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Does students' thought structure in object configuration patterns follow cognitive verbs in learning outcomes?

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Abstract

This study aims to comprehensively investigate students' learning approach in identifying, predicting, and generalizing object and number configuration. A qualitative method was applied with as many as 28 participants from eighth-grade students in Bandung, Indonesia. Students' ability to recognize, predict, and generalize the configuration of objects and numbers was assessed. At the same time, the approaches employed in the process were analyzed. The research showed that students used different approaches to figure out the pattern of objects and number configuration. These approaches were: descriptive, operational, visual-descriptive, visual-operational, and descriptive-visual-operational. Keywords were typically employed in the descriptive approach to predict unknown objects or numbers. However, the descriptive approach could not accurately predict specific patterns. Therefore, students employed other approaches, including operational, visual-descriptive, visual-operational, and descriptive-visual-operational approaches. Unfortunately, all approaches have proven inadequate for formulating generalizations (general rules) independently. These results showed that when teaching students how numbers and objects are configured, they should be encouraged to try different ways of seeing patterns, such as descriptive, visual, operational, or a combination of the three.

Keywords:
Generalizing, Object and numbers configuration, Patterns, Predicting, Recognizing

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1. INTRODUCTION

The 2020 curriculum, also known as the independent curriculum, places the learning outcomes of object and number configurations in the algebraic group. The 2013 curriculum, on the other hand, stated that students should be able to do basic math and understand how numbers work (decree of the head of the Education Standards, Curriculum, and Assessment Agency of the Ministry of Education, Culture, Research, and Technology — Mendikbudristek, 2022). This new composition supports the student's comprehension of symbols and patterns as an effective intermediary between numerical cognition and algebraic reasoning (Somasundram et al., 2019). In addition, many scholars consider that patterns constitute the foundation of mathematics (Liljedahl, 2004); the fundamentals of mathematics reside in the relations and transformations that generate patterns and generalizations (Warren, 2005). Furthermore, patterns serve as a mechanism to assist children in connecting various mathematical concepts and applications (Booker & Windsor, 2010; Reys et al., 1998). Consequently, the learning of object and number configuration needs to have a central role in the development of algebra reasoning, which is essential for learning advanced mathematics (Rittle-Johnson et al., 2017; Zippert et al., 2020).

The material in stage D about objects and number configurations is grouped under algebraic components in the 2020 curriculum. It is then followed by a study of algebraic forms and equations. The learning outcomes explicitly include three cognitive verbs: recognizing, predicting, and generalizing. Having the ability to identify the configuration pattern is the initial step for students in discovering intuitive strategies to identify regularity. Students have to be able to identify the regularity of a configuration pattern and whether the pattern configuration of the analyzed object is repeating or evolving. This process bridges the arithmetic and algebra (Barbosa & Vale, 2015), leading to the three phases that link the arithmetic and algebra, as noted by Kızıltoprak and Yavuzsoy Köse (2017): arithmetic, early algebra, and algebra.

The development of generalization in classroom mathematics encourages students to create algebraic reasoning. Instead of just memorizing strategies and procedures (Booker & Windsor, 2010), algebra reasoning is based on general reasoning, which is also known as imitative reasoning (Lithner, 2008). The process of developing general reasoning requires effort. Meanwhile, the ability to hypothesize or make predictions is challenging for students and difficult to improve (Lazonder et al., 2021). This prediction is related to abduction, which is the first prediction that comes up with a good idea and guides investigations by picking the best pattern or regularity from a group of options (Cramer-Petersen, 2019).

A study of junior high school students that looked at number patterns found that only 34.7% of them could come up with general rules for number patterns and only 29.31% could guess what an object configuration pattern would be (Subekti & Zuhrotunnisa, 2021). In general, students’ results in solving number pattern problems did not differ much across gender, with females making errors by 57.14% compared to 41.86% for males (Siagian et al., 2022). Consequently, students encounter didactic, epistemological, and ontogenic obstacles in the number sequence. The ontogenic obstacles are influenced by students’ inability to recognize the identity number sequence pattern (Lamaizi et al., 2024). Studies by Setiawan and Sa’dijah (2020), and Comendador and Ching (2024) back up these results. They show that students often generalize and use the wrong patterns. These patterns include confusing coincidental patterns with general rules, not giving reasons or evidence for conjectures, not giving enough evidence, not validating arguments well enough, and not giving enough justification and supporting evidence.

These investigations elaborate students’ overall competencies which also highlight the mistakes students make in the process of inductive reasoning, particularly in relation to patterns and generalizations. Understanding how students identify patterns in a configuration of objects and numbers, predict unknown objects and numbers, and develop generalizations is crucial. It is crucial for mathematical learning—specifically the material on object and number configuration—to organize activities that enhance students' abilities in recognizing, predicting, and formulating generalizations.

This article aims to examine students' abilities and diversity in informal reasoning (recognition), synthesizing conjectures, and generalizing object configuration patterns of an assignment that contains open possibilities for pattern recognition. This is supported by studies that argue that scientific reasoning abilities at a certain age are necessary to ascertain students' capabilities and limitations (Lazonder et al., 2021).

2. METHOD

The purpose of this study is to comprehensively examine the students' thinking process in recognizing, predicting, and generalizing the configuration of objects and patterns through a qualitative case study research method. A case study is employed to examine a specific problem, event, and phenomenon, enabling an in-depth exploration of the subject matter (Crowe et al., 2011). In this study, the phenomenon examined was students' thinking process in recognizing, predicting, and generalizing the configuration of objects and numbers.

Participants in this study include 28 students who were purposefully selected based on the results of interviews with their mathematics teachers at the school where this study was conducted. Each student was assigned to solve a series of object and number configuration problems that were designed based on the cognitive work of the learning outcomes. The results of all students' works were analyzed in depth through three steps from Miles and Huberman (1994), namely data reduction, display, and conclusion (generalization): visual and verification.

