems even if they have not encountered them before (Setiyani, Waluya, Sukestiyarno, & Cahyono, 2024). Scaffolding has been found effective in helping students with a IF cognitive style to construct other ideas, thus finding simpler methods to solve problems (Huertas et al., 2017). Based on previous research (Setiyani, Waluya, Sukestiyarno, & Cahyono, 2024), this research will examine the reflective mathematical thinking process of students with a DF cognitive style in solving numerical problems. The characteristics of numeracy problems are chosen because they relate to reflective thinking, which involves consciously reflecting on acquired knowledge, constructing meaning and new skills, and making accurate predictions and decisions (Subekti et al., 2022). Therefore, this study aims to describe the mathematical reflective thinking process of students with a DF cognitive style when solving numerical problems. Based on the obtained reflection process, the researcher attempts to design appropriate scaffolding to enhance the mathematical reflective thinking skills of students with a DF cognitive style. Thus, the scaffolding developed is expected to assist DF students in overcoming difficulties when facing numerical problems and facilitate the more effective development of reflective thinking skills. This gradual approach allows students to learn progressively, with support tailored to their needs, enabling them to build deeper understanding and confidence in solving numerical problems. The findings of this study aim to provide practical strategies for teachers, focusing on designing adaptive learning approaches that effectively utilize scaffolding to support the cognitive development of students with a DF cognitive style.

2. METHOD

This study adopts a descriptive method with a qualitative approach. Qualitative descriptive research, is a method rooted in the philosophy of postpositivism, designed to investigate the conditions of natural objects, where the researcher serves as the primary instrument (Creswell & Creswell, 2017). This approach provides a comprehensive understanding of the data (Wijaya et al., 2024) enabling a more detailed and well-rounded analysis the reflective mathematical thinking ability from DF students in numeracy problems so that the scaffolding design can be identified.

The cognitive styles in this research consist of two characteristics: Dependent Field (DF) and Independent Field (IF), determined using the cognitive style test developed by WITKIN. The classification of characteristics is based on criteria established by Kepner and Neimark (1984), where subjects who provide 0 to 9 correct answers are categorized as students with a DF cognitive style, while those who answer correctly between 10 and 18 are classified as independent field. The subjects of this study are students with a DF cognitive style because these students tend to require more external support, such as guidance or scaffolding, during the mathematical problem-solving process. The stages of subject selection process are illustrated in Figure 1.

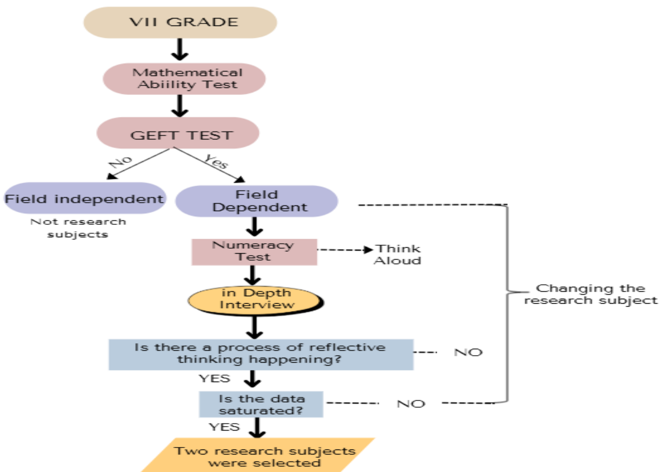


Figure 1. Selecting research subject

This study was conducted from July 2023 to January 2024, involving 92 an initial participants from seventh-grade students at three junior high schools located in Cirebon. The summary of the GEFT test results is presented in Table 1.

Table 1. Test GEFT result in 3 schools

No	School	Cognitive Style	
		Independent Field	**Dependent Field**
1	SMPN 5 Kota Ciebon	4	32
2	SMPN 11 Kota Cirebon	1	32
3	MTs. Husnul Khotimah 2 Pancalang	11	22
	Total	16	76

The selection of DF subjects was based on several considerations, including the level of reflective thinking achieved, willingness to participate as research subjects as expressed in the students' consent statements, fluency in verbal communication, recommendations from mathematics teachers, equivalence in initial mathematical abilities, and data saturation. Details of the research subjects are presented in Table 2.

