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Identifying learning obstacles in proof construction for geometric transformations: Conceptual, procedural, and visualization errors

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Abstract

This study investigates learning obstacles encountered by pre-service mathematics teachers in constructing proofs for geometric transformations, a topic that has not been extensively examined in previous research. In contrast to prior studies, this research identifies specific types of errors, as well as their interconnections, representing the first step in uncovering learning obstacles. The study followed the four steps of phenomenology: bracketing, intuiting, analyzing, and describing, using written tests and interviews to explore students' errors. The findings reveal that errors can be categorized into three types: visualization errors, conceptual errors, and procedural errors. The analysis of their interconnections revealed that conceptual errors were the primary factor contributing to both procedural and visualization errors. Analyzing these errors led to the identification of epistemological obstacles, which manifested when participants struggled to apply fundamental concepts—such as injectivity, surjectivity, and bijectivity—to more complex tasks. Therefore, the study concludes that the primary learning obstacle discovered is an epistemological obstacle.

Keywords:
Conceptual errors, Epistemological obstacles, Geometric transformations, Procedural errors, Visual errors

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1. INTRODUCTION

Geometric transformations are fundamental in the field of mathematics, helping students develop spatial thinking, analytical reasoning, synthesis, and problem-solving skills. As Aktaş and Ünlü (2017) note, understanding geometric transformations supports students' development in spatial thinking and reasoning. Additionally, Kribbs and Rogowsky (2016) highlight that these concepts are crucial for constructing mathematical proofs. Despite its importance, many university students encounter significant difficulties in mastering this topic. A study by Kusuma and Setyaningsih (2015) revealed that they often face challenges in understanding the foundational concepts of geometric transformations.

Several studies have highlighted the challenges students face in understanding geometric transformations. Kusuma and Setyaningsih (2015) found that students struggled to grasp fundamental transformation principles and connect them to functions. Similarly, Noto et al. (2019) reported difficulties in both understanding and applying transformation concepts. Furthermore, Napfiah and Sulistyorini (2021) identified frequent conceptual errors that hindered students' learning progress. Beyond conceptual challenges, other research has focused on difficulties related to proof construction in geometric transformations. Indahwati (2023) noted significant struggles with proof construction, particularly in organizing logical arguments. Likewise, Maifa (2019) observed that students lacked the conceptual clarity needed to construct rigorous proofs and effectively apply transformation properties.

In addition to challenges in proof construction, errors in fundamental geometric tasks have also been documented. Aulia et al. (2023) found that students frequently misplace points and struggle to accurately identify the images of transformed objects, reflecting difficulties with visualization. Furthermore, Anwar et al. (2021) noted that university students often rely on rigid proof formats and face challenges in applying mathematical reasoning beyond predefined templates. Similarly, Sundawan (2018) emphasized that students encounter difficulties in visualizing geometric objects, understanding the principles of mathematical proofs, and employing appropriate reasoning strategies. Taken together, these studies suggest that students' difficulties in geometric transformations encompass conceptual understanding, procedural execution, and visualization.

Despite extensive research on student errors in geometric transformations, previous studies (Kusuma et al., 2024; Napfiah & Sulistyorini, 2021; Nurcahyo et al., 2024; Nurdiana et al., 2021; Sahara et al., 2023; Sunariah & Mulyana, 2020; Uygun, 2020) have primarily focused on categorizing errors into conceptual, procedural, or visualization-based types. While this approach provides valuable insights, it does not fully address the underlying learning obstacles that contribute to these errors. Moreover, existing research has yet to explore how these distinct types of errors may be interconnected. For example, students who make procedural errors in proof construction may do so as a result of deeper conceptual misunderstandings, while those who struggle with visualization may have difficulty fully grasping the transformation properties. Understanding the interrelationship between these errors is essential for identifying the fundamental barriers that hinder students' mastery of geometric transformation proofs.

A study by Noto et al. (2019) investigated learning obstacles in transformation geometry but addressed the topic in a broad manner, identifying general difficulties such as comprehension issues, visualization struggles, challenges in defining transformation principles, problems with proof comprehension, and difficulties in interpreting problems. However, similar to previous studies (Kusuma et al., 2024; Napfiah & Sulistyorini, 2021; Nurcahyo et al., 2024; Nurdiana et al., 2021; Sahara et al., 2023; Sunariah & Mulyana, 2020; Uygun, 2020), this research did not explore the interrelationships between various types of difficulties. Moreover, it did not analyze these learning obstacles within the context of Brousseau's theoretical framework, thereby leaving a gap in understanding how these obstacles correspond to epistemological, ontogenic, or didactical obstacle in mathematical learning (Brousseau, 2002).

Epistemological obstacles emerge when students face challenges integrating new knowledge with their existing understanding, which subsequently hampers their ability to apply learned concepts to more advanced tasks (Araújo & Menezes, 2022; Dewi et al., 2022; Jatisunda et al., 2025; Maknun et al., 2022; Sari et al., 2024; Siagian et al., 2022; Sulastri et al., 2022). Ontogenic obstacles, on the other hand, are related to a student's cognitive development. These obstacles occur when tasks are either too complex or too simplistic, leading to ineffective learning. When tasks are too difficult, students may feel overwhelmed, while tasks that are too easy fail to engage them deeply, thus hindering meaningful learning. Finally, didactical obstacles are linked to the instructional design (Fardian et al., 2025; Fitriani & Widjajanti, 2024; Kandaga et al., 2022; Kuncoro et al., 2024; Nurdiana et al., 2021; Sunariah & Mulyana, 2020; Uygun, 2020). These obstacles arise when the sequencing or structure of learning materials does not align with students' cognitive development, creating gaps in their understanding.