The researchers collected primary data directly from the students using a tool that was made based on their cognitive level and the learning outcomes written in the math curriculum phase D for algebraic elements related to object and number configuration. The instrument for measuring students' ability in the material of object and number configuration encompasses a sequence of repetition and growth that is assessed at the cognitive level of Anderson and Krathwohl (2001) taxonomy: understanding (C2), applying (C3), analyzing (C4), and creating (C6). The researchers designed the following instrument to gain an overview of students' thinking processes (see Table 1).

Table 1. Instrument design to measure the object and number configuration

No	Content	Cognitive Levels	Indicator	Weight	Item Number
1	Repeating Object Configuration Patterns	C2	Given the first three patterns of repeating object configurations are presented, students can identify rules/patterns from the configuration of objects	1	1. a
		C2	Given the first three patterns of repeating object configurations are presented, students can describe the formation of a repeating pattern	1	1. b

No	Content	Cognitive Levels	Indicator	Weight	Item Number
2	Growing Object Configuration Patterns	C2	Given the first 3 arrangements of numbers with a growing pattern, students can identify configuration patterns	1	2. a
		C2	Given the first 3 arrangements of numbers with a growing pattern, students can describe the formation of a growing configuration pattern	1	2. b
3	Object Configuration Patterns	C2	Given a configuration of objects with a general linear pattern, students can identify the number of objects in their configuration arrangement	1	3. a
		C4	Given a configuration of objects with a general linear pattern, students can predict the number of objects in their configuration arrangement	1	3. b
		C6	Given a configuration of objects with a general linear pattern, students can determine general rules for the pattern of the number of objects in their configuration arrangement	3	3. c
4	Number Patterns	C4	Given a number of configurations that contain a combination of growing and shirking patterns, students analyse sequence patterns to determine unknown sequence terms	4	4
5	Application of Object and Number Configurations	C3	Given contextual problems related to object patterns, students can use knowledge in object and number configuration patterns to solve it	4	5

To determine the validity and reliability of the instrument, it was tested on grade IX students. The analysis of the results showed that the lowest item validity was 0.451 (sig. = 0.016), and the highest was 0.875 (sig. = 0.000) for the numerical question. While the Cronbach Alpha reliability score was 0.740, indicating a high level of reliability. Based on these results, the instrument is considered feasible for this research purpose.

In the phase of research data analysis, a systematic approach was undertaken to ensure reliable findings. Initially, the researchers identify different strategies used by students in solving problems about object and number configuration. To obtain reliable data, an independent review was conducted by other researchers, as well as categorizing answer patterns and documenting specific strategies employed by each student. For a thorough finding presentation, the data was summarized descriptively. According to the final step, conclusions were drawn from the data analysis to give a full picture of how the students thought about finding, guessing, and generalizing object and number arrangements. Figure 1 illustrates the data analysis process.

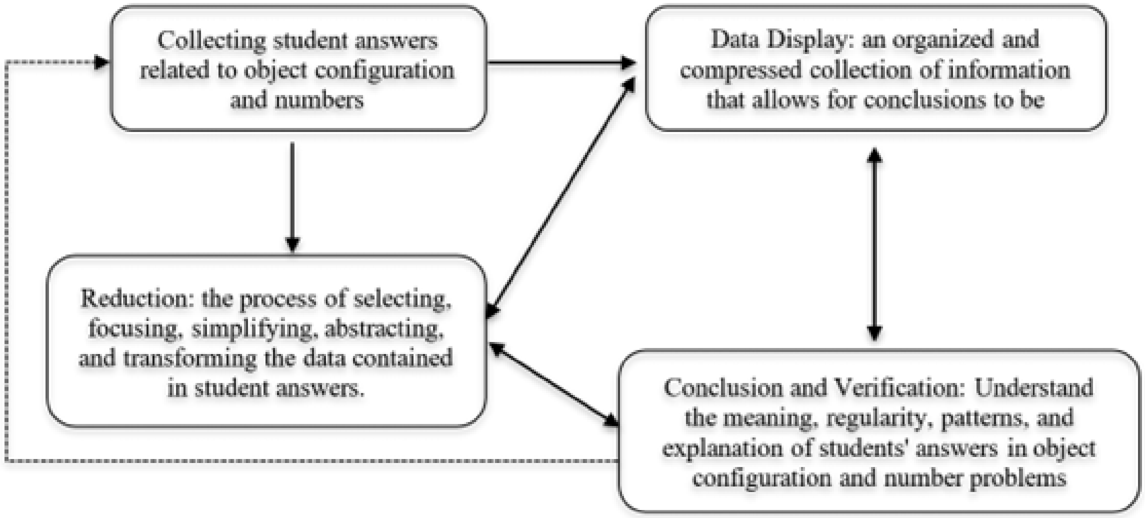


Figure 1. Research data analysis (adapted from Miles & Huberman, 1994)

3. RESULTS AND DISCUSSION

3.1. Results

Descriptively, the results of the analysis of the students' achievements indicated a positive trend. The data is predominantly concentrated at lower scores. Moreover, the distribution exhibits a leptokurtic shape, with a kurtosis of 1.66 exceeding 0.263. The relative frequency achievement (%) for each item is presented in Table 2.

Table 2. Percentage of student achievement scores on each test item

Item	Content	Percent Score
1	Repeating Object Configuration Patterns	82%
2	Growing Object Configuration Patterns	32%
3	Object Configuration Patterns	36%
4	Number Patterns	14%
5	Application of Object and Number Configurations	18%
	Configurations Object and Numbers (Total)	31%

Notes: Score percentage is calculated using the formula: $\frac{(\text{Sum of Score})}{(\text{Weight} \times \text{n})} x100\%$

The initial question is about recognizing the repeating object configuration. The statements given in the question are as follows (see Figure 2).