Table 2. Test GEFT result

Research Subject Codes	Skor GEFT	Cognitive Style	Reflective Thinking Score
S-04	6	Dependent Field	38
S-05	7	Dependent Field	56.7

The numeracy problems in this study were adapted from questions developed by Putra et al. (2016) as an instrument to explore students' mathematical reflective thinking processes. These questions were tested for validity, practicality, and potential effects. After analyzing the research subjects' answers, a think-aloud process was conducted through interviews using a guide previously developed by Setiyani, Waluya, Sukestiyarno and Cahyono (2024) to confirm the results. If the responses to the numeracy problem-solving tasks aligned with the interview statements, the qualitative data were considered valid. In cases of discrepancies, further exploration was conducted with the research subjects to gather additional information and identify findings. The findings from the numeracy problems for each research subject, categorized by cognitive style, were then compared to derive the main discovery about students' mathematical reflective thinking processes. The selection of subjects was repeated until data saturation was achieved, indicated by consistent or recurring patterns across multiple subjects.

To ensure the data obtained were unbiased, this study employed triangulation. Specifically, time triangulation was used by examining the data through written tests and interviews conducted at different times or under different conditions. If the data demonstrated consistency (revealing many similarities), the results from the reflective thinking ability tests and interviews were deemed valid. If inconsistencies remained, additional data collection was conducted at another time and compared with the previous data. The data consistent with the final collection were considered valid.

The data analysis technique followed the stages proposed by Miles and Huberman (1994), which include: data reduction, which involves verifying students' work by discarding irrelevant data, clarifying, selecting, focusing, abstracting, and transforming raw field data into meaningful information; data presentation, which involves organizing and categorizing the reduced data into specific categories to facilitate conclusion drawing; and conclusion drawing/verification, which is the process of summarizing or verifying the data.

In this study, raw data collected from the field were reduced to extract only the information necessary to describe the reflective thinking processes of students with DF cognitive styles in number-related topics. After data reduction, the collected data were organized and categorized. The data were then presented in a more concise narrative form to make drawing conclusions easier. Conclusion drawing involved summarizing the data and verifying its accuracy concerning the reflective thinking processes of students with DF cognitive styles in solving number-related problems.

3. RESULTS AND DISCUSSION

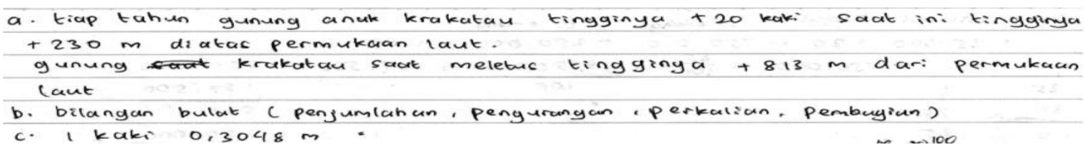
3.1. Results

Students with different cognitive style characteristics have variations in constructing their knowledge when solving numeracy problems. Dependent field students provide an opportunity to explore how scaffolding can assist them in developing mathematical reflective thinking patterns. The detailed thinking process of S-4 and S-5, who have a DF cognitive style and the same initial mathematical ability, is as follows : In the reacting stage, S4 experienced perplexity. The sign that S4 encountered difficulties aligns with S4's statement that "I just saw this problem, it feels like an Olympiad question." The researcher encouraged S4 to still attempt to solve the problem. S4 tried to identify some information contained in the problem. The following is an excerpt from the interview with S4 during the reacting stage:

Resercher (R) : *From the problem you've read, what is the current height of Mount Anak Krakatau, and what was the height of Mount Krakatau at the time of its eruption? Be mindful of the meaning of the "+" sign in the height provided!*

S4 : *Every year, the height of Anak Krakatau volcano increases by 20 feet. Currently, its height is 230 meters above sea level. When Krakatau erupted, it was 813 meters high. I'm marking this with a +, meaning positive, above sea level...*

S4 assumed that the numeracy problem was like a math Olympiad question, making it seem difficult to solve. S4's thought structure didn't match the structure of the problem. Therefore, it can be concluded that S4 experienced perplexity. To overcome this perplexity, S4 read the problem repeatedly, wrote down the information from the problem, as shown in Figure 2.


a. tiap tahun gunung anak krakatau tingginya + 20 kaki saat ini tingginya + 230 m diatas permukaan laut.
gunung ~~saat~~ krakatau saat meletus tingginya + 813 m dari permukaan laut
b. bilangan bulat (penjumlahan , pengurangan , perkalian , pembagian)
c. 1 kaki 0,3048 m

