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Student metacognition levels for solving PISA-like problems: A hierarchy

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Abstract

The metacognitive process is frequently presented in a fragmented manner, and a limited number of integrated learning models are designed to enhance these skills. These gaps underscore the necessity for research that defines metacognitive level hierarchies, clarifies process integration, and develops effective learning models to improve metacognitive abilities in PISA-like problem solving. This study aims to identify and describe the hierarchy of students' metacognitive levels as they solve PISA-like mathematical problems, mapping the stages of metacognition demonstrated throughout the problem-solving process. This qualitative study employed a phenomenological design, involving 76 students from three islands in Indonesia. Data were collected through tests, observations, and in-depth interviews, and were validated by implementing triangulation methods by comparing student answer sheets, observations, and interview results. The data were analyzed using thematic analysis. Then, five metacognition levels were identified: understanding the problem, thinking about the answer, comprehending how to answer, finding the answer, and being confident. Future research may focus on efforts to defragment and explore higher levels of metacognition and develop integrated learning models.

Keywords:
Hierarchy, Level, Metacognition, PISA, Problem-solving

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1. INTRODUCTION

Students employ metacognition in mathematical problem-solving to monitor, control, and assess their ideas, along with the experiences and beliefs underlying their cognitive processes (Jiang et al., 2020; Sutama et al., 2021). Metacognition refers to the process of thinking about thinking (Puente-Díaz et al., 2021). It consists of three aspects: awareness, evaluation, and regulation (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). Awareness arises when problem-solvers are conscious of their thoughts—reflecting on what they know, their progress, what remains to be done, or potential actions. Evaluation occurs when problem-solvers engage in decision-making, recognizing the effectiveness and limitations of their thinking, the implications of their strategic choices, and assessing outcomes. They also determine whether difficulties stem from deficits in ability or understanding (Sutama et al., 2019). Regulation involves strategic planning, goal setting, and selecting appropriate strategies in the context of problem-solving.

Metacognition plays a crucial role in problem-solving by helping students obtain accurate answers (Jiang et al., 2020). Students who employ metacognitive strategies perform significantly better than those who do not (Bae & Kwon, 2019; Kuzle, 2013). In addition, students' metacognition enhances students' ability to solve problems creatively (Chatzipanteli et al., 2014; Hargrove & Nietfeld, 2015). Therefore, excellent and creative problem solvers must be accustomed to using metacognition in solving problems. However, both survey and anecdotal evidence show that teachers often focus solely on students' answers, showing little interest in how those answers were obtained (Masduki et al., 2020). Moreover, students often fail to engage in monitoring while solving procedural problems (Kholid et al., 2022). This may be attributed to teachers' tendency to pose factual rather than critical questions and their difficulty in asking productive questions (Sahin & Kulm, 2008). Teachers often ask questions with yes or no answers rather than questions that require higher-order thinking (Weiland et al., 2014).

Metacognition begins with student awareness in solving problems (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). Problems or questions that stimulate this awareness—such as PISA-like problems with concrete and complex characteristics (Callan et al., 2016)—are valuable for eliciting metacognitive processes. Although metacognition has been widely studied as a global issue in both youth and adult education (Amini et al., 2020; Annevirta & Vauras, 2006; Pennequin et al., 2010), much of the existing research focuses on general metacognitive awareness, strategy use, or the impact of interventions. However, a significant gap remains regarding the structure and progression of metacognitive processes, particularly whether students' metacognitive activity follows a clear hierarchy or is fragmented and inconsistent during problem solving.

Addressing this gap is especially relevant in the Indonesian context, where students' performance in PISA mathematics assessments remains below the international average (OECD, 2022; Salwadila & Hapizah, 2024; Sistyawati et al., 2023; Wijaya et al., 2024). While interventions to improve metacognition are important, a foundational understanding of how students' metacognitive processes are organized—whether they progress through identifiable levels or follow fragmented patterns—remains limited. By investigating the hierarchy of metacognitive levels in the context of PISA-like problems, this study aims to clarify the progression and structure of students' metacognitive activity. Such knowledge is essential for designing more effective interventions and instructional models, as it provides a theoretical basis for targeting specific stages of metacognitive development and addressing fragmentation in students' problem-solving approaches.

Table 1. 2022 PISA results

No.	Countries	Rank	Math Score (Mean)
1.	Singapore	1	575
2.	Brunei Darussalam	40	442
3	Malaysia	47	409
4.	Thailand	54	384
5.	Indonesia	66	366
6.	The Philippines	68	355
International Mean			**472**

Table 1 shows that Indonesia's average mathematics score is below the international mean of 366, placing the country 66th out of 78 countries participating in PISA. Among Southeast Asian countries participating in PISA, Indonesia ranks near the bottom, ahead only of the Philippines. These data indicate that Indonesian students' low performance in PISA mathematics assessments is a complex issue influenced by various factors, including curriculum alignment, teaching quality, socio-economic background, language barriers, and limited exposure to higher-order thinking and problem-solving tasks (Masduki et al., 2020; OECD, 2022; Sari & Valentino, 2017; Sutarni et al., 2024). Several studies have highlighted that Indonesian students often struggle with non-routine, context-based problems that require not only mathematical knowledge but also metacognitive skills such as planning, monitoring, and evaluating their own thinking processes (Kholid & Lestari, 2019; Sutama et al., 2021).

