Infinity Journal of Mathematics Education
Volume 14, No. 3, 2025
p-ISSN 2089-6867
e–ISSN 2460-9285
https://doi.org/10.22460/infinity.v14i3.p781-796

Alternative conceptions about proportional reasoning in high school students

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Received: Mar 18, 2025 | **Revised**: Jun 7, 2025 | **Accepted**: Jun 16, 2025 | **Published Online**: Jul 29, 2025

Abstract

This study aimed to identify alternative conceptions about proportional reasoning among High School Students at a public school located in the state of Guerrero, Mexico. Using a qualitative approach, data were collected through task-based interviews with fifteen students 12th-grade students. Data were analyzed using the thematic analysis method. The findings allowed to identify five alternative conceptions: (1) the variational behavior of a line graph indicates the type of proportional variation; (2) the constant function algebraically represents a proportional variation; (3) a negative slope in the equation of a line indicates an inverse proportional variation; (4) direct proportional variation is conceived as an object; and (5) the constant of proportionality in the graph of a direct proportional variation is interpreted as the length of the line. While this study does not incorporate data from teachers, the findings indicate that instructional strategies prioritizing procedural techniques rather than conceptual understanding. Additional research is required to investigate how teachers' knowledge and instructional methods influence students' development of proportional reasoning. In the same line, the results highlight the need to design instructional strategies that promote the development of more robust proportional reasoning in the High School level.

Keywords:
Alternative conceptions, Proportional reasoning, Qualitative research, Task-based interviews

How to Cite:
Santana-Ramírez, H. F., Salgado-Beltrán, G., García-García, J., & López-González, A. (2025). Alternative conceptions about proportional reasoning in high school students. *Infinity Journal*, *14*(3), 781-796. https://doi.org/10.22460/infinity.v14i3.p781-796

1. INTRODUCTION

Proportional variation (PV) is a fundamental concept in our daily lives, and its importance extends throughout the educational system in Mexico and other countries, from basic level to higher level (Dávila-Araiza & Herrera Garcia, 2021; Ramírez & Hernández, 2017; Reyes-Gasperini, 2016). As students deepen their understanding of this concept, it becomes increasingly complex due to its connections with various disciplines such as biology, chemistry, physics, and others (Lamon, 2007; Ramírez & Block Sevilla, 2009). In these fields, it is common to encounter problems involving PV, which are addressed in the school context to illustrate the diverse applications of this concept (Secretaría de Educación Pública, 2023). Therefore, understanding PV is not an easy task, but it is essential for the development of advanced mathematical thinking (Mochón-Cohen, 2012; Weiland et al., 2021).

From another perspective, those of us dedicated to the teaching and learning of mathematics agree that the concept of PV is often misunderstood, primarily because it is typically taught in a mechanical way in the classroom through the rule of three (Luque-Álvarez & Ibarra-Olmos, 2021a; Ramírez & Block Sevilla, 2009), often referred to as the direct proportionality rule. This rule helps a stutend to solve for an unknown quantity when three other quantities are known and are in direct proportion. However, this approach is insufficient for building a comprehensive understanding of the concept, as it goes beyond the mere application of this algorithm (Ball et al., 2008; Karplus et al., 1983).

On the other hand, PV has been a central topic in mathematics education research for over four decades (Fernández-Verdú & Llinares-Ciscar, 2012). Early studies considered proportional reasoning as a fundamental construct in the consolidation of arithmetic knowledge in primary education, encompassing other concepts such as fractions, ratios, and proportions (Weiland et al., 2015). This reasoning continues to develop in secondary education and fosters logical, heuristic, and creative thinking in students when solving problems. Additionally, proportional reasoning involves understanding the multiplicative relationship between quantities in proportional situations and developing the ability to distinguish between proportional and non-proportional situations (Fernández-Verdú & Llinares-Ciscar, 2012; Lamon, 2012; Moreno-León, 2023).