In English :

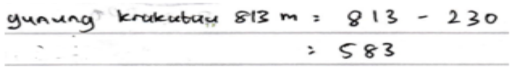
a. Each year, Mount Anak Krakatau increases in height by +20 feet. Currently, it stands at +230 meters above sea level. When Mount Krakatau erupted, it had a height of +813 meters above sea level.
b. Integers (addition, subtraction, multiplication, division)
c. 1 foot = 0.3048 meters

Figure 2. Reacting S4

Based on Figure 2, S4 did not write down what was being asked in the problem and was incomplete in writing the information, such as missing when Krakatau erupted. This impacted the stage of seeking various possible solutions. S4 stated they would find the difference between Anak Krakatau and Krakatau and determine the annual increase in height. This is in line with S4's statement:

S4 : *"The question is, when will the height of Anak Krakatau equal its parent Volcano. It's just about the year, right? Maybe I'll find the difference first"*

In the stage of finding various possible solutions, S4 was confident their answer would be correct. S4 calculated the difference between the two mountains as shown in Figure 3.



In English :
Mount Krakatau 813 m = 813 – 230 = 583

Figure 3. Elaboration S4 (1)

Based on Figure 3, S4 was able to calculate the difference between two integers, then continued by dividing the difference between Krakatau by 0.3048 (1 foot). S4 was unsure about the steps taken, as seen in the following excerpt:

R : *What steps should be taken to ensure consistency between meters and feet when solving the problem?*

S4 : *There's a note that 1 foot equals 0.3048, the difference is 583 meters. So, 583 meters divided by 0.3048 meters… hmmm... but in the problem, the height increase of Krakatau is 20 feet, so 583 divided by 20… wrong… wrong… the units are in feet, right? So, this 583 is in meters… this part is wrong ; I need to change the unit first…*

S4 thought to find the height of Anak Krakatau by finding the difference in height and then dividing it by 1 foot. S4's thought structure was still not aligned with the problem structure. S4 experienced critical reflection, realizing that some information wasn't completely identified, such as the annual height increase. S4 thought it should be divided by the annual height increase. At this point, S4 also realized that the increase was in feet, prompting S4 to convert it to meters. The effort S4 made to overcome perplexity involved trying to divide the difference by the length unit and dividing the difference by the annual height increase. S4 experienced perplexity during the process of elaborating can be seen in Figure 4.

Figure 4. Elaboration S4 (2)

After calculating the annual height increase of Anak Krakatau, S4 proceeded to find the year when Anak Krakatau would reach a height of 583 meters. Next, S4 tried starting from 29 to determine the year when Anak Krakatau reached 583 meters. However, S4 was still unsure about the method used, which involved trial and error, as per the following statement:

R : *After finding the difference in height before and after the eruption, what is the next step?*

S4 : *"I will calculate the annual height increase of Anak Krakatau...The method is 0.3048 multiplied by 20... the result is 6.0960... So if I keep adding 6.0960, it will take a long time... let's start from 29..."*

S4's effort to overcome perplexity was by trial and error with numbers up to 583 meters. This can be seen in Figure 5.

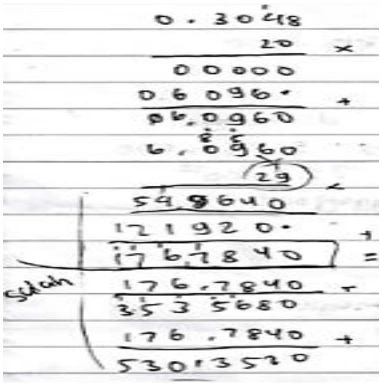


Figure 5. Elaboration S4 (3)

Based on Figure 5, S4 tried different numbers starting from 29 to obtain the year when Anak Krakatau reached 583 meters, resulting in 176.7840. Next, S4 added 176.7480 with 176.7840 to get 353.5680, and so on. S4 became doubtful and confused when the final result was 530.3520. If added again with 176.7840, it would exceed 583 (707.136). This is in line with S4's statement:

S4 : *"I added 176.7840, and the result is 353.5680. Added 176.7840 again, the result is 530.3520... the height difference is 583 meters... if 530.3520 is added with 176.7840, the result is too much, around 700... hmm... what should I do? Should I break it down one by one?"*

R : *"What do you mean by breaking it down, Jasmine?"*

S4 : *"adding it up with 176.7840, maybe I should try adding 6.0960 instead"*

S4's thought structure was still not aligned with the problem structure. After the final result, S4 realized that if added with 176.7840, it would exceed 583 meters. S4's effort to overcome perplexity was by trial and error with the increment of 6.0960, as shown in Figure 6.