Despite these findings, research on how students engage in metacognition while solving PISA-like problems remains limited. Most existing studies focus on general descriptions of metacognitive awareness or the effectiveness of metacognitive strategies (Desoete & De Craene, 2019; Katsantonis, 2024; Laamena & Laurens, 2021; Mevarech & Fan, 2018; Ramlah et al., 2024; Zhang & Lian, 2024), without providing a detailed hierarchy of metacognitive levels or examining how these levels manifest in actual problem-solving situations. This gap is significant, as understanding the specific stages and characteristics of students' metacognitive processes can inform the development of targeted instructional strategies and learning models.

Therefore, investigating the hierarchy of students' metacognitive levels in solving PISA-like problems is crucial for developing a more nuanced understanding of their problem-solving processes. By mapping these levels, this study aims to provide a structured framework that educators and policymakers can use to design interventions that not only improve mathematical achievement but also foster essential metacognitive skills. This contribution addresses the gaps identified in the literature and supports efforts to enhance the quality of mathematics education in Indonesia, ultimately aiming to improve students' performance in international assessments such as PISA.

1.1. Research Position

Studies on metacognition fall into three main categories. The first category contains research that produces instruments to assess student metacognition. These instruments include a test set (Händel et al., 2013), a metacognition assessment interview (Semerari et al., 2012), and a self-report questionnaire (Purnomo et al., 2020). The second category comprises studies that conclude: 1) metacognition supports students success in learning (Sengul & Katranci, 2015; Zohar & Barzilai, 2013); 2) greater emphasis on student metacognition correlates with improved achievement over a learning period (Fisher, 1998; Jayapraba, 2013); and 3) metacognition elevates students' capability beyond their expectation (Zhang, 2010). The third category explores the dimensions of metacognition, including its components (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004) and descriptions of students' metacognitive capabilities (Kholid & Ahadiyati, 2022).

After reviewing metacognitive themes and grouping them into three main categories, the researchers identified potential gaps in the literature. These gaps include establishing the hierarchy of metacognitive levels, defragmenting the metacognitive process, and developing integrated learning models to elevate metacognitive skills. This year, our research focused on the hierarchy of students' metacognitive levels. Figure 1 illustrates the research, including prior studies, potential areas of investigation, and research we have conducted.

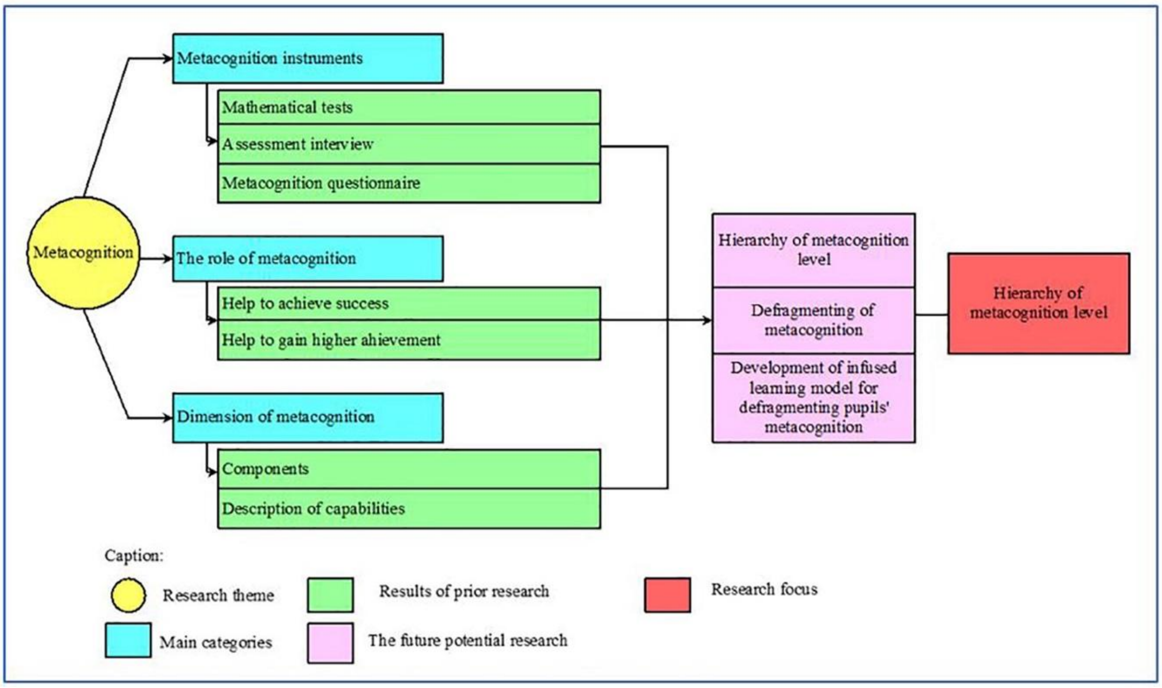

Figure 1. Research position

Research on metacognition in mathematics education has identified several key components, notably awareness, evaluation, and regulation (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). However, to provide a more cohesive theoretical foundation for this study, we draw upon established metacognitive frameworks, particularly those proposed by Flavell (1979) and Schraw and Dennison (1994).

Flavell (1979) conceptualized metacognition as comprising both metacognitive knowledge (knowledge about one's own cognitive processes) and metacognitive regulation (which involves the monitoring and control of these processes). Schraw and Dennison (1994) expanded this framework by distinguishing metacognitive knowledge into declarative, procedural, and conditional knowledge and metacognitive regulation into planning, monitoring, and evaluation. These models have been widely adopted in educational research to analyze how learners understand and manage their thinking during problem solving.