Different studies have pointed out that difficulties with proportional reasoning are not exclusive to students but also affect primary and secondary school teachers, negatively impacting their teaching practices (e.g., Hill, 2007; Hill et al., 2008; Soto-Quiñones et al., 2024). In particular, teachers tend to focus on developing routine procedures, such as the rule of three, when teaching this concept, reflecting limited proportional reasoning similar to that observed in students, as most adults do not possess fully developed proportional reasoning (Soto-Quiñones et al., 2024). This approach is carried out automatically, without adequate reflection on the covariation processes between the variables involved, as noted by Lobato et al. (2011), which could partly explain the low performance of students in the PISA test, especially in problems requiring proportional reasoning (Rivas et al., 2012). In Mexico, studies such as that of Balderas-Robledo et al. (2014) have also highlighted this tendency to rely on the rule of three algorithm, both among teachers and students.

Chhabra and Baveja (2012) emphasize the importance of understanding students' thinking, as their conceptions directly influence their learning. Consequently, these conceptions have emerged as a crucial area of research in mathematics education (Duit & Treagust, 2003). There are studies that focus on Higher Education levels and on topics such as functions and variational thinking (e.g., Aragón Ruiz, 2023; Kaplan et al., 2015; Serhan, 2015). While this literature provides valuable insights into alternative conceptions, these studies do not specifically address proportional reasoning in High School students.

Balderas-Robledo et al. (2014) and Castro-Fernández et al. (2024) argue that mathematics teachers must be aware of students' preconceptions and address them during instruction to help students reorganize their knowledge and correct potential errors, thereby avoiding the formation of alternative conceptions. This is crucial because some alternative conceptions are difficult to modify and may persist even after instruction (Bostan-Sarioglan, 2016; Chi et al., 2012; Denbel, 2014). Similarly, some of these conceptions may persist across different educational stages and ages. These reasons underscore the importance of studying alternative conceptions, especially in key concepts such as proportional reasoning, which serves as a foundation for other concepts like functions, rate of change, limits, derivatives, and continuity, among others. Additionally, identifying these conceptions in students is an essential step in designing strategies to facilitate their change. Therefore, this research aims to answer the following question: *What alternative conceptions do 12th-grade students exhibit when solving tasks involving proportional reasoning?* And as an objective, we seek to identify these alternative conceptions.

2. METHOD

For the purpose of this qualitative research, an alternative conception is understood as knowledge that is inconsistent with the mathematical concepts that the academic community considers mathematically valid (García-García, 2018). These conceptions are manifested in the reasoning students use to solve specific problems and appear in their written productions. Typically, these conceptions originate in the school context (Dolores-Flores, 2004) and emerge when students' beliefs conflict with the accepted mathematical concepts within the discipline (Kastberg, 2002). Often, these conceptions are contradictory or inconsistent with established scientific principles (Salgado-Beltrán & García-García, 2024).

2.1. Participants

This research was conducted in November 2024 at a public High School located in the central region of Guerrero, Mexico. Fifteen students (10 males and 5 females), aged between 17 and 18 years. The participants were selected based on the following criteria: (1) they were legally enrolled in the 12th grade; (2) they had previously worked with the concept of proportional variation in their earlier grades; and (3) they voluntarily agreed to participate in the study and provided consent for the use of their written and verbal contributions in the research. From this point onward, the participants will be identified as E1, E2, ..., E15.

2.2. Research Instrument

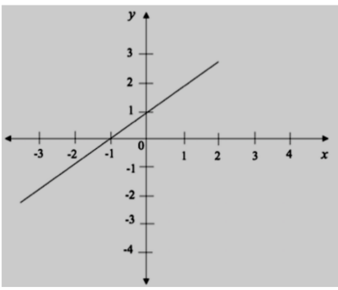
A Task-Based Interview (TBI) was designed and administered, following the principles proposed by Goldin (2012). This method is characterized by an interaction between the interviewee (the person solving the task) and the interviewer (the researcher), focusing on one or more predefined tasks that may include questions, problems, or activities. Five tasks were designed and validated by experts, who evaluated key aspects such as the clarity of wording, the level of difficulty, and the relevance of the tasks in relation to the study's objectives.