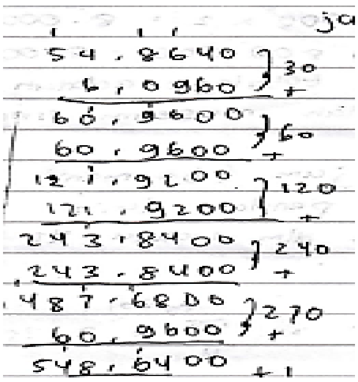


Figure 6. Elaboration S4 (4)

Based on Figure 6, S4 added 54.8640 with 6.0960, resulting in 60.9600. Since this was still far from the expected 583 meters, S4 continued adding the same value, 60.9600, and so on, until reaching 583 meters. When reaching 548.0400, S4 did not add with 548.0400 because it would result in thousands. This is in line with S4's statement:

S4 : *"If I add 548.0400, it will be in the thousands... so I'll add 60.9600... the result is 270 years, so in 2023, the height of Anak Krakatau matches its parent volcano in 2023 + 270, which is 2.293."*

R : *"Are you sure 'current' in the problem means 2023?"*

S4 : *"Yes, ma'am, it's 2023. Hmm, wait a minute, if I add this final result with the annual increment, will it fit the number?"*

S4 realized the mistake in calculating the year when Anak Krakatau would match its parent. This is evidenced by many scribbles. S4 thought 548.6400 could be added with the annual height increase, which is 6.0960. S4's written answer during the critical reflection stage can be seen in Figure 7.

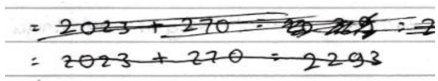


Figure 7. Elaboration S4 (5)

Next, S4 began calculating the final result obtained, which was 548.0400 plus 6.0960 for five increments. The final result obtained was 579.1200. S4 was confident that this final answer was close to 583 meters. This aligns with S4's statement:

R : *What would be the next step you would take after obtaining the results from the yearly height increase calculations?*

S4 : *"From here I continued adding the annual height increase...so 548.0400 plus 6.0960 equals 554.7360. Added 6.0960 again to get 560.8320. Added 6.0960 again to get 566.9280, and let's add two years' worth, shall we? So adding 12.1920 results in 579.1200..This is the closest to 583 meters"*

This statement indicates that S4 was confident the answer was correct because it was close to 583 meters. S4's exploration when continuing the calculation by adding the annual height increment can be seen in Figure 8.

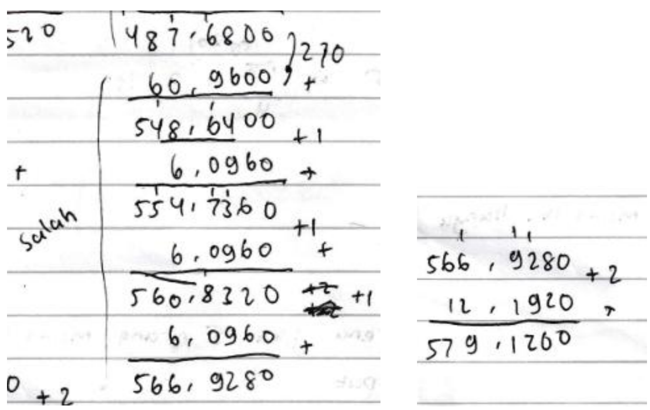


Figure 8. Elaboration S4 (6)

Next, S4 added the "current" year, 2023, with 275, because when Anak Krakatau reached 579.1200 meters, it took 275 years. S4's calculation can be seen in Figure 9.