To address these gaps, this study investigates the interconnections between conceptual errors, procedural mistakes, and visualization difficulties in the process of constructing proofs for geometric transformations. By examining these interrelationships, this study seeks to uncover the learning obstacles underlying students' struggles, providing a comprehensive understanding of the cognitive barriers they face in proving transformations. Unlike previous studies that merely identify student errors, this research aims to bridge the gap by identifying the learning obstacles that contribute to these difficulties, thereby offering deeper insights into the challenges that hinder students' success in mastering proof construction in geometric transformations.

2. METHOD

This study adopts qualitative research methodology with a phenomenological approach to explore and describes the specific errors made by pre-service mathematics teachers in constructing proofs of transformation. The primary focus of this research is to analyze and understand the challenges encountered by pre-service mathematics teachers in mastering the formal concept of geometric transformations as bijective mappings and in applying these concepts to proof construction. Furthermore, this study aims to identify the underlying learning obstacles that contribute to these errors.

The phenomenological approach is grounded in a strong philosophical foundation, as articulated by Edmund Husserl, emphasizing the importance of suspending preconceptions to understand phenomena in their purest form (Tuffour, 2017). This approach offers a framework for exploring how individuals interpret and make sense of their experiences through descriptive and interpretative analyses. To support this approach, the phenomenological research process is conducted through four main steps:

Bracketing

In this phase, Sanders emphasizes that researchers must suspend any initial assumptions or beliefs regarding the phenomenon under investigation to ensure objective data analysis (Greening, 2019). Bracketing is thus a key process in phenomenological reduction, enabling phenomena to be understood in their purest form. During this stage, the researcher identifies initial preconceptions, designs the test, administers it to students, and collects their responses.

a. Identifying Initial Preconceptions: The researcher assumes that participants make errors at three critical stages of proof construction: proving that a mapping is a function, proving the injectivity property, and proving the surjectivity property.
b. Designing an Unbiased Test: The researcher selects problems from relevant geometric transformation textbooks. The test includes tasks requiring formal proof of a mapping's qualification as a geometric transformation, as well as the identification of the injectivity and surjectivity properties of the given mapping.
c. Documenting Student Responses: All participants' responses are systematically collected for subsequent analysis.

Intuiting

Following the bracketing process, the intuiting stage centers on understanding the meaning that students assign to the phenomena they encounter. Moustakas underscores the significance of the researcher's active engagement in deeply comprehending the participants' experiences (Greening, 2019). This stage is primarily concerned with exploring participants' perspectives and identifying the errors they make.

a. Interviews: After the test, participants are interviewed to delve into their thought processes.
b. Documenting Interview Dialogues: Relevant excerpts from the interviews are recorded and utilized as supporting data for the research findings.

Analyzing

In this stage, the collected data undergoes analysis through coding and categorization processes. According to Polkinghorne, phenomenological analysis involves identifying overarching themes within the data (Greening, 2019). During this phase, data from both tests and interviews are examined to detect error patterns and uncover underlying learning obstacles.

a. Error Categorization: participants' responses and interview transcripts are systematically coded and classified according to error categories.
b. Data Presentation in Tables: The findings are summarized in tables, listing, types of errors, frequency of errors, identified learning obstacles.

Describing

The final stage entails constructing a descriptive narrative derived from the analyzed data. According to Greening (2019), phenomenological description seeks to offer a profound and comprehensive understanding of the phenomena under study. This stage is dedicated to synthesizing the research findings into a cohesive narrative.

a. Explaining findings from tables: This step involves elucidating the findings presented in the tables, with a focus on identifying the most prevalent error types and the emerging categories of errors.

b. Describing the relationship between error types and learning obstacles: In this step, the relationship between error types and learning obstacles is articulated, illustrating how specific errors are linked to particular learning obstacle.

2.1. Participants

The participants of this study were selected using criterion sampling, a purposive sampling method where individuals are chosen based on specific predetermined criteria (Tuffour, 2017). This method intentionally prioritizes participants who can provide meaningful insights into specific experiences relevant to the study (Alase, 2017). The study involved 19 pre-service mathematics teachers enrolled in the Mathematics Education program at Universitas Timor, Indonesia.

Participant selection was based on a set of criteria related to their academic background and course enrollment, ensuring that they had sufficient exposure to the topic of geometric transformations. The inclusion criteria were as follows:

a. Enrollment in the Geometric Transformations course, ensuring participants had prior exposure to geometric transformation proofs.
b. Completion of coursework on geometric transformations or related topics, confirming their familiarity with relevant theories.
c. Demonstrated knowledge of mathematical proof techniques, validated through academic records and lecturer confirmation.
d. Willingness to to engage in problem-solving tasks and participate in interviews as part of the study.

To validate these criteria, participants were selected from students who had successfully completed the Geometric Transformations course, ensuring their exposure to formal proof techniques related to geometric transformations. Eligibility was confirmed through course enrollment records and lecturer verification, ensuring that participants met the necessary knowledge requirements.

2.2. Data Collection

Data were collected using two primary methods: tasks and semi-structured interviews.