The final instrument consisted of the tasks described in Table 1, designed to identify students' alternative conceptions about proportional reasoning. The interviewer's protocol included general questions applicable to all tasks, such as: "What does this term mean?", "Can you explain more about what you mentioned?", and "Can you draw it?" Additionally, specific questions were incorporated to delve deeper into the participants' reasoning and knowledge about the concept of proportional variation. These questions were used when the interviewee's responses were ambiguous, when their reasoning lacked clear foundations, or to encourage the exploration of other possible solutions they might know.

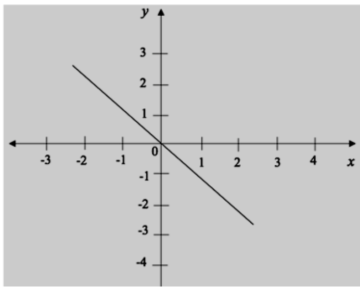
Table 1. Tasks proposed for the students

Task 1.	Explain, what do you understand by proportional variation, and what types do you know?
Task 2.	Propose and describe real-world situations where you can identify and explain the types of proportional variation.
Task 3.	Draw a graph on the Cartesian plane that represents each type of proportional variation (direct and inverse). Explain your answer.
Task 4.	Analyze the following algebraic expressions and indicate which represent direct proportional variation and which correspond to inverse proportional variation. Explain your answer. a) $y = 2$ b) $y = -3$ c) $y = -4x$ d) $y = 6x$ e) $y = x^2$ f)
Task 5.	Examine the graphs provided and determine if they represent any type of proportional variation. Identify the type of variation and explain the meaning of the constant of proportionality.

a)



b)



2.3. Data analysis

The participants' answers were recorded both on worksheets and in video recordings. To analyze this information, the thematic analysis method proposed by Braun and Clarke (2012) was employed. This approach included triangulation among the researchers, ensuring the reliability, validity, and rigor of the results. Through this method, patterns of meaning (themes) reflecting the alternative conceptions present in the students' answer to the tasks in the TBI protocol were identified. The process consists of six phases. In the first three phases, each researcher conducted the analysis independently; from the fourth phase onward, discussion sessions were held to reach a consensus on the alternative conceptions identified in the student group. Below, the phases of the process are detailed.

Phase 1. Familiarization with the data

The interviews were transcribed, and the written answers were digitized. Next, each researcher conducted an initial reading of the answers to gain a preliminary understanding of the alternative conceptions expressed by the students, as well as to familiarize themselves with the language and expressions used by the participants.

Phase 2. Generation of initial codes

Words, phrases, and/or calculations used by the students to refer to the concept of proportional variation were identified. For example, in the production from E1 for Task 5, key phrases were highlighted in italics, allowing the generation of two initial codes: "the constant of proportionality is the length of the line" and "the measurement of the line indicates the constant of proportionality".

In this phase, 147 initial codes werw identified based on the ideas students expressed about the concept of proportional variation. These codes were generated through the analysis of all written responses and verbal arguments provided by participants during the interviews. In some cases, students mentioned more than two ideas, leading to different coding. This analysis was conducted with the answers of all fifteen participants.

Once all the codes had been identified, we carefully reviewed them to find patterns, similarities, and common ways of thinking among the students. This allowed us to group several codes into subthemes in the next phase.

Phase 3. Search for themes and subthemes

The initial codes were reviewed to group those that shared a similar meaning or showed a common answer pattern, resulting in the formation of subthemes. These subthemes were associated with a main theme reflecting an ACPR (Alternative Conception of Proportional Reasoning). For example, from the subthemes "the graph of an increasing line represents direct proportional variation (DPV)" and "the graph of a decreasing line represents inverse proportional variation (IPV)," the theme was constructed: "the variational behavior of the graph of a line indicates the type of proportional variation (PV)".