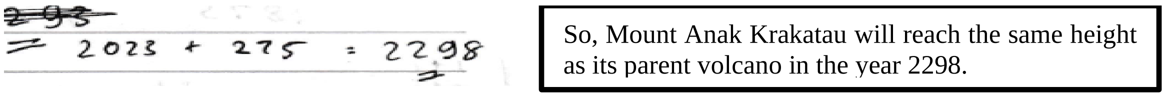


Figure 9. Critical Reflection S4

Based on Figure 9, S4 performed an addition operation of the current year with the obtained year when the height reached 583 meters and wrote a conclusion. In the critical reflection stage, perplexity occurred, as per the following interview excerpt:

R : *"After obtaining the answer, did you check your calculations again?"*

S4 : *"Yes, I looked at the calculation again, especially this part, Miss… Hmm… wait, I think I made a mistake… yaah, wrong again, ma'am…"I calculated 54.8640... that's just the result of 6.0960 multiplied by 9, ma'am... not 29. It should be 176.7840 plus 6.0960... I'll calculate it again.*

S4 realized the mistake in the calculation but did not recognize the incomplete information in the problem. S4 did not try to reread the problem but only rechecked the calculations. S4's attempt to address perplexity was to recalculate from a height of 176.7840 (starting from 29 years), as shown in Figure 10.

Figure 10. Elaboration S4 (8)

Based on Figure 10, S4 started the calculation at a height of 176.7840, then added the annual increase, and so on. The height increase was the result of estimating a number where the final result did not exceed 583 meters. To verify this, when S4 obtained a result of 579.1200 meters, they tried adding 6.0960 again and obtained a result that was more than 583 meters, which was 585.2160. S3 was still unsure whether the calculation was correct, so they rechecked it, as shown in Figure 11.

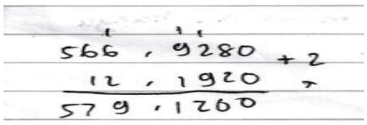


Figure 11. Elaboration S4 (9)

Next, S4 was convinced that it would take 95 years for the height of the child volcano to reach 583 meters because if they took 96 years, the result would be more than 583 meters, which was 585.2160. To find out when Anak Krakatau's height would match its parent, S3 added the current year, 2023, to 95 years, as shown in Figure 12.

$2023 + 95 = 2118$

Figure 12. Elaboration S4 (10)

Based on Figure 12, S4 was confident that in the year 2118, the height of Anak Krakatau would be the same as its parent. This aligns with the following excerpt:

R : *"According to Jasmine, does the answer make sense?"*
S4 : *"Yes, ma'am, I've checked the calculation again, and there's nothing wrong... this time I'm sure, ma'am."*

The excerpt from the interview indicates that S3 was confident that the obtained answer was reasonable. Therefore, according to S3, the difficulty in finding the year when Anak Krakatau would reach 583 meters was resolved by rechecking the calculations. S3 also made a conclusion, as shown in Figure 13.

jadi, gunung anak krakatau akan setinggi induknya
pada tahun 2118

So, Mount Anak Krakatau will reach the same height as its parent volcano in the year 2118.

Figure 13. Critical Reflection S4

Based on Figure 13, S4 believed that the final answer obtained was reasonable. The description of S4's reflective thinking process represents an incomplete clarificative reflection. The effort to overcome obstacles focused more on rechecking the calculations/procedural aspects without paying attention to detailed information in the problem. In the final answer obtained, S4's thought structure was still not aligned with the problem. Based on the analysis of S4's reflective thinking process, the pattern found by the researcher was incomplete or pseudo clarificative reflection. The detailed process of S4's mathematical reflective thinking while solving the problem is explained in Table 3.

Table 3. Explanation of S4's reflective thinking process

Code	Remarks
MS	Determine the year when the height of Anak Krakatau equals that of Krakatau.
a	Identify the height of Anak Krakatau.
I_1	Identify the height of Krakatau.
I_2	Identify the units (1 foot = 0.3048 meters).
I_3	Identify the annual increase of Anak Krakatau, which is 20 feet per year.
I_4	Find the height difference between Krakatau and Anak Krakatau.
S_1	Calculate the height difference between Krakatau and Anak Krakatau.

Code	Remarks
E_1	Divide the height difference by the unit in feet.
E_2	Recognize the mistake in the approach used.
RK_1	Divide the height difference by the annual increase of Anak Krakatau (in feet).
E_3	Recognize the mistake in the approach used.
RK_2	Calculate the annual height increase of Anak Krakatau in meters (convert unit concepts: 1 foot = 0.3048 meters using rational number operations, 20 × 0.3048 m).
E_4	Calculate the height when Anak Krakatau reaches 583 meters by trial and error.
E_5	Recognize the mistake in the approach used.
RK_3	Calculate the height when Anak Krakatau reaches 583 meters by trial and error again.
$E_{5'}$	Identify the current year as 2023.
I_5	Add the "current year" to the result obtained in step E5'.
E_6	Recognize the mistake in the approach used.
RK_4	Calculate the height when Anak Krakatau reaches 583 meters by trial and error again.
$E_{5''}$	Add the "current year" to the result obtained in step E5''.
$E_{6'}$	Conclude the answer obtained within the context of the problem.
RK_5	Reflective Thinking Process
- - - - -	Determine the year when the height of Anak Krakatau equals that of Krakatau.