Tasks

Participants were given a structured task designed to assess their ability to construct mathematical proofs related to geometric transformations. The task aimed to explore how participants approach proof construction and identify potential challenges in applying formal mathematical definitions and logical reasoning. The selection of this task was justified by its inclusion in widely used university-level geometry textbooks. A review of four such textbooks commonly used in Indonesia revealed that three out of the four included this problem as a primary example for explaining transformation proofs. The following are the titles of the university-level geometric transformation textbooks that feature this type of problem: *Geometri Transformasi* (Budiarto, 2015); *Tangkas geometri transformasi: Cepat tepat menguasai geometri transformasi* (Kurniasih & Handayani, 2017); *Geometri*

Transformasi (Darhim & Rasmedi, 2014); and *Geometri Transformasi* (Nugroho et al., 2018).

To ensure its relevance to this study, only minor modifications were made to the naming of points, lines, and functions, without altering the fundamental structure of the problem. These adjustments aimed to preserve the integrity of the problem while enabling the exploration of participants' approaches and reasoning in proof construction. The task was designed to prompt students' understanding and problem-solving strategies by requiring participants to engage in two primary activities. The two primary activities required of participants were:

a. Constructing visual representations to illustrate the mapping.
b. Formally proving whether the given mapping qualifies as a transformation.

These two tasks provide insights into how students connect abstract mathematical definitions with concrete problem-solving strategies, which may, in turn, help identify errors and potential learning obstacles in proof construction. The task used in this study is presented in Figure 1.

Let $S \in \mathbb{R}^2$. A mapping with the domain in $\mathbb{R}^2$ and codomain also in $\mathbb{R}^2$ is defined as follows:

1. $G(S) = S$.
2. If $M \neq S$, then $G(M) = W$, where W is the midpoint of the line segment $\overline{SM}$.

Based on the given definition:

Task 1: Visualize the mapping in the form of a diagram that you can construct.

Task2 : Investigate whether the given mapping qualifies as a transformation.

Figure 1. Task of proving a transformation

Semi-Structured Interviews

After completing the task, participants participated in semi-structured interviews designed to gain deeper insights into their engagement with the task and their problem-solving processes during proof construction. The interviews specifically aimed to explore their understanding of the task requirements and the strategies they employed in constructing the proof. The interview structure focused on investigating:

a. Participants' understanding of the mapping and task requirements.
b. The steps they took to verify the properties of the transformation, specifically injectivity and surjectivity.
c. Challenges they encountered during the proof process.

2.3. Data Collection Process

Participants completed the tasks individually in a quiet, distraction-free environment to ensure full concentration. Each participant was allotted 60 minutes to complete the assigned task, which was considered sufficient based on the problem's complexity and preliminary testing. After task completion, the researcher reviewed the responses over the course of one day to prepare targeted interview questions based on the students' answers. The following day, face-to-face interviews were conducted to explore participants' approaches to the task and the reasoning behind any difficulties or errors they encountered. Each interview lasted between 30 and 45 minutes and was audio-recorded with the participants' informed consent. The focus was on understanding how they approached the task and the underlying reasons for any errors made. These steps ensured the thoroughness and consistency of the data collection process, providing a detailed understanding of participants' experiences and strategies during the proof construction process.

2.4. Data Analysis

In line with the phenomenological approach employed in this study, data analysis was conducted in accordance with the four key phases of phenomenological research: Bracketing, Intuiting, Analyzing, and Describing. Following the administration of the test and the collection of participants' responses during the Bracketing phase, as well as the interviews conducted in the Intuiting phase, the analysis then progressed through the Analyzing and Describing phases. These latter stages involved coding, categorization, interpretation, and presentation, ensuring a structured and systematic examination of the data. The analysis proceeded through the following steps:

Coding (Analyzing)

The responses from participants' task solutions and interviews were carefully reviewed to identify recurring elements and key insights related to their approaches to proof construction. Coding was applied systematically to extract important features from the data.

Categorization (Analyzing)

The identified codes were grouped into broader categories representing the key challenges in proof construction. These categories were formed based on recurring patterns observed in the data, reflecting common themes related to difficulties students faced during proof construction. The categorization process allowed for a deeper examination of the underlying challenges encountered by participants.

Interpretation (Describing)

After coding and categorization, the data were further analyzed to explore the connections between the identified challenges and potential learning obstacles. This phase involved interpreting how errors or difficulties in proof construction were linked to specific barriers in learning, such as didactical, epistemological, or ontogenetic obstacles.

Presentation (Describing)

The findings were synthesized into comprehensive narratives and tables, emphasizing recurring errors, their frequency, and the corresponding learning obstacles. The results were systematically presented in both structured narratives and tables, offering a clear and comprehensive summary of the findings while supporting their interpretation.

3. RESULTS AND DISCUSSION

3.1. Results

This section presents the findings from the analysis of participants' responses to the given task, focusing on the types of errors identified in their attempts to: (1) visualize the mapping as specified in the task, and (2) determine whether the mapping qualifies as a transformation. The data, collected from 19 pre-service mathematics teachers, were categorized into three main error types—conceptual, procedural, and visual—based on the framework outlined in the methodology.

3.1.1. Bracketing and Intuiting Phases: Identification and Exploration of Student Responses

Participants completed the written test individually, and their responses were analyzed to identify emerging error patterns. The responses were classified into three categories: correct responses, responses with errors, and no responses. Table 1 provides a summary of participants' performance across both tasks.