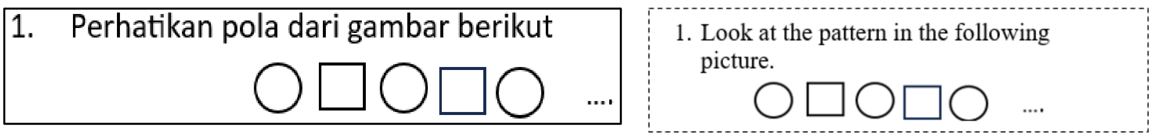
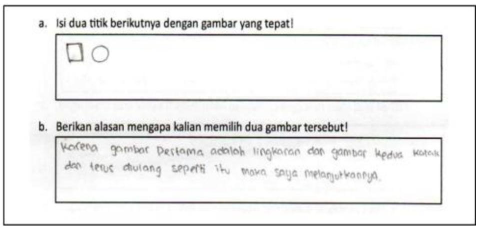


Figure 2. Questions of object configuration with repeating patterns

The presented questions asked students to illustrate the next object with the reasons. Generally, as described in Table 2, students could recognize the patterns of object repetition and predicted them by drawing the next object. It can be inferred that students could effectively identify and predict simple patterns of object repetition. The predictions of object configurations and students' recognition can be seen in the following example of students' responses in Figure 3.



Since the first image is a circle, the second is a square and this pattern repeats, I continued it accordingly.

Figure 3. Thinking patterns in repetitive configuration patterns

The second question tasked students with recognizing and predicting the growing pattern in object configurations. The second question is as follows (see Figure 4).

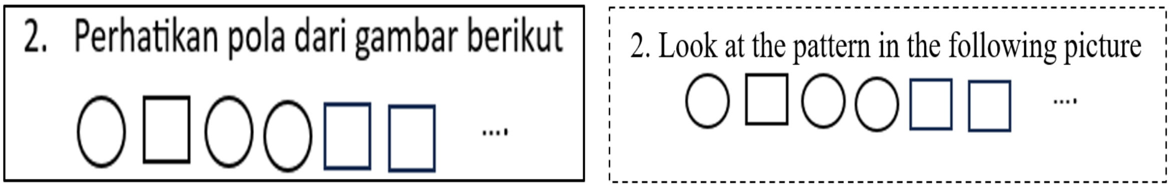
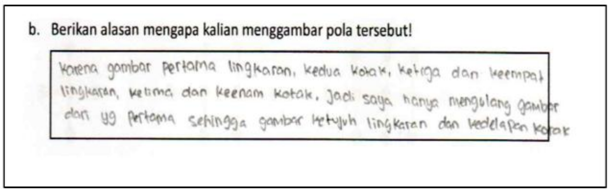


Figure 4. Questions of object configuration with growing patterns

In answering the questions of recurring object configurations, there were only 32% of students could draw the next configuration. Meanwhile, there were only 30% of students that could explain the pattern accurately. This means that 2% of students could draw the unknown object pattern but could not explain the configuration pattern.

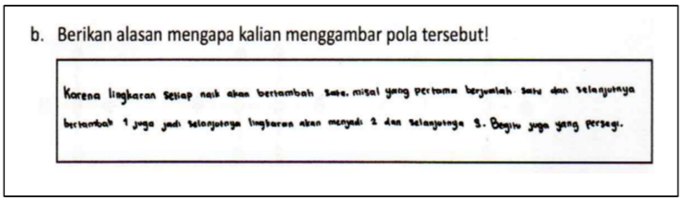
The analysis of students' explanations illustrated that the approaches used by students were descriptive, visual, and descriptive-visual. Figure 5 illustrates the students' in recognizing object configuration patterns.



Because the first image is a circle, the second is a square, the third and fourth are circles, and the fifth is a square. I repeated the sequence. So, the seventh and eighth images were squares.

Figure 5. Descriptive Approach in Recognizing Patterns with Error Prediction

The Figure 5 illustrates that students were able to recognize the pattern descriptively. However, they were not predicting accurately the next unknown object. The following Figure 6 is an example of a descriptive pattern recognition approach with accurate predictions.

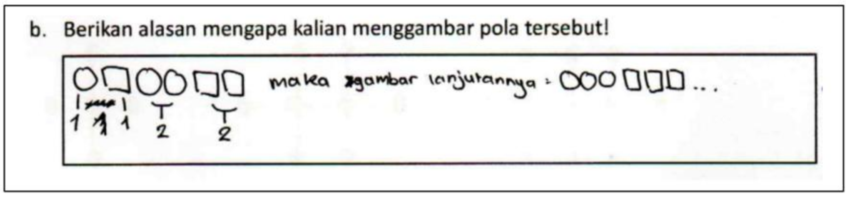


Each circle increases by one. For example, the number of circles starts at one, then it increases by two and then 3. The same pattern applies to the square.

Figure 6. Descriptive recognition approach with correct predictions

From the Figure 6, it can be illustrated that students could recognize the patterns properly, by using the keyword "increase by 1 too", and it was followed by their decision-making skills as the way to predict the objects in the given configuration. It can be inferred that students were able to recognize the object configuration patterns properly and employ an inductive reasoning process in their predictions.

The third approach to recognizing the object configuration is visual-operational. Students employed diagrams to identify the object configuration pattern. The arrangements of the diagram were linked to the number patterns. With this pattern recognition, it could be predicted the unknown objects from their configurations accurately. Examples of student answers can be seen in Figure 7 as follows.