In this study, the hierarchy of metacognitive levels is examined through the lens of these frameworks. Specifically, we investigate how students demonstrate metacognitive knowledge and regulation while solving PISA-like mathematical problems. By mapping students' responses and strategies to the components outlined by Flavell (1979) and Schraw and Dennison (1994), this research aims to clarify whether students' metacognitive processes progress through identifiable levels or remain fragmented. This integrated framework not only guides the analysis but also ensures that the findings contribute meaningfully to the broader discourse on metacognition in mathematics education.

1.2. Research Roadmap

The researchers involved in this work have not only reviewed prior studies on metacognition but have also conducted research in recent years. In 2018, they investigated students' metacognition abilities in solving PISA-like problems (Kholid & Lestari, 2019). The results indicated that the students with stronger mathematical skills also demonstrated higher metacognition abilities. In the same year, a worksheet template was developed to stimulate students' metacognition. In 2019, the researchers examined the role of students' adversity quotients (AQ) in relation to their metacognition abilities (Kholid & Yuhana, 2019). The findings show that students with good AQ can implement metacognition skills more effectively, while quieter students require additional guidance. In 2020, students' problem-solving abilities and responses were investigated in relation to their use of metacognitive strategies in learning. The findings concluded that the application of metacognitive strategies effectively supports students' problem-solving skills. Based on these studies, in 2024, researchers examined the hierarchy of students' metacognition levels in solving PISA-like problems. The findings served as a foundation for guiding future research in 2025-2027 to develop defragmenting methods to elevate students' metacognitive abilities. The roadmap completed research, ongoing studies, and future analyses is presented in Figure 2.

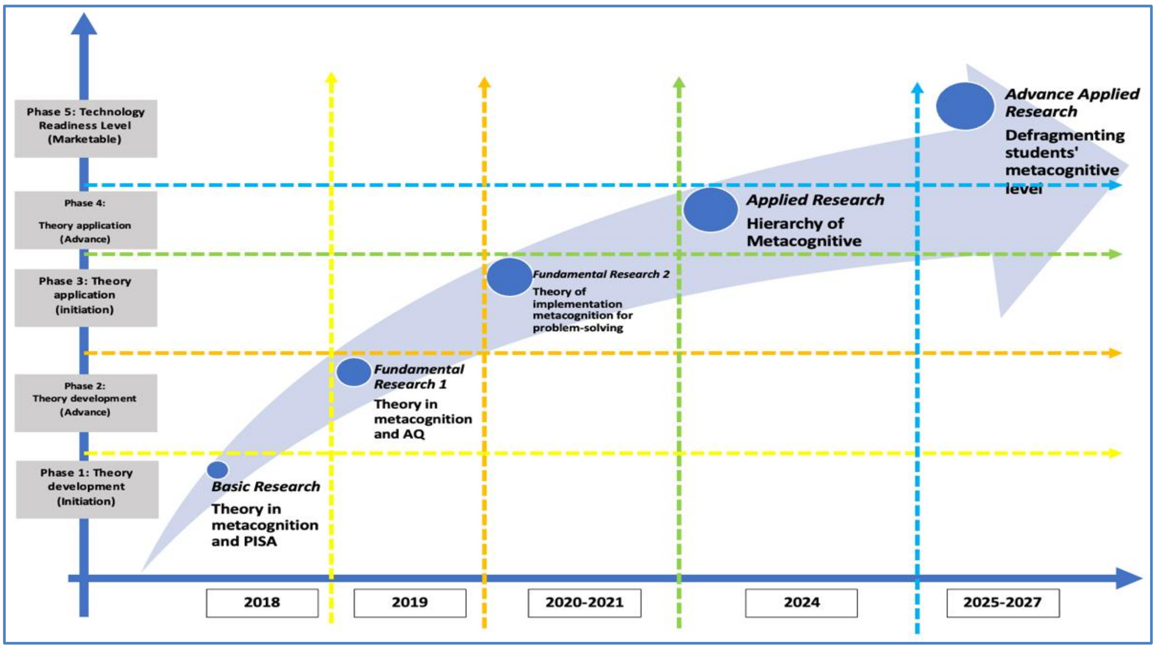

Figure 2. Research roadmap

1.3. Objective and Urgency

This study aims to determine the metacognition-level hierarchy in students when solving PISA-like problems. The results will be employed as a theoretical framework model for two subsequent research directions. This research carries a sense of urgency, aligning with the goal of shaping competitive Indonesian human capital as outlined in the National Research Master Plan (NRMP) (*Rencana Induk Riset Nasional (RIRN)*). As stated on page 84 of the NRMP document, research in Indonesia from 2017 to 2045 is directed toward the thematic area of social humanities, arts, culture, and education. Figure 3 depicts the detailed Indonesian research plan related to this theme.