The detailed process of S5's mathematical reflective thinking while solving the numeracy problem is explained in Table 4.

Table 4. Explanation of S5's reflective thinking process

Code	Remarks
MS	Determine the year when the height of Anak Krakatau equals that of Krakatau.
a	Identify the year when Krakatau erupted.
I_1	Identify the annual increase of Anak Krakatau, which is 20 feet per year.
I_2	Identify the height of Anak Krakatau.
I_3	Identify the height of Krakatau before it erupted (813 meters).
I_4	Identify the unit conversion, where 1 foot = 0.3048 meters.
I_5	Calculate the annual height increase of Anak Krakatau.
S_1	Find the height difference between Krakatau and Anak Krakatau.
S_2	Determine the year when Anak Krakatau reaches a height of 583 meters
S_3	Calculate the annual height increase of Anak Krakatau by converting units.
E_1	Calculate the height difference between Krakatau and Anak Krakatau.
E_2	Recognize the mistake in the approach used.
RK_1	Recalculate the height difference between Krakatau and Anak Krakatau.
$E_{2'}$	Determine the year when Anak Krakatau reaches a height of 583 meters (the difference) by multiplying both numerator and denominator by the same number, 10,000.
E_3	Recognize the mistake in the approach used.
RK_2	Determine the year when Anak Krakatau reaches a height of 583 meters (the difference) by rounding the numbers.
$E_{3'}$	Identify the current year as 2023.
I_6	Add the "current year" to the result obtained in step E3'.

Code	Remarks
E_4	Conclude the answer obtained within the context of the problem
RK_3	Determine the year when the height of Anak Krakatau equals that of Krakatau.
– – – – –	Identify the year when Krakatau erupted.

The process of S4's and S5'sincomplete clarificative reflection while solving numerical problems can be seen in Figure 14.