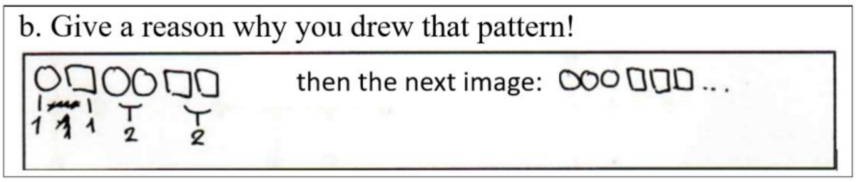
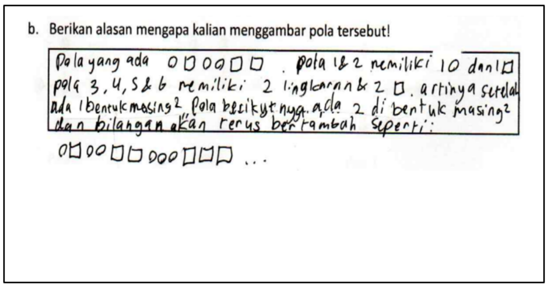


Figure 7. Visual-operational recognition approach

The third approach to students' thinking process in answering the repeated configuration pattern is the combination of descriptive-visual-operational. Students explained the configuration pattern in detail as in the following students' responses in Figure 8 as follows.



Existing patterns ○□○○□□
Patterns 1 and 2 consist of one circle and one square, while patterns 3, 4, 5, and 6 consist of two circles and two squares. It means, after that, the number of each shape alternates between increasing by one and two in subsequent patterns, and the total number of shapes will continue to grow accordingly
○□○○□□○○○□□□

Figure 8. Visual-descriptive recognition approach

Students combined descriptive and visual explanations of the configuration pattern. Then, by using the number patterns associated with visual objects, it obtained the keyword “there are two formed each, and the number will continue to increase”, to predict the unknown object in the given configuration accurately.

From the findings above, it can be inferred that some approaches used by students in recognizing the growth objects configuration are descriptive, visual-operational, and descriptive-visual-operational. By using the descriptive approach, could be accurate and inaccurate predictions, whereas the visual-operational and descriptive-visual-operational approaches consistently produced accurate predictions.

Question number three relates to the configuration pattern of circular objects with a linearly growing base pattern. This object configuration is also associated with the number patterns. Students are required to find the general rule for some circles in the object configuration. The question is shown in Figure 9.

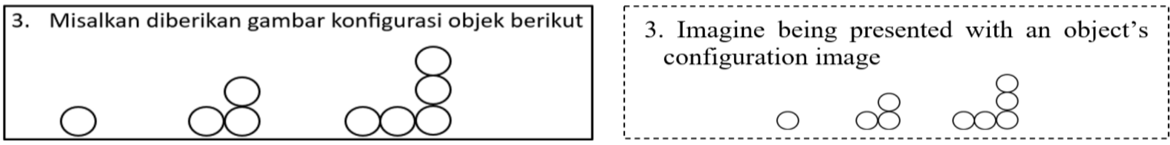


Figure 9. Questions for configuration patterns related to number sequences

The question in item number 3 involved determining the number of circles in the 4th arrangement, establishing regularity, predicting, and formulating the general pattern of some circles. The analysis of students’ responses identified four approaches; recognizing the pattern with its explanation, operational-descriptive combination, and visual-descriptive, operational (general rules). The first approach students employed in recognizing the configuration pattern is to provide a descriptive explanation of the increasing pattern of objects (circle) as follows (see Figure 10).

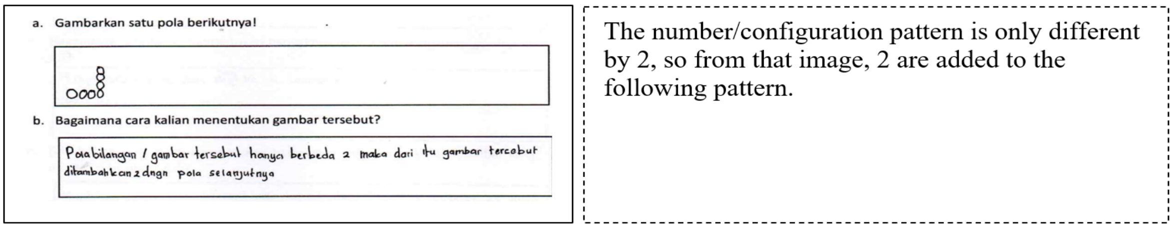
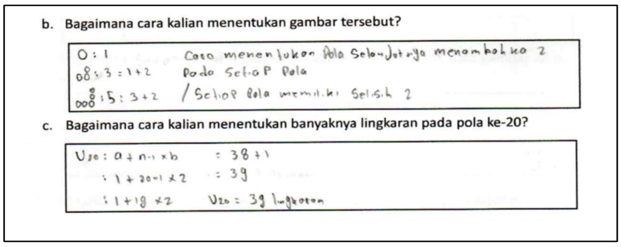


Figure 10. Descriptive recognizes approach in growing pattern

The Figure 10 illustrates students’ explanation of the additional objects in each configuration. Their descriptive recognition was correct. They could draw the next object but they could not predict as well as generalize. This failure was due to a lack of the synthesis process of students’ thinking skills. Therefore, for more complex configurations, the descriptive recognition failed in predicting and generalizing the object configuration patterns.

The second approach was the use of operational methods combined with the description, as seen in the following Figure 11.