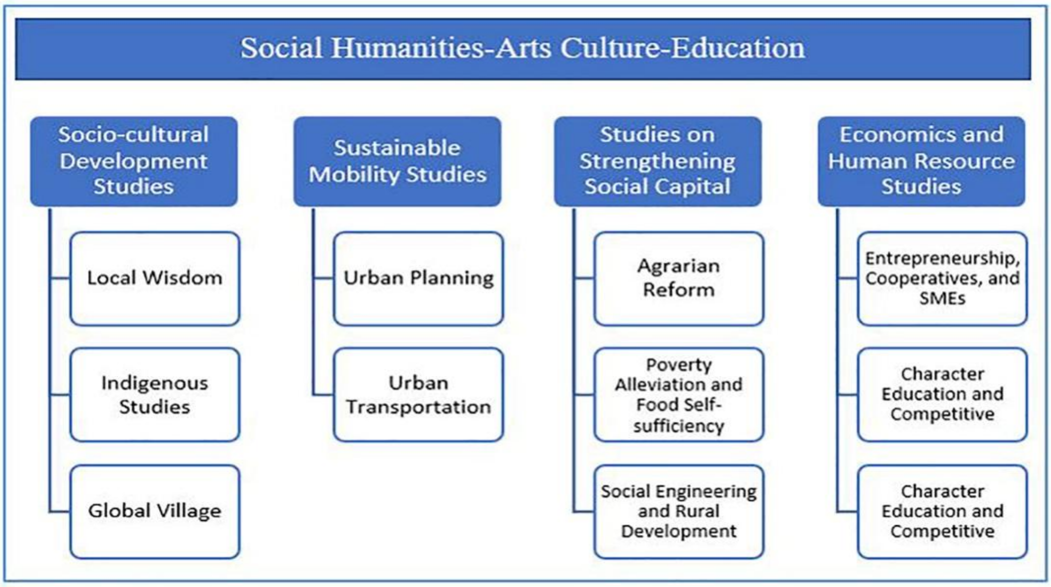

Figure 3. Themes and topics for the focus of social research: humanities, arts, culture and education

Based on Figure 3, Indonesia is prioritizing participatory technology research to strengthen national identity through several research areas: socio-cultural development, sustainable mobility, the strengthening of social capital, and the development of economic and human resources. In this context, the study of metacognition as a sub-focus of educational research aims to produce human resources with strong character and high competitiveness. These priorities highlight the need for increased national attention to metacognitive research.

2. METHOD

A qualitative phenomenological design (Creswell, 2014) was adopted to explore the hierarchy of students' metacognitive levels while solving Programme for International Student Assessment (PISA)-like problems. Phenomenology was chosen because it emphasizes describing participants' lived cognitive experiences rather than measuring predetermined variables.

2.1. Context of study

This research continues the previous metacognition study road map shown in Figure 2. That study identified a gap in understanding how students apply metacognition when solving PISA-like problems. In response, the present study focuses on students' metacognitive traits in this context. A PISA-like problem was used to answer the hierarchy of student metacognitive levels students employ during problem-solving. It was expected that students would demonstrate a level of metacognition sufficient to effectively solve such problems.

2.2. Participants

Seventy-six 15-year-old secondary school students from three Indonesian islands (Java, Sumatra, and Sulawesi) completed the written PISA-like task. Geographical diversity was incorporated to capture potential curricular and cultural variations that may shape metacognitive development (Vygotsky, 1978). For the in-depth phase, eight "information-rich" students (Patton, 2014) were purposively selected based on the following criteria: (1) completion of the entire task, (2) evidence of metacognitive behavior in written work (e.g., self-corrections, strategy annotations), and (3) willingness to participate in follow-up interviews.

The final interview sample (5 females, 3 males) was sufficient to reach analytic saturation (Guest et al., 2020). Gender was not treated as an analytic variable; as existing literature suggests no systematic gender differences in metacognitive regulation at this age (Zimmerman, 1990), only aggregate gender frequencies are reported to illustrate sample diversity. Figure 4 presents the distribution of participants by island of origin.

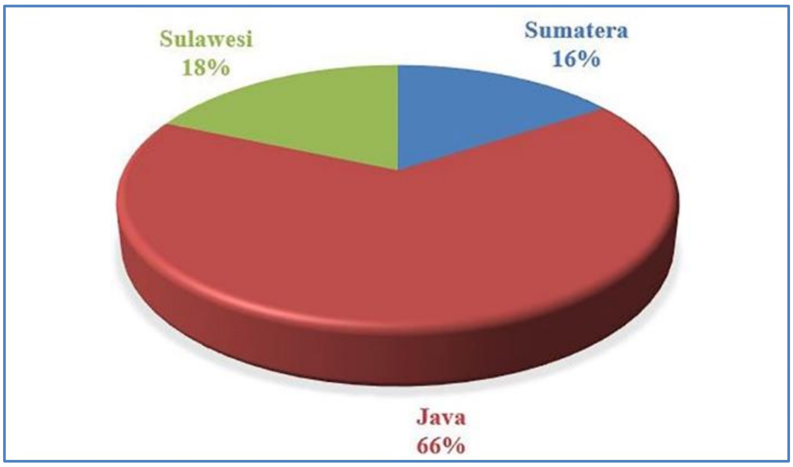

Figure 4. Demographic structures of the participants

2.3. Task Design/Data Collection Instruments

A single mathematics problem mirroring PISA framework specification (OECD, 2022) was constructed and validated by three experts in mathematics education. The item required interpretation of contextual data using set relationships—an authentic scenario known to elicit metacognitive monitoring and evaluation (Callan et al., 2016). Prior to data collection the item was piloted with a comparable cohort (n = 12); minor wording revisions followed.

Students solved the task individually under standard classroom conditions (30 min). Think-aloud protocols were introduced with a brief rehearsal example. Subsequent stimulated-recall interviews were conducted with the eight information-rich students (45–60 min each). All sessions were video-recorded using two synchronised cameras: (1) camera 1 captured facial expressions and verbal protocols, supplying observable cues of planning, monitoring, and evaluation, (2) camera 2 captured the written workspace, enabling fine-grained linkage between verbalised strategies and on-paper actions. Dual-angle recording enhanced data triangulation and reduced reliance on self-report. Figure 5 depicts the PISA-like problem.