Table 1. Summary of participants' responses to the tasks

Task	Category	Number of Students	Percentage (%)
Task 1 – Visualizing the mapping	Correct	13	68
	Incorrect	6	31
	No. respon	0	0
Task 2 – Profing Transformastion	Correct	2	10
	Incorrect	13	68
	No response	4	21

Based on Table 1, there is a notable difference in participants' performance between visualizing the mapping and proving the transformation. A total of 68% of students successfully visualized the transformation, while 10% provided a correct proof. In Task 1 (Visualization), all students attempted the task, with no non-responses. However, in Task 2 (Proof Construction), 21% of students did not respond.

Although all participants attempted the visualization task, not all representations were correct. Some participants misinterpreted the transformation, leading to incorrect visual depictions. Figure 2 presents an example of an incorrect visualization, where the student incorrectly placed M' as the midpoint. Instead of correctly identifying the midpoint as the center between points S and M, the student positioned it closer to one endpoint, disrupting the symmetry required for an accurate transformation representation.

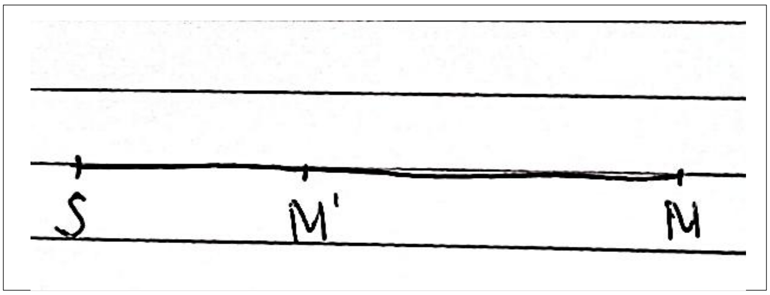

Figure 2. Answer to visualize the transformastion

To further investigate the reasoning behind this mistake, follow-up interviews were conducted with participant who made that error, as shown in Figure 3.

R: *Can you explain how you determined the position of M'?*
S1: *"I thought the midpoint should simply be somewhere between the two points. I placed it in between without considering the exact distances."*

Figure 3. Interview with participant about the answer in Figure 1

Recognizing that 21% of participants (4 participants) did not provide any response in Task 2 (Proof Construction), further interviews were conducted to understand the difficulties they faced in initiating or constructing the proof. The interviews revealed that participants who left the question unanswered generally felt they lacked sufficient understanding to begin the proof, both in terms of the definition of transformation and the proof strategies required. Figure 4 presents selected excerpts from the interviews, illustrating the reasons behind their struggles. Of the 4 participants who did not respond at all in Task 2, one participant successfully visualized the mapping in Task 1. This participant is represented by S3 in the following interview.

R: *"You successfully visualized the mapping, but you didn't attempt the proof. Why?*
S1: *"I understood how to visualize the mapping because it seemed easy, but when it came to proving it, I had no idea where to start, so I didn't answer at all."*
R: *"Do you know that the initial step in the proof is to verify whether this mapping is a function and then determine whether it is a bijective?"*
S2: *"I didn't know that. I just illustrated the mapping based on the given information, but beyond that, I had no idea what to do.*
R: *"But do you know what a function is?"*
S3: *"Yes, I do. A function means that each element in the domain is paired with exactly one element in the codomain."*

Figure 4. Interview with the participants who failed to respond to task 2

While some participants did not attempt Task 2 at all, the majority (68%) attempted the proof but made errors. One common mistake observed in s responses was their tendency to identify the mapping as a function without verifying the required properties of a transformation. As shown in Figure 5, one student correctly stated that each point in the domain has exactly one corresponding image in the codomain but did not address the injectivity or surjectivity of the mapping. When asked why he only stated that the mapping was a function, the participant explained that he assumed this was sufficient to qualify as a transformation.

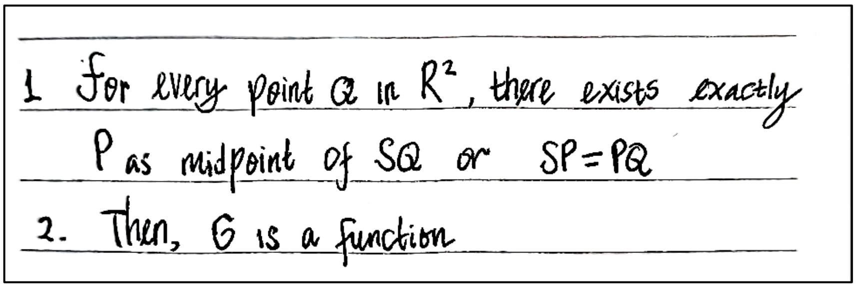
1 For every point Q in R², there exists exactly
P as midpoint of SQ or SP = PQ
2. Then, G is a function

Figure 5. Participant 1's answer to prove the transformation

Another common error observed in participant's responses was the misalignment between the assigned domain and codomain elements and the given mapping definition. As illustrated in Figure 6, the student provided a structured representation of the mapping, specifying the domain and codomain sets. However, the assigned elements did not conform to the actual transformation rule. Additionally, in points (a) and (b), the student incorrectly stated function properties as transformation criteria. While these conditions ensure that the mapping is a function, they do not establish the necessary conditions for it to be classified as a transformation.