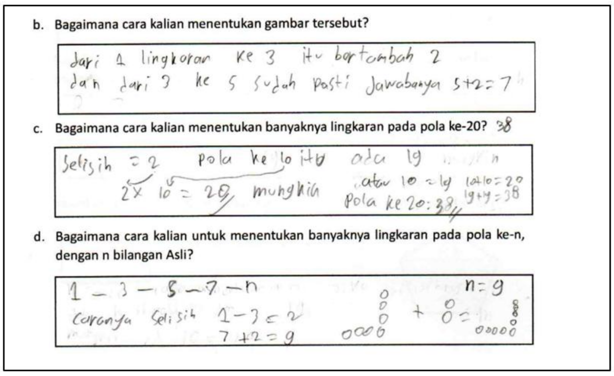


The next way to determine the pattern is by adding 2 to each pattern/each pattern has a difference of 2

Figure 11. Descriptive-operational prediction approach

Students began by recognizing patterns by counting all the circles in each configuration. Then, they arranged the regularity operationally as follows; 1, 1+2, 3+2, and so on. Next, they employed the keyword “each pattern has a difference of 2”. However, the pattern recognition constructed by students failed to be used in predicting the 20th term of the object configuration. Students relied on the general rules of an arithmetic sequence, such as U_20=1+ (20-1)2=39. Thus, the operational approach used by students could not build their thinking skills to predict, as well as arrange the general object configuration pattern.

The third approach identified from the data analysis was a combination of descriptive-operational methods. The initial pattern configuration of this approach was similar to the first approach, where students described the pattern from the object configuration. Each object pattern increased by two, allowing students to determine the number of objects in the next arrangement such as 5+2= 7 (see Figure 12).

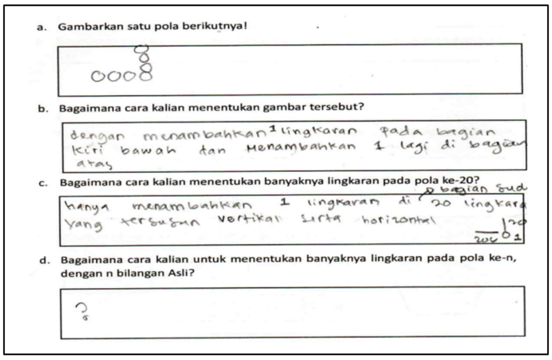


From 1 circle (figure 1) to 3 circles (figure 2), the increase is 2; from 3 circles to 5 circles, the answer is 5 + 2 = 7.

Figure 12. Visual-operational prediction approach

In the next step, students employed an operational approach to predict the 20th pattern. It was started by calculating the difference between patterns, which was 2. Then, they attempted to use the keyword “maybe” to determine the number of objects in the 20th term by using some alternative, such as "or 19", "20", "or 38". Finally, it was determined that the pattern of object number in the 20th was 38.

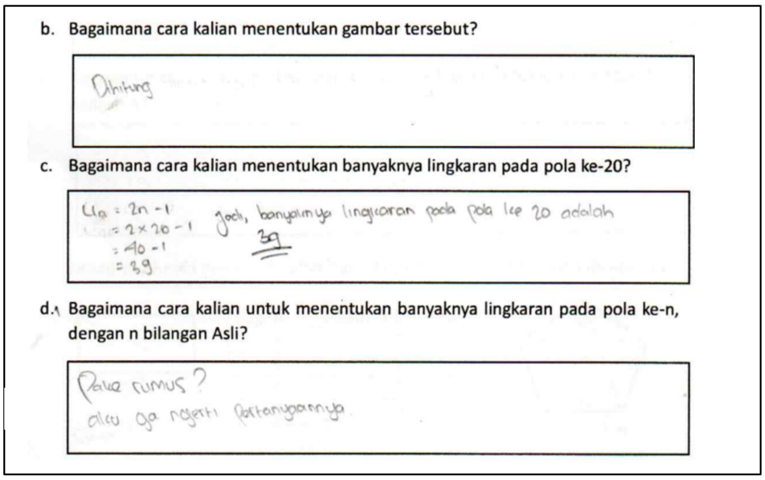
The next approach developed by students was descriptive-visual. Students recognized the pattern by explaining the growth pattern from object configuration but it was different with the first descriptive, as each object in configuration increased by one in the horizontal arrangement and one in the vertical arrangement. The descriptive recognition which was developed by students effectively utilized visual representations, enabling them to predict using a visual (sketch) approach. Unfortunately, students failed to formulate the general pattern from the object configuration that was provided. The following Figure 13 illustrates the fourth approach to students' thinking process.



- By adding 1 circle at the bottom left and adding 1 more at the top.
- Only add 1 circle at the corners of the object (circle) that is arranged vertically and horizontally (included diagram).

Figure 13. Visual-descriptive prediction approach

The fifth approach identified from the result of data reduction was the use of general rules. This pattern was similar to the second pattern. However, in this approach, students directly applied the general formula for arithmetic sequence derived from the series circle counts by calculating them. In making predictions, the students solved the problem in reverse-shifting from inductive to deductive reasoning by using general rules for arithmetic sequences. The approach taken by these students can be seen in Figure 14.



$$U_n = 2n - 1$$
$$= 2 \times 20 - 1$$
$$= 40 - 1$$
$$= 39$$

So, the number of circles in the 20th pattern is 39

Figure 14. Operational approach for prediction

The correct answer, which was answered by students in determining the 20th term, was considered not the prediction process, but rather the application of the existing rules (deduction). Consequently, students struggled to answer subsequent questions related to generalizing the provided object configuration pattern.

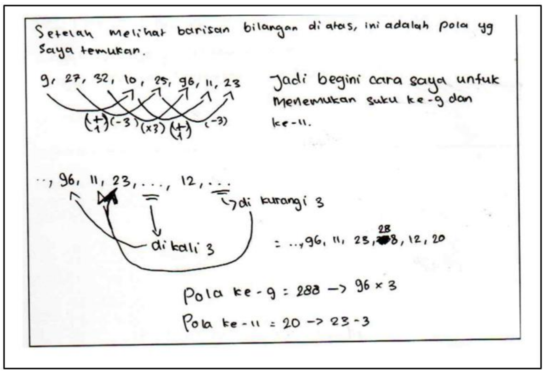
Question number four measured students' ability to find some unknown terms in a number sequence. Students needed to analyze three number sequences and synthesize them to derive the rules for each number sequence. This question adopted the reasoning model of state university entrance exams. This means that this question is included in the high-level measurement category. The problem is as follows (see Figure 15).