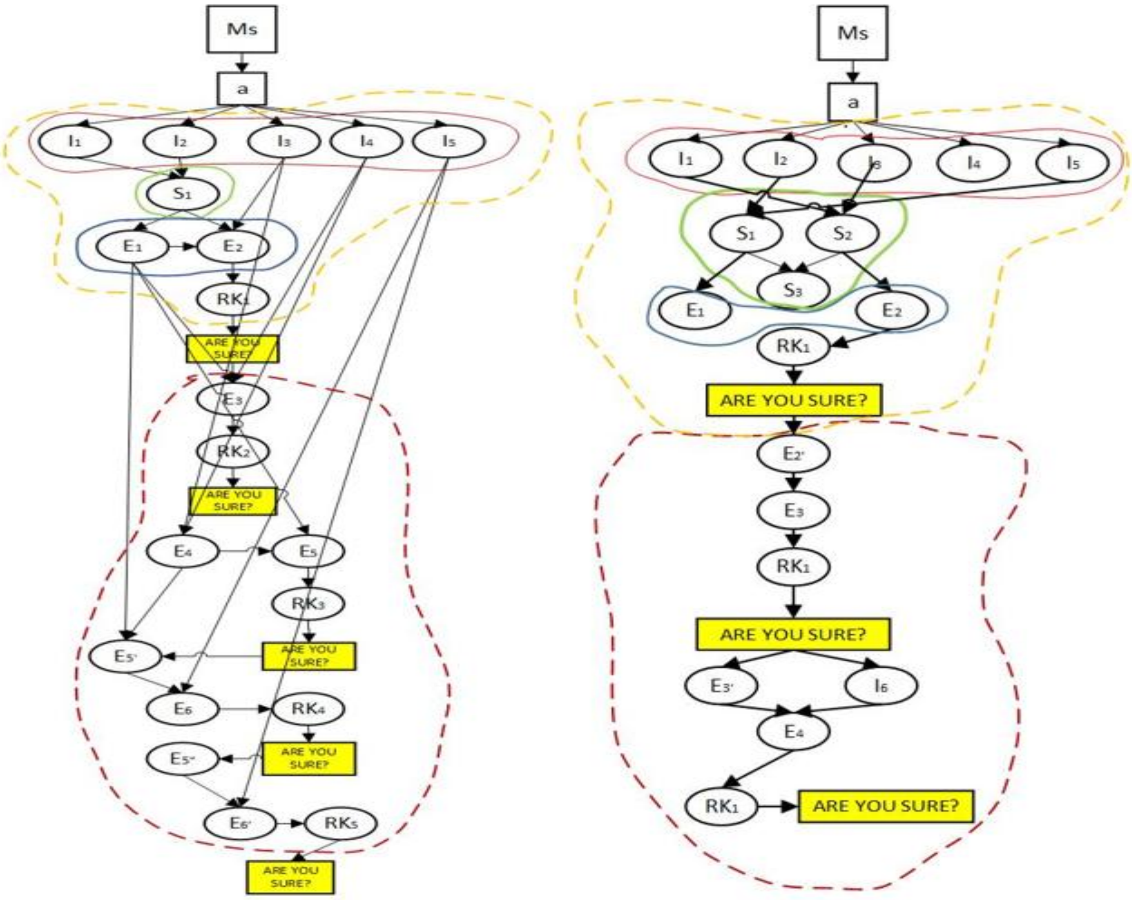


Figure 14. Mathematical reflective thinking S4 and S5

Next, incomplete or pseudo-clarificatory reflection was observed in DF students, specifically S4 and S5. When faced with perplexity, S4 and S5 repeatedly conducted investigations by clarifying information when identifying it, clarifying while choosing the operations to use, clarifying during calculations involving division of integers and rational numbers, recalling formulas needed to solve the problem, but did not monitor the solution process. DF students made mistakes during the reaction stage, resulting in inaccurate solutions. Upon reaching the final answer, DF students did not monitor the solution process or think it through thoroughly. For instance, when DF students obtained a final result of 2120.16 in the context of a year, they could not decide to round it to 2121, as the remaining 0.16 indicated the following year. This means that DF students were unable to analyze arguments from multiple perspectives and assess if there were deeper implications. This became a research finding on DF students' characteristics, showing they do not engage in the process of "result in context," which means they lack the ability to understand, interpret, and use numerical results (numbers or data) in concrete or situational contexts. It also includes the ability to link numbers to real situations, make appropriate interpretations, and take corresponding actions based on those numerical results.

3.2. Discussion

Seventh-grade students, most of whom have a DF cognitive style, often face difficulties in solving numeracy problems. To help them achieve the expected competencies, scaffolding can be provided (Reiser, 2003). Generally, scaffolding can take the form of modeling, providing hints, asking questions, instructing, explaining, and giving feedback (van de Pol et al., 2015). Scaffolding guidelines are structured based on the students' difficulties or problems, enabling them to reach higher levels of ability. The scaffolding guidelines in this study serve as a form of learning assistance for DF students to improve their reflective thinking skills in solving numeracy problems.

In this study, scaffolding was integrated with the Project-Based Learning (PjBL) model and was included in the teacher's guidebook, which had been validated by experts. The implementation took place over eight sessions, conducted in the classroom and utilizing the laboratory. Initially, students were not accustomed to the application of this innovative model and often complained about the numeracy problems given. To address this, researchers provided engaging learning resources and guidance to help students access learning materials at home. This strategy facilitated FI students who tend to learn independently and rely less on others.

For the first syntax, identifying fundamental questions, the teacher introduced integers using students' prior knowledge of natural numbers. This approach aligns with Gallardo (2002), who stated that students can learn about integers by transitioning from natural numbers, from arithmetic to algebra. For example, we know that the addition of natural numbers will result in a natural number, such as $4 + 5 = 9$, where 4, 5, and 9 are members of the set of natural numbers. If presented in a general form, we get $a + b = ...$, which will also yield a natural number, later called c, so the equation becomes $a + b = c$ where a, b, and c are natural numbers. However, a different fact is found when presenting an equation in the form $a + ... = c$, where a and c are natural numbers. To complete this equation, it is not always possible to do so with natural numbers. For example, $4 + ... = 9$ can be solved using fingers to find 5 to complete the equation. Next, if presented with $9 + ... = 4$, how can this equation be solved to be true? Obviously, this cannot be done using only the domain of natural numbers; we need to expand the domain of numbers used. This activity stimulates students to think reflectively by connecting or relating integers to natural numbers. Scaffolding at this stage is provided at level 2, and the teacher also prepares concrete and digital media (Scaffolding level 1) to help relate integer learning to real life. The importance of linking mathematical processes with real life or familiar contexts for students has been revealed in several studies.