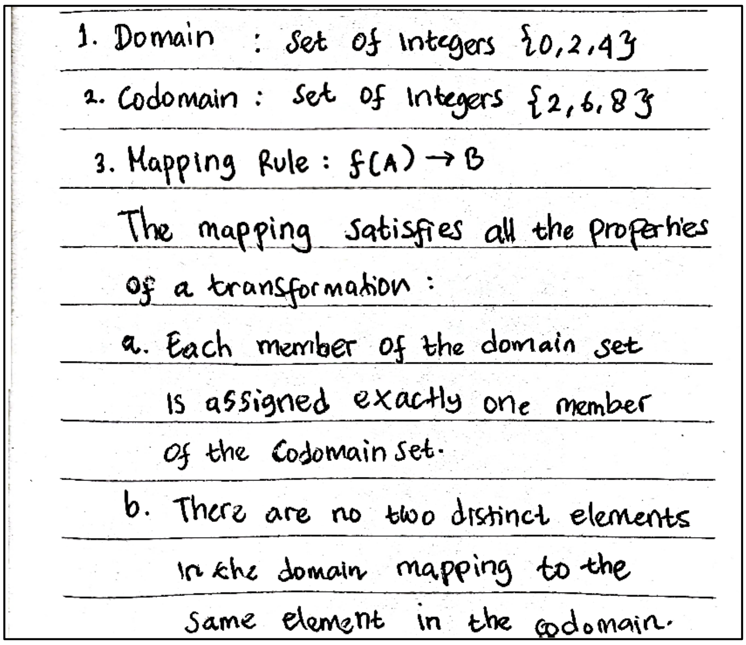
1. Domain : Set of Integers {0,2,4}
2. Codomain : Set of Integers {2,6,8}
3. Mapping Rule : f(A) → B
The mapping satisfies all the properties
of a transformation :
a. Each member of the domain set
is assigned exactly one member
of the Codomain set.
b. There are no two distinct elements
in the domain mapping to the
same element in the codomain.

Figure 6. Participant 2's answer to prove transformation

To gain deeper insights into why students misinterpreted the conditions of a transformation, interviews were conducted, as shown in Figure 7.

R: *"Why did you assign specific elements to the domain and codomain? Why did you choose integers? And why did you claim that these function criteria define a transformation?"*

S4: *"I just wanted to provide an example of the domain and codomain because integers are commonly used. As for the transformation, I understood that a transformation is simply a function."*

Figure 7. Interview with the student about the answer in Figure 6

The last example represents a common incorrect response frequently observed among students. As shown in Figure 8, the student attempted to prove only one property of the transformation—injectivity—while completely neglecting surjectivity. However, even within the proof of injectivity, errors were present. The participant who provided this response explained that they struggled with proof by contradiction, as it was their first encounter with such a method, and they were unable to grasp its logical flow. When asked about the concept of injectivity in relation to the "if P≠R then G(P) ≠ G(R)", the student did not recognize it as a formal condition for an injective function. Instead, they wrote this statement at the end of the proof, mistakenly assuming it was a natural outcome rather than a fundamental condition that needed to be stated at the beginning of the proof.

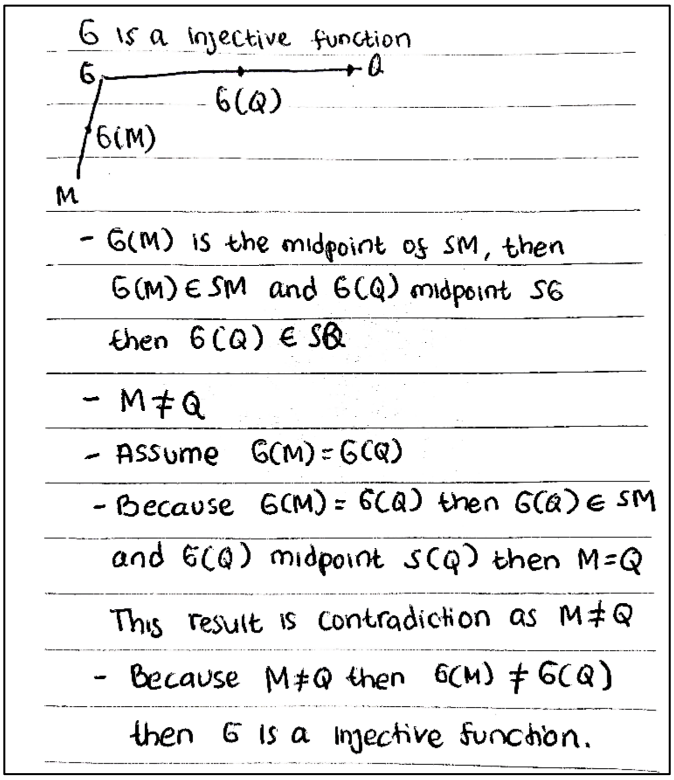

Figure 8. Answer to prove the injective function

3.1.2. Analyzing Phase: Systematic Coding and Categorization of Errors

Based on students' responses and interview findings, a systematic coding process was employed to classify errors into three main types: conceptual errors, procedural errors, and visual errors (see Tables 2 to 4). This classification was guided by recurring patterns observed in students' incorrect answers and their explanations during interviews. The categorization process enabled a structured analysis of students' difficulties, highlighting specific areas where misconceptions or weaknesses in reasoning occurred.