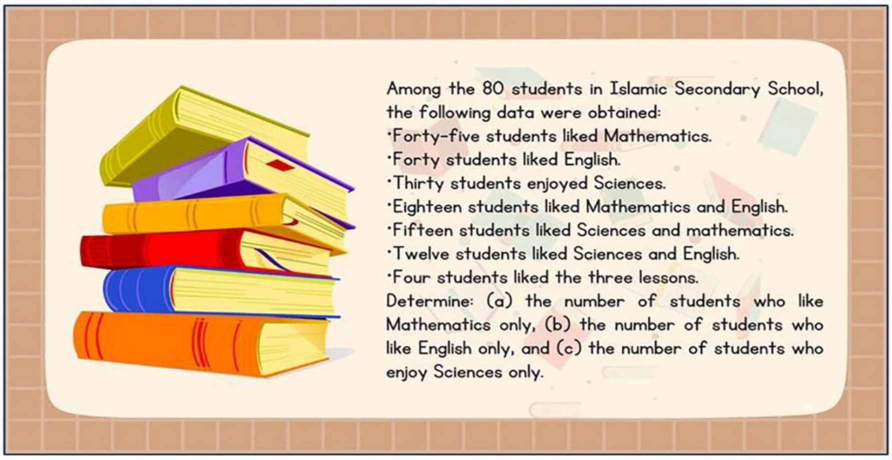

Figure 5. The PISA-like problem

2.4. Data Analysis

The analysis of the collected data commenced with the verbatim transcription of all audio-visual files, which were then meticulously synchronized with screenshots of the students' written work. This process resulted in a comprehensive corpus of approximately 92,000 words, forming the foundation for subsequent analysis.

Thematic analysis, following Braun and Clarke (2019) six-phase procedure, was employed to identify patterns and themes within the students' responses. A hybrid deductive–inductive strategy guided this process. Overarching categories were initially informed by established metacognitive frameworks, specifically those proposed by Flavell (1979) and Schraw and Dennison (1994). However, the more granular sub-themes and the hierarchical refinements of students' metacognitive levels emerged inductively from iterative engagement with the data itself. This dual approach allowed the researchers to leverage existing theoretical understanding while remaining open to novel insights derived directly from the empirical evidence.

To ensure the trustworthiness and consistency of the coding process, two trained coders independently analyzed a significant portion of the corpus, specifically 25% (approximately 23,000 words). Inter-coder reliability was then computed using Cohen's kappa, which yielded a value of .82, indicating substantial agreement between the coders (Landis & Koch, 1977). Any discrepancies identified during this initial independent coding phase were thoroughly discussed and resolved through consensus, leading to a refined and consensual codebook that was subsequently applied to the remaining data.

Reflexivity was a continuous practice throughout the entire analysis. The researchers, both possessing prior experience as mathematics educators, acknowledged that their backgrounds could potentially influence their interpretation of the data. To mitigate any potential expectancy bias, reflexive memos were diligently maintained after each coding session. Furthermore, weekly peer debriefings were scheduled, providing a structured platform to critically interrogate personal assumptions and ensure that the generation of themes remained rigorously data-driven and objective.

Through this iterative comparison and refinement process, five hierarchical metacognitive levels were generated: Understanding, Strategising, Executing, Verifying, and Concluding. These levels were retained as distinct themes only when supporting evidence was consistently observed across at least 75% of the participants. Crucially, the transitions between adjacent levels had to be clearly traceable within individual student protocols, demonstrating a logical progression. The emergence of specific sub-themes, such as "strategy rehearsal" or "error-driven revision," served to extend and enrich the preliminary theoretical framework rather than merely confirming it, thereby actively mitigating the risk of confirmation bias. Table 2 depicts the level of students' metacognition.

Table 2. Levels of student metacognition

Level	Description
Level 1: Understanding the problem	The problem solver fully understands the problem. The problem solver marks the essential part of the problem. The problem solver explores the information and understands the meaning of the problems.

Level	Description
Level 2: Thinking about the answer	The problem solver recalls the math concepts that they have studied and strategies that they have experienced.
	The problem solver decides on the concepts and strategies used to solve the problem.
Level 3: Comprehending how to answer	The problem solver carries out the math concepts and effective strategies to solve the problem.
	The problem solver conducts monitoring to identify errors.
	The problem solver thinks about how to fix these errors. The problem solver fixes the errors.
Level 4: Finding the answer	The problem solver discovers the answer to the problem.
	The problem solver checks the correctness of the answer.
Level 5: Confidence in the answer	The problem solver gives a conclusion to every answer.
	The problem solver gains confidence in their answer.

In addition, we triangulated the data to improve the objectivity and reliability of the findings. To ensure the trustworthiness of our assessment, all authors collaborated in discussions with experts in mathematics education until a mutual agreement was reached. We ensured the accuracy and completeness of the data by administering the task in written form and transcribing each interview immediately after recording. The process of coding and recording the categories was also validated through discussions with several mathematics education experts.

3. RESULTS AND DISCUSSION

3.1. Results

After data analysis, the findings showed five levels of student metacognition in solving PISA-like problems: understanding the problem, considering possible answers, comprehending solution strategies, identifying the correct answer, and demonstrating confidence in the answer.