Next, in steps 2, 3, and 4 of the PjBL model, which are designing project plans, creating schedules, and monitoring, students worked on projects related to integers and rational numbers in groups, such as finding temperature values in various countries, daily pocket money, mobile phone battery usage within a day (related to percentages), etc. While working in teams, students develop skills in planning, organizing, negotiating, and solving problems related to the tasks to be completed, determining who is responsible for each task, and how information will be gathered and presented (Noviyana, 2017). If students encounter difficulties in completing tasks on their own, full and continuous instructional support is needed; in this case, a guidance approach (scaffolding) helps students build understanding of new knowledge and processes (Bature & Jibrin, 2015). Projects are designed to allow students to learn by doing and apply ideas in real-world activities similar to those performed by professionals (Krajcik & Shin, 2014). The scaffolding provided at this stage is level 2.

In the fifth stage, which is evaluating project work, students present their projects to review the work done collaboratively. Project-based learning is a learning approach that gives students the freedom to plan learning activities, carry out projects collaboratively, and ultimately produce work products that can be presented to others (Greenier, 2020; Jacobsen & Børsen, 2019; Kokotsaki et al., 2016; MacLeod & van der Veen, 2020). At this stage, the teacher's role is to assign and provide tasks that actively engage students in thinking, encourage and carefully listen to ideas presented by students both verbally and in writing, consider and provide information on what is explored during discussions, and monitor, assess, and encourage students to participate actively, which can be presented so that students' problem-solving skills improve (Gunawan et al., 2023).

Evaluating students' learning experiences is the final stage of the PjBL model. The teacher and students reflect on the activities and results of the projects that have been completed. The teacher provides feedback, which ultimately leads to new findings to address the problems posed in the first stage of learning. The scaffolding provided at this stage is level 3, which involves encouraging reflective discussions. Teachers can facilitate reflective discussions among students to help them gain a deeper understanding of their experiences in the project or task. Teachers can ask questions that help students reflect on their experiences and think about ways to improve future projects or tasks. Scaffolding in PjBL should be flexible and adjusted to the students' level of understanding and ability. This allows them to progressively take responsibility for their own learning in the context of the projects they are working on (Waiyakoon et al., 2015). The findings of this study can provide a basis for designing differentiated learning that focuses on enhancing reflective thinking skills, especially in the context of numeracy, through technology, models, pedagogy, or other learning strategies.

4. CONCLUSION

In the Reacting stage, DF students read the problem in its entirety and are able to absorb the information well, but they struggle to record it completely. They identify information by recalling previously learned concepts and applying them based on experience. However, they are unable to interpret the problem using mathematical symbols. DF students find it difficult to extract key information from word problems because they are overly focused on the overall context. As a result, they tend to be slow in identifying the essential elements needed to solve the problem. In the Seeking Possible Solution stage, these students rely heavily on teacher instructions before they begin solving the problem. They may not trust their ability to tackle the problem independently without external help. Consequently, DF students depend on examples and written steps. They tend to seek concrete examples or written steps provided by the teacher or textbooks to understand how to solve numeracy problems. These students struggle to develop mathematical concept understanding on their own and are often not flexible in adapting strategies when faced with slightly different numeracy problems because they tend to follow what has been previously taught.

In the Elaboration stage, DF students solve problems without prior consideration and directly execute the solution. Students with reflective thinking ability are willing to correct mistakes and solve problems by recalling methods they have observed from previous experiences. In the Critical Reflection stage, DF students believe the final answer they have obtained is correct but are unable to connect this final answer to real-life situations. This study is limited to problem-solving in the numeracy domain of numbers. Other contexts, such as algebra, geometry, data, and uncertainty, have not been explored, leaving opportunities for future researchers to develop suitable scaffolding. Furthermore, the scaffolding designed in this study has not been implemented in a classroom setting to assess its effectiveness in enhancing mathematical reflective thinking skills.

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Author Contribution	: S: Conceptualization, Visualization, Writing - original draft, and Writing - review & editing; G: Conceptualization, Visualization, Writing - original draft, and Writing - review & editing; SBW: Formal analysis, Methodology, and Writing - review & editing; YLS: Formal analysis, Methodology, and Writing - review & editing; ANC: Formal analysis, Methodology, and Writing - review & editing; M: Conceptualization, Visualization, Writing - original draft, and Writing - review & editing; WY: Writing - review & editing.
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