4. Perhatikan barisan bilangan berikut 9, 27, 32, 10, 25, 96, 11, 23, ..., 12, ...
Jelaskan strategi kalian menemukan suku yang masih kosong (suku ke-9 dan ke-11)

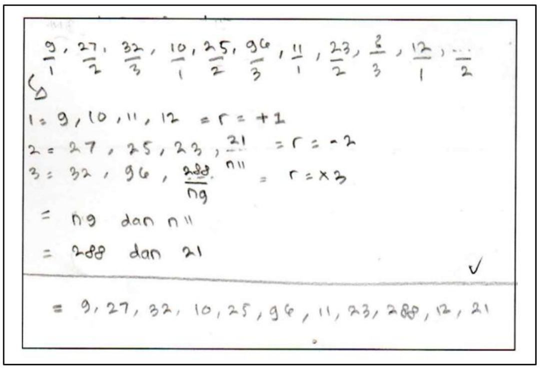
4. Look at the following number sequence:
9, 27, 32, 10, 25, 96, 11, 23, ..., 12,...
Explain your strategy for finding empty terms (9th and 11th terms)

Figure 15. Questions for number sequence

In general, students failed to solve the given questions, as reflected by the percentage of students' achievement scores which was 14%. Many students did not answer this question. Then, among those who did, there were two approaches employed by students such as visual and operational. Figure 16 shows the two approaches used by students.



(a)



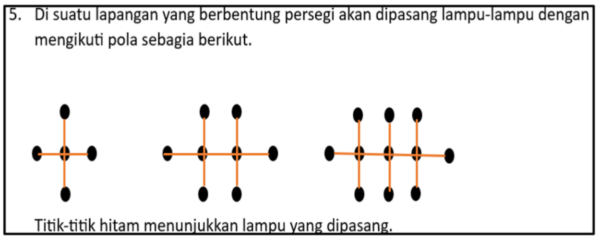
(b)

Figure 16. Solution approach of numbers configuration: (a) Visual approach; (b) Operational approach

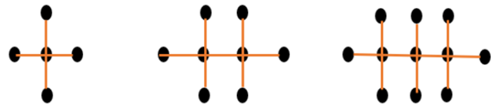
The first pattern recognition approach employed by students was creating a visual diagram with arrows connecting to the sequential relationships between the numbers in the series (see Figure 16a). This visual approach was followed by an operational approach, namely identifying the regularities: the first number sequence was "plus 1", the second number sequence was "minus 3", and the third sequence was "multiplied by 3". These patterns were then used to predict unknown numbers in the provided sequence such as the 9th term, which equals 288 = (96 x 3), and the 11th term, which equals 20 = (23-3). The combination of visual and operational approaches encouraged students to recognize and predict unknown numbers in the provided number series.

The second pattern recognition approach (see Figure 16b) was investigated by students using operational technique. The students were assigned sequence numbers as 1,2,3,1,2,3, and so on. The results were divided into three groups of sequences which were accompanied by their symbolic sequence rule; the rules in the first sequence "r= +1", the second sequence followed the rules "r= -2", and the third sequence followed the rule "r= x 3". Although the symbolic rules written by students were not entirely accurate, they were still able to use them correctly to predict unknown numbers in the provided sequences.

Question number five involved the implementation of object configuration. This question is related to installing lights in a certain pattern. The problem scenario is as Figure 17.



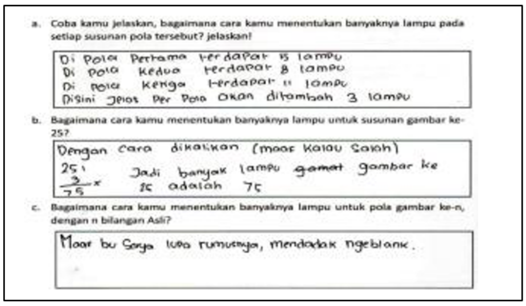
5. In a square-shaped field, lights will be installed in the manner shown below.



The black dots indicate the installed lights.

Figure 17. Question for applying object configuration

The provided problems involved recognizing, predicting, and generalizing patterns of object and number configurations from contextual problems. The analysis of students' responses revealed that the average score was 0.28 (28 %) out of 4. Further analysis identified two groups of approaches such as descriptive and operational (formal). The approaches employed by the students are shown in the following Figure 18.



a. Coba kamu jelaskan, bagaimana cara kamu menentukan banyaknya lampu pada setiap susunan pola tersebut? jelaskan!

Di Pola Pertama terdapat 5 lampu
Di Pola Kedua terdapat 8 lampu
Di Pola Ketiga terdapat 11 lampu
Disini Jelas per Pola akan ditambah 3 lampu

b. Bagaimana cara kamu menentukan banyaknya lampu untuk susunan gambar ke-25?

Dengan cara dikalikan (maaf kalau salah) 25 x 3 = 75 Jadi banyak lampu gambar ke 25 adalah 75

c. Bagaimana cara kamu menentukan banyaknya lampu untuk pola gambar ke-n, dengan n bilangan Asli?

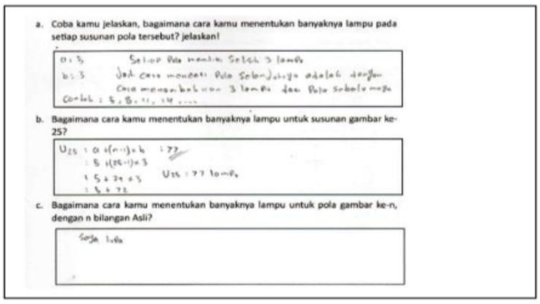
Maaf bu Saya lupa rumusnya, mendadak ngeblank.

a. In the first pattern, there are five lights.
In the second pattern, there are eight lights.
In the third pattern, there are eleven lights
Here, it is clear that each pattern, there will be an increase of 3 lights
b. By multiplying (sorry if it's wrong) 25 x 3 = 75.
So there are 75 lights.
c. Sorry, ma'am, I forgot the formula; it suddenly went blank.