Table 3. Distribution of the subjects

Hierarchy Level	Origin Island	Numbers	Sum
First level: Understanding the problem	Sumatera	4	22
	Java	14	
	Sulawesi	4	
Second level: Thinking about the answer	Sumatera	3	18
	Java	11	
	Sulawesi	4	
Third level: Comprehending how to answer	Sumatera	2	16
	Java	11	
	Sulawesi	3	
Fourth level: Finding the answer	Sumatera	2	12
	Java	8	
	Sulawesi	2	

Hierarchy Level	Origin Island	Numbers	Sum
Fifth level: Being confident in the answer	Sumatera	1	8
	Java	6	
	Sulawesi	1	

Table 3 depicts the distribution of participants across metacognition levels by island of origin. In general, the data showed that as the metacognition level increases, the number of participants decreases. Thus, the hierarchy of student metacognition in solving PISA-like problems can be represented as a pyramid, as illustrated in Figure 6. The following section presents the results for each level of metacognition.

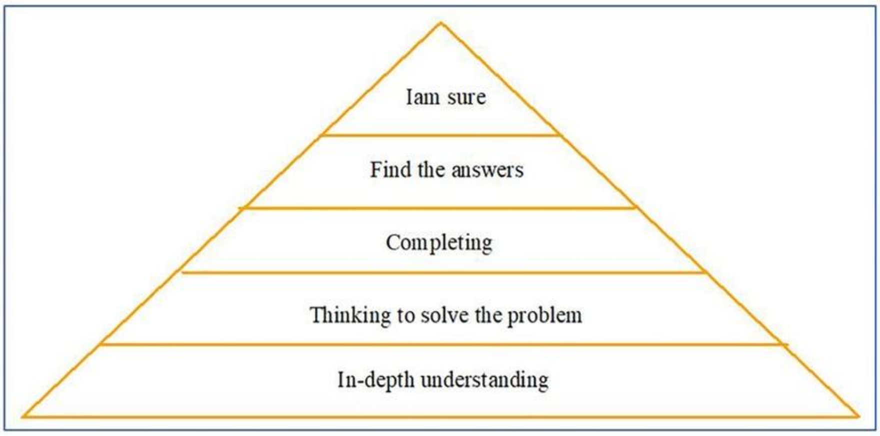

Figure 6. The hierarchy of student metacognition levels

3.1.1. Level 1: Understanding the Problem

To reach this level, the subject read the task carefully. The think-aloud transcript showed that after reading the task, the subject stated, “Well, I understand what is known and what is being asked.” This statement illustrated that the subject understood the given and required information. The subject then rewrote the known data to understand the problem. Unfortunately, the subject did not highlight key parts of the task or use mathematical symbols when reporting the data. Nevertheless, the subject understood the problem’s significance and was able to solve the problem properly. The supporting answer sheet is illustrated in Figure 7.

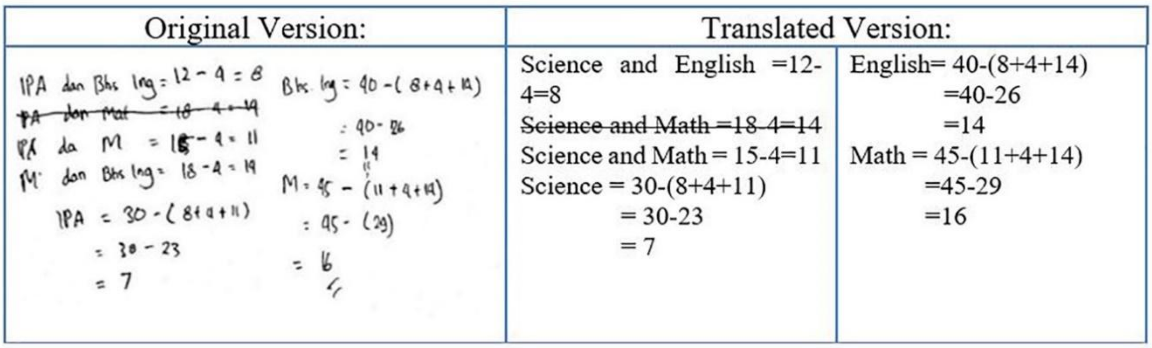

Figure 7. The data and problems as written by the subject

Figure 7 displays the subject's answer sheet for writing the task's data and problems. The subject wrote the essential data concisely to enhance understanding of the problem. They then reported the number of students who liked the lessons. In addition to considering the data and questions, the subject investigated alternative ways to understand the problem by rereading the task. These findings were obtained through in-depth interviews, as presented in the following interview transcripts:

Researcher : *"Did you understand the significance of the task?"*
Subject : *"Yes, sir."*
Researcher : *"What efforts did you do to understand the task?"*
Subject : *"First, I rewrote the data and questions (the subject shows the intended section on the answer sheet). Then I also read the task until I understood it well so that no understanding is missed."*
Researcher : *"What does it mean that you read the problem more than once?"*
Subject : *"Of course, I read the problem up to four times. I always do it to get a solid understanding of the meaning of the problem."*

3.1.2. Level 2: Thinking About the Answer

At this level, the subject considered and selected a problem-solving strategy based on the data obtained from the task. In determining their approach, the subject recalled relevant concepts previously learned, which facilitated the formulation of appropriate equations as a first step. This process enabled the subject to choose an effective strategy, specifically by applying the concept of Venn diagrams. The following think-aloud transcripts support the findings: "*To solve this problem, I employed diagrams, more precisely the concept of Venn diagrams. We first created the diagram (the subject drawing Venn diagram)*".