Phase 4. Review of themes and subthemes

Two levels of review were conducted. In the first level, it was verified that all themes were clear and adequately reflected the generated codes. The goal of this stage was to ensure that each theme was supported by a coherent set of data and that the codes within each theme were closely related. In the second level, a more thorough review of all data was conducted based on the identified themes. This step ensured that the themes accurately represented the complete dataset, avoiding overlaps and inconsistencies between themes and subthemes. If necessary, new themes were created. Additionally, in this phase, the researchers met for peer review, which contributed to improving the quality and validity of the data analysis. This process allowed for refining the identification of themes, which in this study correspond to alternative conceptions, and helped reduce bias from a single researcher.

Phase 5. Defining and naming themes and subthemes

In this stage, names were assigned to the identified themes to facilitate the presentation of the findings, which, in this study, correspond to ACPRs. Each theme was given a clear and precise definition reflecting the ideas expressed by the students, based on their written or verbal responses.

Phase 6. Report writing

In this phase, the alternative conceptions (themes) are presented along with their corresponding subthemes (see Table 2). For the purposes of the research, the results are described according to each identified alternative conception.

3. RESULTS AND DISCUSSION

3.1. Results

The analysis of the participants' answers allowed for the identification of five ACPRs (Alternative Conceptions of Proportional Reasoning), which emerged during the process of solving the tasks by the High School Level students (see Table 2).

Table 2. Alternative conceptions identified

Alternative Conceptions	Definition	Subthemes	Frequency
The variational behavior of a line graph indicates the type of PV	Refers to the relationship established between the behavior of a line graph and the type of PV it represents.	- The graph of an increasing line represents a DPV. - The graph of a decreasing line represents an IPV.	40
The constant function algebraically represents a PV	Refers to conceiving the function $y = k$ as the algebraic model of a PV.	- The function of the form $y = k$, with $k > 0$, represents a DPV. - The function of the form $y = k$, with $k < 0$, represents an IPV.	35
A negative slope in the equation of a line indicates an IPV	This conception indicates that if $m < 0$ in $y = mx$, the model represents an IPV.		29
DPV is an object	In everyday situations, it refers to a surface or an object with the shape of a line.		24
The constant of proportionality in the graph of a DPV is the length of the line	This conception indicates that the constant of proportionality corresponds to the measurement of the line representing the proportional relationship.		19

3.1.1. The Variational Behavior of a Line Graph Indicates the Type of PV

This alternative conception refers to the relationship between the behavior of a line graph and the type of PV it represents. It emerged from the answers of the majority of students, with the exception of E8, E9, E13, and E15, in Tasks 3 and 5. Since the students expressed two ideas linked to the notion of PV, two subthemes were identified, which are detailed below.

The Graph of an Increasing Line Represents a DPV

This subtheme was constructed from the answers provided by eleven participants in Tasks 3 and 5-a. First, when graphing the different types of PV on the Cartesian plane, the students associated DPV with the representation of an increasing line. Although this association is not entirely incorrect within the mathematical context for this type of PV, their reasoning did not include the fact that the line should pass through the origin of the plane. This same idea was replicated when they identified that the graph in Task 5-a, being an increasing line, represented a DPV. Below are some of the phrases and statements used by the students, which guided the construction of this subtheme: "If it is an increasing line, then it is a DPV", "The graph of a DPV is a line that increases" or "The line that goes up represents a DPV". To illustrate part of the verbal evidence that contributed to the construction of this alternative conception, an excerpt from the dialogue with E2 is presented.

E2 : My drawing will be a line [constructs the Cartesian plane], since these represent a direct proportional variation.
I : Does any line represent a direct proportional variation?
E2 : Well [...], it should be going up [draws an increasing line on the plane]. How do you say it? Increasing.
I : Is it only required that the line be increasing?
E2 : I think so.