Figure 18. Descriptive approach for solution of applying object configuration

The students described the number of lights in each configuration arrangement to identify the pattern, concluding that "it is clear that per pattern, there will be an increase of 3 lights." Unfortunately, the student altered the identified pattern when making predictions, applying the rule "multiplying by 3," which led to an incorrect total of 75 lights. Because the pattern recognition was not based on the configuration of the objects, the students were unable to accurately comprehend the configuration pattern of the number of lights. As a result, they failed to make accurate predictions and generalizations.

The second approach identified is a combination of descriptive and operational approaches, as illustrated in the following Figure 19.



a. Coba kamu jelaskan, bagaimana cara kamu menentukan banyaknya lampu pada setiap susunan pola tersebut? jelaskan!

b. Bagaimana cara kamu menentukan banyaknya lampu untuk susunan gambar ke-25?

c. Bagaimana cara kamu menentukan banyaknya lampu untuk pola gambar ke-n, dengan n bilangan Asli?

a. Each pattern has a difference of 3 lights. So, the way to look for the next pattern is to add three lights from the previous pattern, for example, 5, 8, 11, 14, etc.
b. By using the general formula for arithmetic sequences, we obtain

$$U_n = a + (n-1)b$$
$$= 5 + (25-1)3$$
$$= 5 + 24 \times 3$$
$$= 5 + 72$$
$$= 77$$
$$U_{25} = 77 lights$$

Figure 19. Operational approach for solution of applying object configuration

The students recognized the pattern in the number of lights in each configuration, leading to the number sequence of lights: 5, 8, 11, 14, ... When asked to predict the number of lights in the 25th configuration, they used the general formula for an arithmetic sequence, $U_n=a+(n-1)b$. This means the students applied deductive reasoning (formal operational) to predict the value of the sequence that had not yet been determined. However, the students were unable to form the general expression for the number sequence that represented the configuration of the objects.

From the findings above, it can be concluded that in solving contextual problems involving object configurations, the students attempted to recognize the configuration pattern both descriptively and operationally by identifying the number sequence formed by the objects in each configuration. The next step involved using a formal method, namely applying the general rule for number sequences to predict the unknown numbers. As a result of using this formal approach, the students were unable to make generalizations from the contextual problem related to the object configuration.

3.2. Discussion

The repeating pattern of object configurations can be appropriately recognized by students in a descriptive form. With this approach, students are able to make predictions about unknown object configurations. The recognition of regularity in a descriptive manner, commonly used in object configurations, may be due to the limited opportunities students have to recognize patterns from different perspectives. As stated by Reys et al. (1998), teachers need to understand that students may think about a pattern in different ways, and frequently, there is more than one correct way to extend that pattern. Therefore, even though the pattern of object or number configurations may be simple or complex (Reys et al., 1998; Rittle-Johnson et al., 2017), teachers need to provide opportunities for students to recognize the repeating patterns of object configurations in various ways.

In the case of object configurations with a growth pattern, it was found that there were some ways in which students recognize the pattern of object configurations, including descriptive, visual-operational, a combination of description and visual, and operational approaches. The emergence of these varied approaches is very beneficial for students in aiding the inductive reasoning process. The development of visual approach allows students to see information more clearly (Geçici & Türnüklü, 2021), and thus, recognition using visual approach combined with operational or descriptive approaches enables students to predict the pattern of object configurations. Additionally, the process of recognizing patterns through visual approach is closely linked to mathematical thinking (Geçici & Türnüklü, 2021), with a connection between spatial visualization, patterns, and mathematics (Rittle-Johnson et al., 2018). Reasoning through visuals will assist students in the discovery process, enhancing comprehension of mathematical concepts (Geçici & Türnüklü, 2021). Therefore, recognizing object configuration patterns through visual-operational, or even audiovisual approach should be considered by teachers in the context of literacy skills (Kandir et al., 2018), as this recognition will encourage students to abstract mathematical principles (Pasnak, 2017).

The use of various approaches by students in recognizing object and number configuration patterns is highly effective in facilitating mathematics learning. As stated by Mielicki et al. (2021), the development of pattern-based mathematics learning greatly contributes to further mathematical learning, thereby improving students' ability to develop patterns. Hunter and Miller (2018) recommend using real-life tasks, such as Pacific patterns, which refer to object configurations of plants and flowers given as gifts for special occasions like weddings or birthdays. Suherman and Vidákovich (2022) used object configurations in the Lampung Tapis. Essentially, through activities involving the recognition and prediction of object configuration patterns, students can engage in mathematical thinking processes. Mathematical thinking is crucial in mathematics education, as it equips individuals to adapt in real life (Geçici & Türnüklü, 2021).

The recognition of object configuration patterns can determine the success of students in making predictions and generalizations. The research findings revealed that the students' approach of recognizing descriptive patterns in growing object and number configurations can contribute to make predictions. Students use key phrases, such as "increases by 1 too" or "2 are added to the following pattern". With these key phrases, students can develop predictions for the next configuration, but unfortunately, some fail to predict accurately from the given object configuration. As a result, students with a descriptive recognition approach fail to formulate generalizations. In the case of recognizing visual patterns or combining visuals with descriptions, as well as visual-operational approaches, students recognize the pattern using diagrams or relying on the diagrams. Initially, the recognition was examined descriptively by utilizing the provided object configuration images, then students combined it with a visual approach (sketches) or operational approach (numbers). As a result, students could predict well both objects that were "close" and those that were "far" from the given object configuration. However, this thinking process has not been able to develop a general pattern for the evolving object configurations. Although they failed to develop generalizations, the visual recognition process in object configurations was slightly more advanced than the descriptive approach. These findings confirm the research by Warren and Cooper (2007), and Angraini et al. (2023), that visual representation is easier to transform into numerical representation by students, helping them identify object and number configuration patterns. Additionally, Arcavi's (2003) finding that visualization encompasses capability, process, and product of creating, interpreting, using, and reflecting of images or diagrams in a person's mind. It aimed to depict and communicate information, to think and develop previous unknown ideas. Thus, this process can encourage and support someone to develop the understanding.