The subject understood that the Venn diagram concept was very suitable for solving this problem, as further supported by the interview transcript.

Researcher : *"Why did you choose the Venn diagram concept to solve the problem?"*
Subject : *"Because I have experience solving similar problems. The idea of the Venn diagram was exact. I employed it at the time."*
Researcher : *"Are there other concepts that might also be suitable for employment?"*
Subject : *"Maybe, but I don't know it. I choose to involve my experience in solving problems."*

The interview transcripts revealed that the subject selected the Venn diagram concept based on prior experience. Venn diagrams were perceived as effective tools for solving equivalent problems. Although the subject acknowledged that alternative methods might exist, they were unfamiliar with them. The findings show that once problem-solvers understand the problem, they can think of the correct solution.

3.1.3. Level 3: Comprehending How to Answer

At the third level, the subject applied the chosen strategy systematically. Unfortunately, the subject did not monitor the answer during the problem-solving process. Figure 8 illustrates that the subject was able to substitute data into the Venn diagram, indicating an understanding that the solution was progressing appropriately.

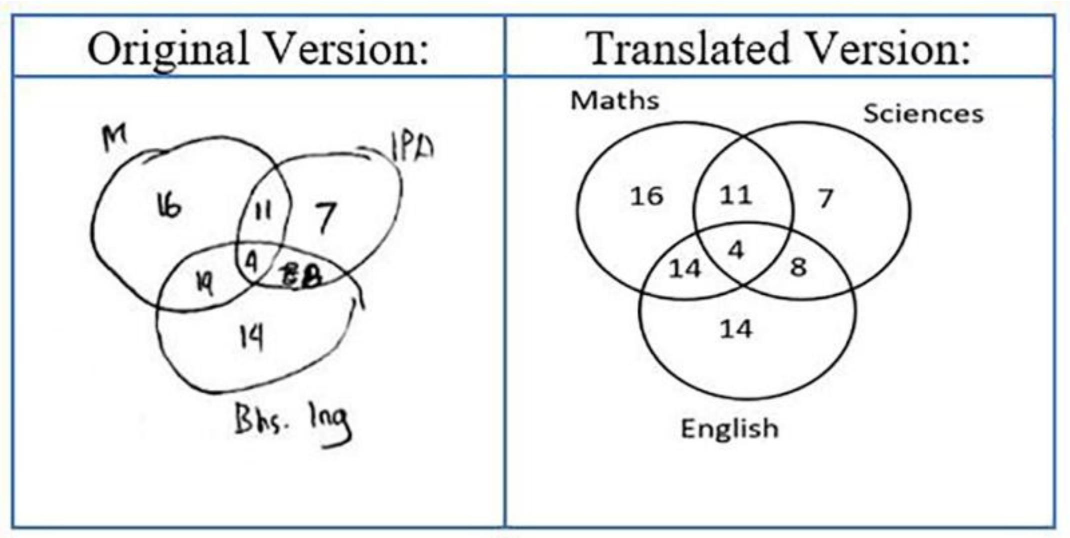

Figure 8. The subject understood the concept of Venn diagrams

The interview transcripts show that the subject mastered the completion process, as illustrated in the following excerpt:

Researcher : *"Do you have difficulty solving this problem? The completion steps that you continue are appropriate."*
Subject : *"I have no difficulty. Only my answer was wrong, then I rethought the answer."*
Researcher : *"Where were the mistakes?"*
Subject : *"When I determine the number of students who like Sciences and Mathematics."*
Researcher : *"What efforts did you make to overcome these difficulties?"*
Subject : *"I did a monitoring and correction to gain confidence in my answers."*

3.1.4. Level 4: Finding the Answer

To reach this level, the subject wrote down the final results of the problem, indicating that the problem was solved appropriately. According to the answer guidelines, the subjects' answer sheet shows the ability to solve the problem:

Original Version	Translated Version
jumlah siswa = 80 menyukai matematika = 45 " bahasa inggris = 40 " IPA = 30 matematika dan b. inggris = 18 IPA dan matematika = 15 IPA dan b. inggris = 12 IPA, b. Inggris, matematika = 4 Ditanya : a. siswa yang menyukai matematik saja b. " B. Inggris saja c. " IPA saja	The number of all students: 80 Students who like Math: 45 Students who like English: 40 Students who like Science: 30 Students who like Math and English: 18 Students who like Science and Math: 15 Students who like Science and English: 12 Students who like Science, English, and Math: 4 Determine the number of: a. Students who like Math only b. Students who like English only c. Students who like Science only

Figure 9. The subject solved the problem appropriately

Figure 9 illustrates that the subject solved the problem correctly. To indicate completion, the subject underlined the answer with two lines. The answers were obtained by following the problem-solving steps using the given data. The findings show that the subject understood the problem, identified the correct steps, executed the strategy, and arrived at the correct solution, although it required more time.

3.1.5. Level 5: Confidence in the Answer

To be at this level, after finding the answer, the subject wrote a conclusion, indicating confidence in the answer, as shown in Figure 10.