The Graph of a Decreasing Line Represents an IPV

This subtheme was developed from the answers provided by eleven participants in Tasks 3 and 5-b. In their answers, the students included the drawing of a decreasing line to represent an IPV, which was highlighted when these same case studies attributed the same PV relationship to part b of Task 5. In this context, the students showed that they had adopted certain correlation conditions, such as those associated with line-proportional variation and decreasing-inverse. This tendency was evidenced in phrases such as: "The decreasing line represents an IPV", "Inverse goes with decreasing" or "Any line that goes down expresses an IPV" (see Figure 1). This occurs because students tend to associate visual characteristics of graphs with types of variation directly, without considering all the necessary aspects that integrate their definition.

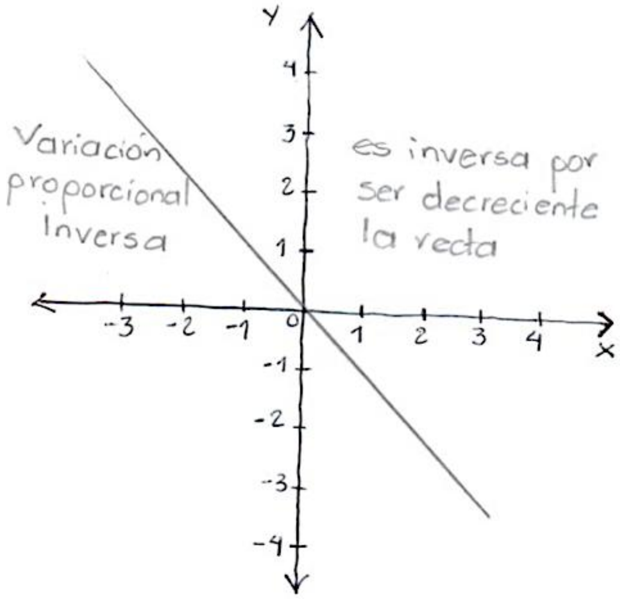


Figure 1. E3 represents an IPV through a decreasing line in Task 3

3.1.2. The constant function algebraically represents a PV

Less frequent than the previous one, this alternative conception involves identifying the constant function y=k as the algebraic model representing a PV. This idea emerged in the answers of the students, with the exception of E8, E11, E13, E14, and E15, in Task 4. Based on their answers, two different approaches were identified, allowing them to be grouped into two subthemes, which are detailed below.

The function of the form y =k, with k>0, represents a DPV

This subtheme was constructed from the answers of ten students in Task 4. In this context, when analyzing algebraic representations of different functions, the students identified those that corresponded to some type of DPV. In their justifications, they highlighted the constant function y=2 as the model representing a DPV, arguing that since it is a positive number, this property made it representative of a DPV. Below are some of the students' statements that guided the construction of this subtheme: "y=2 is the equation of a DPV", "A DPV is one that has a positive number", "The sign of the number is what makes it a DPV" or "Any constant function, as long as it is positive, will give us a DPV". To illustrate part of the verbal evidence that contributed to the construction of this alternative conception, an excerpt from the dialogue with student E1 is included.

I : Look at the expressions, which one represents a direct proportional variation?

E1 : Let's see, I think a, because it is a positive number, and in fact, any function like this [writes y=3 and y=8] would also represent a DPV.

I : Can you explain your answer a bit more?

E1 : Yes, well, I look at the sign of the constant function, the others have a variable integrated, while this one does not [points to a] and b would be an IPV. Well, that's how I understand it, and also, they are lines.