Conceptual errors were identified when participants demonstrated fundamental misunderstandings of transformation properties, such as failing to recognize that a valid transformation must be bijective, meaning it satisfies both injectivity and surjectivity (see Table 2). These errors were evident in students' inability to connect definitions to the structure of the proof. The detailed classification is provided in Table 2.

Table 2. Indicators of conceptual errors

Aspect of Analysis	Error Indicators	Error Code
Understanding of Bijective Functions	participants fail to understand that transformations are bijective functions, meaning they satisfy the properties of injectivity and surjectivity.	CE1
Understanding of Injectivity	participants cannot explain or demonstrate that if $P \neq R$ then $G(P) \neq G(R)$	CE2
Understanding of Surjectivity	Participants are unable to show that every element in the codomain has a corresponding preimage in the domain.	CE3
Understanding of function	Participants Fail to understand functions, their conditions, and how to apply function concepts.	CE4

Procedural errors were observed when participants struggled with the application of logical proof strategies, particularly in constructing a proof by contradiction (see Table 3). Many students failed to provide a structured argument or omitted key logical steps in their reasoning. The detailed classification is provided in Table 3.

Table 3. Indicators of procedural errors

Aspect of Analysis	Error Indicators	Error Code
Function Verification	participants fail to verify that each element in the domain has exactly one corresponding image.	PE1
Injectivity Proof with Contradiction	participants fail to construct a logical argument by contradiction to prove the injectivity property.	PE2
Surjectivity Proof	participants do not use a systematic approach to demonstrate that every element in the codomain has a preimage in the domain.	PE3

Visual errors were characterized by mistakes in representing the mapping geometrically, such as misplacing critical points or incorrectly identifying relationships between elements in the domain and codomain (see Table 4). These errors often stemmed from intuitive reasoning rather than formal mathematical analysis. The detailed classification is provided in Table 4.

Table 4. Indicators of visual errors

Aspect of Analysis	Error Indicators	Error Code
Breaking Down the Source	participants fail to extract key information from the given mapping definition or misunderstand the relationship between the points involved in the mapping	VE1
Initial Coordination	participants do not correctly identify the domain elements or the codomain elements to construct an initial visual representation.	VE2
Setting the Target	participants are unable to construct a proper diagram showing the mapping process, including the correct positioning and relationships of points.	VE3

To provide a quantitative overview of the distribution of categorized error types, Table 5 presents the frequency of conceptual, procedural, and visual errors observed in each task. This classification is based on the coding process conducted earlier, which identified common error patterns in students' responses to the visualization of the mapping (Task 1) and the proof of the transformation (Task 2).

Table 5. Distribution of conceptual errors across tasks

Error Code	Task 1	Task 2	Percentage (%)
CE1	-	4	21
CE2	-	11	58
CE3	-	6	31
CE4	6	-	
PE1	-	4	21
PE2	-	11	58
PE3	-	6	31
VE1	3	-	15
VE2	0	-	0
VE3	3	-	15

3.1.3. Describing Phase: Interpretation and Presentation of Findings

As shown in Figure 2, several students struggled to accurately position the mapped points in their visual representation. Additionally, some responses demonstrated misplacement of domain and codomain elements, leading to incorrect representations of the given transformation. These errors were categorized as visualization errors, which were then coded into three specific indicators presented in Table 4.

Errors related to conceptual understanding were also prevalent. Figures 5 and 6 illustrate responses in which participants identified the mapping as a function but failed to recognize that transformations require bijectivity. These conceptual errors reflect misunderstandings of transformation properties and were further classified into four indicators (see Table 2). Similarly, Figure 8 presents an example in which participants failed to complete the proof, proving only injectivity or surjectivity, but not both. In some cases, students attempted proof by contradiction but made logical errors in structuring their argument. These responses were classified as procedural errors, which were subsequently broken down into key indicators in Table 3.

To quantify the frequency of these errors across participants, a systematic data recording process was implemented. The total number of participants making each type of error was compiled and summarized in Table 5, providing an overview of the distribution of errors across both tasks. In particular, visual errors were only observed in Task 1, as this task focused on how students represented the given mapping. Three students made VE1 errors, while another three made VE3 errors, resulting in a total of six students with incorrect visualizations.

Conceptual errors (CE) and procedural errors (PE) were observed in Task 2, as this task required formal proof construction. The data show that 4 participants committed CE1 errors, meaning they failed to understand that transformations must be bijective. These participants left the proof section blank, as confirmed through interviews. No students were classified under PE1 errors, as all those who attempted the proof successfully established that each domain element had a unique image in the codomain. Additionally, conceptual errors were also evident in Task 1. The data reveal that 6 participants, who struggled with visualization (VE1 and VE2), were unable to articulate their understanding of the function concept when questioned (CE4). Interestingly, a participant who left Task 2 blank was still able to answer Task 1 correctly.

As presented in Tables 2 and 3, a strong relationship was observed between CE2 and PE2 (errors in proving injectivity), as well as between CE3 and PE3 (errors in proving surjectivity). Specifically, 11 students made both CE2 and PE2 errors, indicating that the same individuals struggled with proving injectivity. Additionally, 6 participants made both CE3 and PE3 errors by failing to prove surjectivity. Among these, 4 participants made all four errors (CE2, CE3, PE2, and PE3), indicating difficulty in both proof components. In total, 17 students made errors in Task 2.