Original Version	Translated Version
a. Siswa yang menyukai mtk saja = 16 siswa b. Siswa yang menyukai b.ing saja = 14 siswa c. Siswa yang menyukai IPA saja = 7 siswa	The number of a. Students who like Math only: 16 b. Students who like English only: 14 c. Students who like Science only: 7

Figure 10. The subject wrote the conclusion of the answer

Figure 10 illustrates that the subject wrote the final result in complete sentences. The following think-aloud transcript supports this: "So, students who like mathematics have 16 students, English 14 students, Science 7 students". The word "so" implies that the subject had confidence in the answer. The findings show that the subject understood the problem, considered appropriate steps, correctly executed the strategy, appropriately solved the problem, and demonstrated confidence in the answer.

3.2. Discussion

In level 1, the interview transcripts indicate that the subject understood the task well. The problem-solving activities included contemplating the data questions and rereading the task. Indicators for level 1, understanding the problem, were adopted from the awareness aspect of metacognition (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). Awareness is displayed by a student's ability to understand the problems based on previous experience (Christ & Dreesmann, 2022). In mathematical problem-solving, students can discover problems because they can understand concepts. The students, in general, displayed indicators of level 1. They understood the problem by reading the text carefully and repeatedly (Sari & Valentino, 2017) and determined appropriate solution strategies based on their understanding of the problem and its purpose (Yorulmaz et al., 2021). These findings align with Purnomo et al. (2017), who found that the capability to determine problem-solving strategies emerges in diverse approaches that ultimately lead to judgments.

The second metacognition level, thinking about the answer, draws from both awareness and regulation aspects, which are key drivers in mathematical problem-solving (Anggo, 2011). Students' answers are influenced by their awareness of the problem and their ability to process their memory. This aligns with Kuzle (2013), who emphasized the importance of connecting new information with existing understanding. Effective problem-solving requires students to recall appropriate concepts and apply them efficiently. Students typically demonstrate competence in selecting suitable strategies such as formulas, tables, or other methods to solve the problems (Puente-Díaz et al., 2021). However, due to varying individual abilities, the chosen strategies or formulas often differ (Jiang et al., 2020).

The interview transcript from level 3 reveals that the subject's problem-solving steps are correct. The findings show that after understanding the problem and identifying an appropriate solution, the subject was able to implement the problem-solving strategy effectively (See Figure 8). However, the subject did not revisit or monitor the answer during the process. Once a student encounters a way to answer a problem, they execute their strategies (Sengul & Katranci, 2015). At the third metacognition level, comprehending how to answer, the student applies their formula. Their final steps may involve written explanations, procedures, or images. Indicators for this level are adopted from elements of regulation and evaluation, focusing on the student's calculation proficiencies. The student can replace the known data in the formula. This aligns with Jayapraba (2013), who stated that in implementing a solution strategy, a student must be able to substitute information into the settlement formula. The student can then proceed with the working process by applying their knowledge and abilities. It is necessary to process previously known information regarding the concepts, operations, and formulas identified in the problem (Kuzle, 2013).

Students find the answer at the fourth metacognition level. The indicator for this level is an embraced answer, demonstrating the metacognitive activity of evaluation. Evaluation involves assessing decisions, recognizing the significance of the selected strategies, and scrutinizing limitations in the problem-solving process (Bakar & Ismail, 2019; Lingel et al., 2019; Magiera & Zawojewski, 2011; Schneider & Artelt, 2010; Toit & Kotze, 2009). In general, students can find answers to each question. This aligns with Masduki et al. (2020), who state that students find the solutions or answers by implementing measured strategies.

However, the student was unable to solve the second problem. According to Sutama et al. (2021), failure to find a solution may be influenced by a lack of passions and motivations. Another issue is students' lack of mindfulness (Shute, 2019) and their tendency to spend more time evaluating their strategies (Wong, 1992). Consequently, students may struggle to understand or determine appropriate strategies (Vorhölter, 2021). Thus, improvements are needed in completing problem exercises related to PISA content.

The fifth metacognition level is confidence in the answer. A study by Sa'dijah et al. (2020) showed that the subject's conclusions are influenced by their capability to examine and evaluate problems. By writing a conclusion, students showed confidence in the results obtained (Purnomo et al., 2017). According to our analysis, students' confidence can be observed through the think-aloud video recordings or their answer sheet. Interview results also showed that the students were confident in their answers. Another indicator of this level of metacognition is whether students re-examine their work as a whole to determine the correctness of their answers. This may occur because metacognition helps reduce anxiety (Dragan et al., 2012). In general, the students did not recheck their answers, although interviews revealed they were confident about them. This aligns with other research stating that confident students who can solve problems tend not to recheck their written answers (Fleur et al., 2021).

4. CONCLUSION

This research explored students' metacognition and produced a hierarchy of metacognition levels in solving PISA-like problems. The first level involves understanding what is known and what is being asked in the problem. The second level entails connecting new concepts and strategies to prior knowledge. The third level comprises utilizing the concepts and executing the strategy to find the answer. The fourth level involves obtaining the answer and verifying its correctness. The fifth level is marked by confidence in the answer, supported by justification and a written conclusion.

Further research can focus on defragmenting student metacognition during problem-solving to improve it. To achieve this, it is necessary to identify specific steps for refining student metacognition at each level.

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Writing - review and Editing; KF: Supervision, and Validation; SM: Formal analysis, Methodology, and Writing - review and Editing; MI: Formal analysis, Methodology, and Writing - review and Editing; APW: Formal analysis, Methodology, and Writing - review and Editing; AP: Formal analysis, Methodology, and Writing - review and Editing; AH: Formal analysis, Methodology, and Writing - review and Editing.

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