The function of the form y= k, with k<0, represents an IPV

This subtheme was developed from the answers provided by ten students in Task 4. In their answers, the students indicated that the negative sign in the formula of the constant function was the determining factor for the model to represent an IPV, leading them to choose the function y=-3 as an example of an IPV. This reasoning revealed a limited understanding of the concept, as it reflected a superficial treatment of the different registers in which the concept can be represented. Phrases such as "y=-3 represents an IPV", "The negative sign [in y=-3] must indicate an IPV", or "The plus corresponds to a DPV and the minus to an IPV" (see Figure 2) evidence this tendency. These findings suggest that, in a school context, students do not have sufficient opportunities to conceptualize proportional variation through different representation registers, which limits their understanding of the concept.

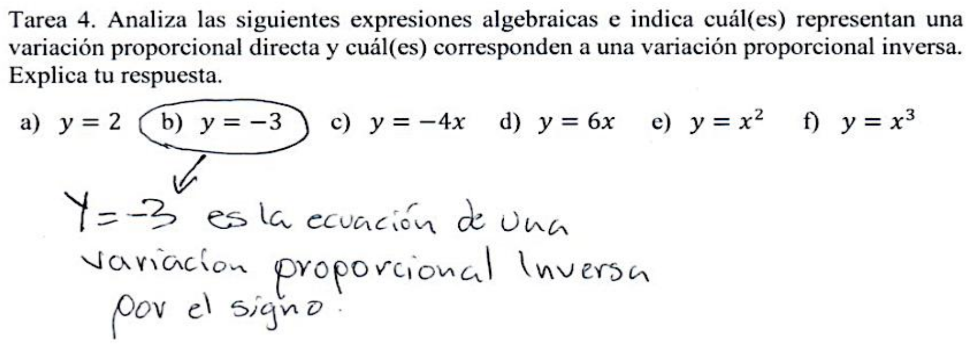


Figure 2. E5 identifies the algebraic model of an IPV in Task 4

3.1.3. DPV is an object

This alternative conception refers to the interpretation of DPV as a surface or an object with a linear shape. It was observed in the answers of students E1, E3, E4, E6, E7, E8, and E9 in Task 2, when, while proposing and describing real-world situations to identify types of proportional variation, these students associated DPV with objects that can have a linear shape. Among the examples mentioned are streets and roads, ramps, stairs, roofs, and others (see dialogue excerpt with E3). For these students, DPV is not understood as a functional relationship between two variables that satisfy the condition of their quotient being a positive constant, but rather as the linear shape of objects in their everyday environment. These findings highlight a limited understanding of the concept of DPV among students, which focuses more on its visual form than on its mathematical foundation.

E3 : Let's see, direct proportionality in mathematics is like a line [draws an increasing line on the Cartesian plane], and it must be like this.
I : Why do you consider it this way?
E3 : That's what I understand, and that's how I was taught… So, in a real-world situation, it would have to have that line shape. Right now, I think of an incline, like a street. Even house roofs have a line shape.
I : Are you saying that DPV can be each object you mention?
E3 : Yes, and others. We just need to see that they are like lines.

3.1.4. The constant of proportionality in the graph of a DPV is the length of the line

This alternative conception involves interpreting the constant of proportionality as the measurement of the line representing the DPV. It was observed in the answers of students E1, E4, E7, E8, and E14 in Task 5, when they identified the type of PV represented by the graphs of two lines on the plane and based on this, established the meaning of the constant of proportionality. Phrases such as "The constant of proportionality is the length of the line", "What the line measures is the value of the constant of proportionality", and "The constant of proportionality is linked to the length of the line", among others, reflect this tendency and were key to the construction of this subtheme (see Figure 3). This alternative conception may originate from the fact that students give greater importance to the visual representation and geometric context of the graphs, prioritizing these aspects over the algebraic or formal properties of the proportional relationship.