These findings systematically present the frequency of errors observed in participants responses, offering a quantitative overview of their difficulties in both visualizing and proving transformations. This analysis lays the groundwork for further discussion on the potential learning obstacles that contributed to these errors. Theoretical perspectives on learning obstacles, as outlined by (Brousseau, 2002), categorize these obstacles into three main types: (1) Epistemological Obstacles – These obstacles arise when students struggle to apply learned knowledge to new or complex contexts. Difficulties in proof construction, such as an inability to transfer the concepts of injectivity, surjectivity, and bijectivity to formal reasoning, may indicate the presence of epistemological obstacles; (2) Didactical Obstacles – These obstacles stem from instructional design or the sequencing of learning materials. If fragmented instruction contributes to students' misconceptions—such as learning transformation properties without reinforcing their logical implications—didactical obstacles may be a contributing factor; and (3) Ontogenic Obstacles – These obstacles are related to students' cognitive development and readiness to engage with abstract mathematical reasoning. Persistent difficulties in synthesizing conceptual, procedural, and visualization skills despite repeated exposure may suggest the presence of ontogenic obstacles linked to cognitive limitations.

3.2. Discussion

The Conceptual errors (CE) emerged as the most prevalent type of mistake in this study, significantly contributing to students' difficulties with proof construction. As shown in Table 5, CE1 was observed in 4 participants, CE2 was the most frequent, affecting 11 students, followed by CE3, which was observed in 6 participants. Conceptual misunderstandings were also evident in CE4 (misconceptions about the function concept), which was noted in 6 participants, particularly those who struggled with visualization errors (VE1 and VE2) in Task 1. This suggests that difficulties in conceptualizing the fundamental properties of transformations constitute a major barrier for many participants, hindering their ability to effectively apply procedural and visual reasoning.

This study highlights the interconnection of conceptual, procedural, and visual errors. The data reveal that students who struggled with visualization (VE1 and VE2) were unable to articulate their understanding of the function concept (CE4). This suggests that challenges in representing a transformation visually may be linked to deeper conceptual misunderstandings regarding functions and their properties. In Task 1, errors such as misplacing points or incorrectly aligning domain and codomain elements contributed to these visualization errors, further emphasizing the interplay between conceptual comprehension and the ability to accurately represent mathematical relationships. These findings align with previous research, such as Aulia et al. (2023), who reported that students frequently misplace points and struggle to determine the image of a transformation, indicating persistent difficulties in visualization. Similarly, Sundawan (2018) noted that students face challenges in visualizing geometric objects and understanding mathematical proof principles, reinforcing the idea that visualization errors are closely tied to conceptual misunderstandings.

Furthermore, the study found that conceptual errors were strongly linked to procedural difficulties in proof construction. As presented in Tables 2 and 3, all 11 students who struggled with understanding injectivity (CE2) also made errors in proving injectivity (PE2), and all 6 students who misunderstood surjectivity (CE3) also encountered difficulties in proving surjectivity (PE3). For example, students who mistakenly treated the statement "if $P \neq R$, then $G(P) \neq G(R)$" as a final outcome, rather than recognizing it as a defining property of injective functions, faced significant challenges in completing their proofs. This misunderstanding led to incomplete proof attempts or the omission of crucial logical steps. This observation aligns with prior studies, such as Indahwati (2023) and Maifa (2019), who found that students struggle significantly in proof construction, particularly in structuring logical arguments and correctly applying transformation properties. Anwar et al. (2021)

further emphasized that Indonesian mathematics students tend to rely on rigid proof formats, suggesting a lack of deep conceptual engagement in proof reasoning.

Our findings align with those of Noto et al. (2019), who observed that pre-service mathematics teachers tend to focus more on procedural knowledge than on developing a deeper conceptual understanding. However, the current study emphasizes that conceptual errors were the most prevalent, affecting students' ability to construct proofs. As shown in our data, students who struggled with the conceptual understanding of bijectivity, injectivity, and surjectivity also faced significant challenges in applying procedural proof strategies. This finding supports the notion that a solid understanding of the underlying concepts is crucial for successfully constructing valid mathematical proofs. While procedural knowledge is important, the interdependence between conceptual and procedural errors underscores the need for a deeper conceptual foundation to facilitate effective proof construction.

The result of this study also align with the work of Kusuma and Setyaningsih (2015), who identified comparable difficulties in participants' understanding of geometric transformations. They observed that participants often struggled with applying formal transformation rules and constructing logical steps in proof construction. This finding consistent with the procedural difficulties we observed in this study, where students not only struggled to understand the concepts of injectivity and surjectivity but also faced challenges in applying these concepts in constructing valid proofs.

Similarly, Aktaş and Ünlü (2017) found that students frequently struggled with visualizing geometric properties when working with transformations. In this study, this challenge was evident in Task 1, where 6 students made visualization errors (VE1 and VE3), particularly in accurately positioning points and aligning domain and codomain elements. These errors were not isolated; they were linked to a lack of understanding of fundamental mathematical properties, such as functions, as demonstrated by students who struggled to articulate the definition of a function (CE4). This suggests that visualization errors in this study were not merely technical mistakes but were influenced by deeper conceptual misunderstandings about the nature of geometric transformations.