The research findings showed that, in general, students have not been able to formulate a general pattern or generalization from the given object and number configurations. According to Ramdhani (2018), the low ability to generalize is caused by several factors, such as difficulty in recognizing patterns, expressing patterns or rules in verbal language, and formulating patterns or rules in symbolic language. Additionally, there is variation in pattern recognition and a lack of attention to their relationships. Students tend to recognize patterns from only one perspective. In line with the findings of Hunter and Miller (2018), students generally learn or recognize patterns using only one perspective (variable) of object or number configurations and fail to consider the covariation relationship between two patterns. In fact, the ability to link two quantities in object and number configurations is a factor that influences the formulation of generalizations (Syawahid et al., 2024). Therefore, in mathematics learning practice, especially in teaching object and number configurations, there needs to be reinforcement and encouragement from the teacher so that students can recognize object and number configuration patterns from various perspectives, identify relationships, and eventually make generalizations. General patterns or generalizations will guide students to the basic of mathematics, which lies in the relationships and transformations that lead to patterns and generalizations of objects (Carpenter et al., 2003).

The results of this study also found that recognizing and predicting number sequences containing more than two patterns of number sequences are difficult for students. Only 2 out of 28 students (7.1%) were able to identify, find, and predict the values of the unknown sequences. As indicated by the previous research, the pattern recognition performed by the students used different approaches. The first approach used a visualization technique with arrow diagrams linking the regularities between the numbers in the series sequentially. The second approach used an operational technique. Students assigned sequence numbers with 1, 2, 3, 1, 2, 3, and so on. As a result, it was divided into three groups of number sequences, each accompanied by its respective symbolic sequence pattern: the first sequence had the rule "r = +1", the second sequence had the rule "r = -2," and the third sequence had the rule "r = x 3". According to Warren and Cooper (2007), the students' difficulty in recognizing sequences that contain sub-sequences is the identification of cross-rules between sub-sequences. Students tend to focus on one sequence rule and pay less attention to potential patterns within the number sequence. The two answers found by students in this study could serve as inspiration for teachers when teaching how to recognize number sequences containing sub-sequences.

Solving contextual problems related to object configurations and number sequences mostly used a descriptive approach with key phrases such as "it is clear that per pattern, there will be an increase of 3 lights". Unfortunately, this approach could not be used by students to predict and formulate generalizations. Another approach used by students was the application of the general rules of number sequences that they had learned, specifically the formal deductive approach. The formal deductive approach could be applied by students to determine the values of number sequences from the object configurations provided, by applying the general rules, rather than as a prediction. Another consequence of using this formal deductive approach was that students became confused when trying to formulate a generalization of the patterns from the presented object and number configurations. The difficulty in solving contextual problems related to number patterns, according to Subekti and Zuhrotunnisa (2021), is caused by students' lack of precision, such as incorrectly applying principle and procedural knowledge.

From the findings, students' ability to recognize, predict and generalize the materials of object configuration is needed in learning practices. It can make students easier to find regularities through various approaches, such as visual, tabular, and operational approaches. Pattern examination is potential to facilitate the detection of new mathematical principles for students (Naccache & Yifrach-Stav, 2022). Although the process of formulating generalizations from a pattern is not easy (Permatasari et al., 2021), teachers can assist students by asking questions that provide references for students to form generalizations (Miliyawati, 2014).

Finally, the learning pattern should emphasize on a series of introductions, predictions, and generalizations according to the learning outcomes in the national curriculum and highlight the role of analogy, learning strategies, and constructivist approaches (Angraini et al., 2023) because learning about patterns can enhance student's cognitive development and academic performance (Pasnak, 2017). In practice, teachers need to use object configurations related to context such as batik patterns that contain repeating and developing patterns. In Truntun batik pattern (Nurcahyo et al., 2024), the algebraic thinking, particularly the generalization pattern, can be facilitated (Andini & Suryadi, 2017). In fact, research by Kandir et al. (2018) recommends that pattern-based mathematics learning programs should be integrated into the school mathematics curriculum.

4. CONCLUSION

Based on the research finding, it can be concluded that several approaches were used by students in recognizing the pattern of object and number configurations, including descriptive, operational, visual-descriptive, visual-operational, and descriptive-visual-operational approaches. The descriptive approach is employed in recognizing patterns of repeating and growing object configurations. In recognizing the pattern of object and numbers configurations, the descriptive approach utilizes key phrases, enabling students to predict unknown objects. Approaches involving visuals (visual-descriptive, visual-operational, and descriptive-visual-operational) are used by students in recognizing object configurations with growing patterns, as well as number sequences containing sub-sequences. The approaches involving visuals are employed by students to predict unknown objects or numbers from the given configuration. However, unfortunately, all the approaches developed by students in this study have not been effective in helping students formulate generalizations of the general patterns from the object and number configurations.

Based on the conclusion of this study, it is recommended that learning about object and number configurations should encourage students to explore various approaches to recognize patterns, such as descriptive, visual, operational, and combinations of these approaches. The patterns discovered by students should be employed to help them predict and formulate general patterns from the configurations. In addition, teacher should consider to include the object and number configurations that related to real-life contexts in the learning material.

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