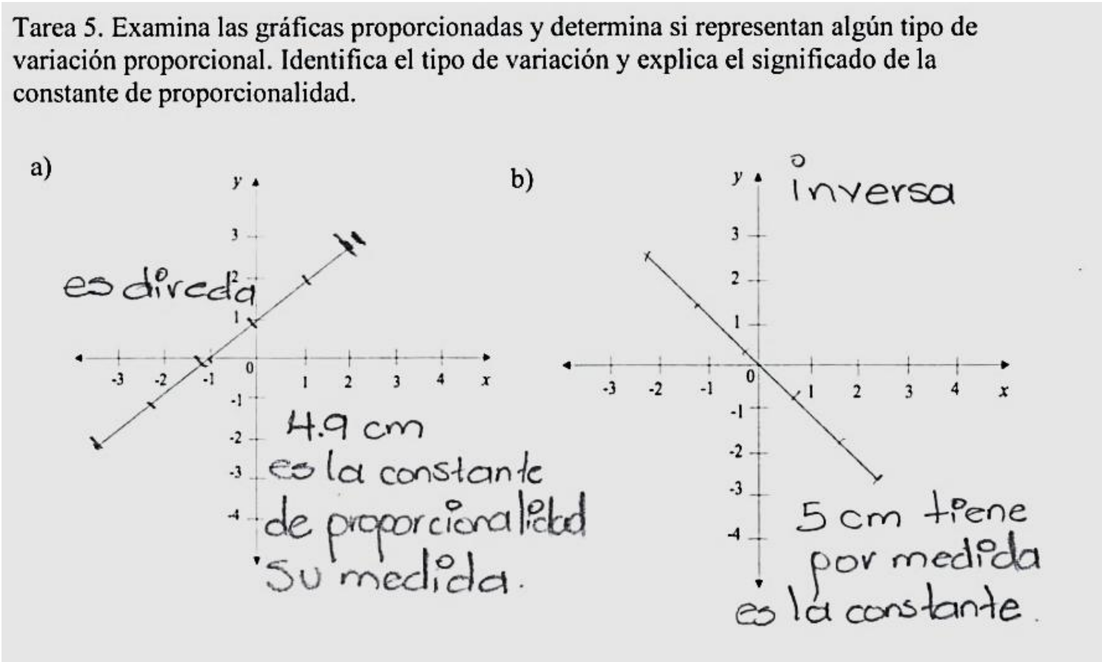


Figure 3. E8 interpreting the constant of proportionality in the graph of a line in Task 5

3.2. Discussion

The findings revealed that students fail to identify the PV in other semiotic registers, such as graphs, and consequently, they conceptualize the line as the graph corresponding to a PV, associating growth with DPV and decline with IPV (Moss & McNab, 2011). This result highlights a limited understanding of the concept of proportionality and underdeveloped proportional reasoning in students, which are essential skills for understanding more advanced mathematical concepts (Lugo-Lugo et al., 2023; Mochón-Cohen, 2012).

Another alternative conception identified in the students' responses and arguments is the interpretation of the constant function as the algebraic model of a PV. This idea emerged when students asserted that the sign of the function denoted the type of PV, indicating that a plus (+) is linked to a DPV and a minus (–) to an IPV. This result is consistent with findings from other studies (e.g., Butto et al., 2019; Dolores-Flores, 2004). It also underscores those students, lacking solid proportional reasoning, tend to rely on visual or superficial characteristics to establish relationships between mathematical concepts, as in this case, the constant function and PV. This type of superficial reasoning may arise when students have not fully developed an understanding of the underlying properties and concepts. Instead of analyzing how PV behaves in terms of its algebraic or graphical definition, they rely on a more immediate or visual interpretation of the sign. This highlights the need for teaching that promotes deeper reasoning about proportionality, enabling students to understand not only the visual characteristics but also the relationships and structures underlying the mathematical concept (Luque-Álvarez & Ibarra-Olmos, 2021b; Secretaría de Educación Pública, 2023).