While previous studies have highlighted conceptual, procedural, and visualization difficulties in geometric transformations, this study extends their findings by explicitly analyzing the interconnections between these errors. Unlike prior research that examined these difficulties in isolation, our study demonstrates that conceptual misunderstandings are the root cause, influencing both procedural errors and visualization struggles. This represents a significant contribution, as it emphasizes the need to examine learning difficulties holistically rather than treating them as separate issues.

Exploring potential learning obstacles that participants may encounter when studying transformation proofs is essential. This approach mirrors the work of Jatisunda et al. (2025), who identified learning obstacles faced by participants when learning geometric transformations. The purpose of such research is to minimize these obstacles in the future through various strategies, such as refining teaching materials based on identified learning barriers. As Rosita et al. (2019) suggest, one solution to improve learning is to design instructional materials tailored to the specific obstacles students encounter. This underscores the importance of investigating learning obstacles in specific subjects to effectively address and mitigate them in future learning experiences.

The results of this study indicate that the most significant learning obstacle encountered by participants is epistemological. Epistemological obstacles arise when participants are unable to apply their existing knowledge to more complex situations, even though they have acquired the necessary concepts. This was clearly observed in this study, where participants struggled to apply the concepts of injectivity, surjectivity, and bijectivity when constructing proofs or visualizing geometric transformations. Unlike prior research that merely categorized participants' difficulties, this study systematically analyzes how these errors contribute to epistemological obstacles, which further hinder participants' ability to construct proofs effectively. Noto et al. (2019) examined learning obstacles in a general sense without linking them to Brousseau's framework, failing to categorize these obstacles specifically as ontogenic, didactical, or epistemological. Thus, this study contributes by establishing a connection between conceptual, procedural, and visualization errors and epistemological obstacles, offering a deeper understanding of the challenges pre-service mathematics teachers face in proving transformations.

Suryadi (2019) explains that epistemological obstacles arise when students are unable to transfer or apply their acquired knowledge effectively to new situations. In this study, while participants had learned the basic definitions of transformations and their properties, they struggled to apply these concepts when constructing proofs or creating accurate visual representations. For example, many participants correctly identified that transformations are functions but failed to use this knowledge to prove injectivity or surjectivity. Similarly, when visualizing transformations, participants faced difficulty in using their understanding of functions to accurately depict the transformation in diagrams. These findings support the argument that conceptual knowledge, when not fully internalized, leads to further challenges in both procedural and visual reasoning (Beeler et al., 2024; Braithwaite & Sprague, 2021).

The lack of alignment between participants' difficulties and didactical obstacles is evident from the data, as this study did not analyze instructional materials or teaching methods. Instead, the study relied on written tests and interviews to identify participants' difficulties. The primary challenge appeared to lie in participants' understanding and internalization of fundamental concepts, such as injectivity, surjectivity, and bijectivity, which were essential for constructing proofs and visualizing geometric transformations. While didactical obstacles can arise from teaching methods or material design, this study did not find evidence suggesting that these factors were significant contributors to students' struggles. Rather, the difficulties seemed to stem primarily from conceptual misunderstandings and the inability to transfer learned concepts to more complex tasks.

Similarly, ontogenic obstacles, related to participants' cognitive development, typically emerge when learning tasks exceed or fall below students' developmental capabilities. While it is possible that some participants may have faced cognitive limitations, the errors observed were more indicative of difficulties in applying abstract concepts rather than failure to engage with tasks at an appropriate cognitive level. For example, many students struggled with fundamental concepts such as injectivity, surjectivity, and bijectivity, which are essential for proof construction. This suggests that their cognitive development was not the primary obstacle, as these concepts are typically taught at this stage of their academic journey. The students' difficulties appear to be more rooted in their understanding and internalization of these concepts, pointing to epistemological obstacles rather than cognitive developmental barriers.

The evidence gathered from this study strongly supports the conclusion that epistemological obstacles were the most significant learning obstacle for participants. These obstacles hindered participants' ability to construct valid proofs and accurately visualize geometric transformations, as they struggled to apply their conceptual understanding in new contexts. While didactical and ontogenic obstacles may contribute to certain learning difficulties, the findings of this study suggest that the primary challenge for students lies in their epistemological difficulties, particularly in applying learned concepts to more complex tasks. This distinction is critical, as it underscores the importance of addressing students' conceptual understanding to overcome the barriers that impede their success in mastering geometric transformations.

4. CONCLUSION

The findings of this study revealed three distinct types of errors encountered by pre-service mathematics teachers when constructing proofs for geometric transformations: visualization errors, conceptual errors, and procedural errors. Further analysis demonstrated that conceptual errors served as the fundamental source of difficulty, as they directly influenced both procedural errors and visualization errors. This deeper investigation into the errors indicated that the primary learning obstacle faced by participants is epistemological obstacle, as they were unable to transfer their conceptual knowledge to new contexts, particularly in the domain of proof construction and geometric representation. However, this study is limited to identifying learning obstacles without providing instructional interventions to address them. Additionally, this research did not include a document analysis of the instructional materials used by the lecturers, which could offer valuable insights into didactical obstacles. Therefore, future research could further expand by exploring potential didactical obstacles in the teaching of geometric transformations, specifically by examining how teaching materials and strategies might contribute to students' difficulties in mastering the topic. Future research should build upon these findings by developing targeted teaching strategies that specifically address epistemological obstacles, enabling students to strengthen their conceptual understanding and apply it effectively in proof construction and visualization tasks.

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