Another relevant finding of this research was the alternative conception that associates a negative slope in the equation of a line with an IPV. This idea is closely linked to the previous ones, as it persists in the notion that both the graph and the algebraic model of a line always represent a PV, and that certain characteristics can determine the specific type of PV being reflected. In this case, the negative slope of the line was used as the indicator to link the algebraic expression with an IPV, a result consistent with those reported by Fernández-Verdú and Llinares-Ciscar (2012) and Hernández-Solís et al. (2024). This phenomenon constitutes clear evidence of the limited proportional reasoning that students develop in the school context, as well as their lack of awareness that an IPV is graphically represented by a hyperbola, indicating that as one variable grows, the other decreases nonlinearly.

On the other hand, the alternative conception of DPV as an object was identified in students when they associated this type of variation with everyday situations, such as surfaces (streets or roads) or other objects (house roofs), due to their linear shape. This constitutes clear evidence of students' visual and intuitive thinking about the concept of PV, as well as their tendency to centralize ideas around the line. Additionally, these interpretations may be attributed to a traditional approach to teaching proportional variation, which in many cases is limited to the use of the rule of three, without promoting conceptual understanding or its connection to real-world situations (Butto et al., 2019; Cuevas-Vallejo et al., 2023).

Furthermore, the alternative conception that the constant of proportionality in the graph of a DPV is its length was observed less frequently compared to the others. However, the lack of clarity regarding the concept of PV led students to interpret the constant of proportionality as a visual attribute of the graph of a line, specifically its measurement. This reflects an intuitive interpretation, where students associate visible properties of the graphical representation with algebraic concepts (Cuevas-Vallejo et al., 2023). It also highlights the difficulties students experience in interpreting the constant of proportionality, which affects their ability to solve mathematical problems involving proportionality (Butto et al., 2019; Cuevas-Vallejo et al., 2023).

The findings of this study reveal several alternative conceptions among students, indicating limited development of proportional reasoning. While this research focuses solely on student responses, it is worth considering the broader instructional context in which these misconceptions may have formed. Previous studies (e.g., Byerley & Thompson, 2017; Salgado-Beltrán, 2020) have reported challenges related to teacher preparation and conceptual understanding among High School mathematics teachers in Mexico. Although this study does not include data from teachers, such findings suggest that instructional approaches emphasizing procedural methods over conceptual understanding might contribute to the persistence of these misconceptions. Further research is needed to directly examine the role of teacher knowledge and instructional practices in shaping students' proportional reasoning.

4. CONCLUSION

The results of the study revealed that students exhibited different alternative conceptions. In total, five were identified: (1) the variational behavior of a line graph indicates the type of PV, (2) the constant function algebraically represents a PV, (3) a negative slope in the equation of a line indicates an IPV, (4) DPV as an object, and (5) the constant of proportionality in the graph of a DPV is the length of the line. A significant finding in this study was the prevalence of an alternative conception among students regarding the relationship between the graphical representation of a line and the type of PV it represents. Specifically, most students associated an increasing line with a DPV, regardless of whether it passed through the origin, and a decreasing line with an IPV.

On the other hand, the findings of this study underscore the need to design instructional approaches aimed to change the alternative conceptions identified among the participants regarding proportional reasoning. Likewise, mathematics teachers' professional development could focus on offering opportunities to broaden and deepen their understanding of proportional reasoning, thereby enabling them to foster more meaningful learning in their students. Therefore, it is essential for future research to investigate the knowledge that teachers draw upon to promote proportional reasoning at different educational levels, in order to explain some of the possible origins of the findings reported in this study. Additionally, it would be valuable to develop classroom proposals that support a more robust understanding of proportionality among upper secondary students.

Acknowledgments

The authors would like to thank the research participants that allowed us to work with them.

Declarations

Author Contribution	: HFS-R: Conceptualization, Investigation, Visualization, Writing – original draft, and Writing – review and editing; GS-B: Formal analysis, Methodology, and Writing – review and editing; JG-G: Formal analysis, Supervision, Validation and, Writing – review and editing; AL-G: Formal analysis, and Writing – review and editing.
Funding Statement	: This research did not receive any funding or public private.
Conflict of Interest	: The authors declare no conflict of interest.
Additional Information	: Additional information is available for this paper.

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