

## Mapping errors in solving linear equations: A hermeneutic phenomenological study

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### Abstract

Many students struggle with solving linear equations, especially in translating word problems into algebraic expressions. While previous studies have focused on identifying procedural errors, they often overlook deeper cognitive and interpretative factors that influence students' problem-solving capabilities. This study addresses that gap through a hermeneutic phenomenological approach to examine how students' perceptions and experiences shape their understanding and approach to linear equations. Data were collected from 37 seventh-grade students at a public junior high school in West Sumatra, Indonesia, through written tests and semi-structured interviews. As a qualitative phenomenological study, the participants were selected based on the relevance of their experiences. Analysis revealed that students primarily committed conceptual, procedural, and resultant errors. Conceptual errors stemmed from misunderstandings of mathematical concepts, procedural errors incorrect application of mathematical operations, and resultant errors occurred in the final solutions due to earlier mistakes. The findings emphasize the importance of addressing both cognitive and interpretative challenges in teaching linear equations. This study contributes to the existing literature by offering insights into factors influencing students' learning processes and highlighting teaching strategies that go beyond merely correcting technical errors. These findings can inform educators in designing more effective approaches that consider students' cognitive and interpretative needs, ultimately improving problem-solving skills and mathematical understanding.

### Keywords:

Conceptual understanding, Hermeneutic phenomenology, Linear equations, Procedural mistakes, Student errors

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## 1. INTRODUCTION

Linear equations are a fundamental skill in algebra and a core component of the mathematics curriculum in secondary schools worldwide (Smith et al., 2022). They play a crucial role in developing problem-solving skills, cognitive abilities, and abstract reasoning,

which form the foundation for learning more complex mathematical concepts (del Carmen et al., 2024; Supianti et al., 2022). In addition, linear equations prepare students for success in highly demanded fields such as science, technology, engineering, and mathematics (STEM), which are becoming increasingly important in the modern era (Seage & Türegün, 2020). Despite the widespread recognition of their importance, many students face significant challenges in solving linear equations, particularly in translating word problems into the correct algebraic expressions and applying mathematical operations accurately. These difficulties often lead to recurring mistakes that hinder students' overall understanding of algebraic concepts (Jupri & Drijvers, 2016). Mathematical errors, whether conceptual or computational, can disrupt the learning process and limit students' ability to grasp more advanced concepts, which are crucial for real-world applications (Hu et al., 2022; Putri et al., 2024).

Although linear equations are foundational to mathematics education, many students encounter persistent difficulties. Specifically, they often struggle to convert word problems into correct algebraic expressions (Santos, 2022; Tatira, 2023). Such errors are not limited to calculations but frequently reflect deeper misunderstandings of basic algebraic concepts (Johari & Shahrill, 2020). In this context, a more profound challenge lies in how students interpret and understand mathematical problems, which ultimately affects how they solve linear equations (Gryaznov et al., 2024). Research has identified various types of errors, including misapplications of mathematical operations or incorrect formulation of equations from word problems (Siregar et al., 2025). However, most studies focus on procedural or technical aspects of error analysis. This highlights the need to explore more deeply the cognitive and interpretative factors that influence how students approach and solve mathematical problems, especially in the context of linear equations (Azizah et al., 2022; Fardian, Suryadi, Prabawanto, et al., 2025; Putri, Juandi, Herman, et al., 2025).

Previous research has significantly contributed to identifying the types of errors students make in solving linear equations. del Carmen et al. (2024) found that one of the most prevalent challenges is difficulty in formulating equations from word problems, often due to a lack of understanding of the relationship between real-world situations and algebraic representations. While this study provides an overview of the frequency of errors, it does not examine the underlying causes in depth. Expanding on these findings, Izsák and Beckmann (2022) investigated students' conceptual understanding of linear equations, focusing on the errors arising from misunderstandings related to coefficients and variables. Using interviews, the study identified conceptual errors that influence how students solve algebraic problems in general.

Additionally, Sandoval et al. (2023) found that despite receiving more detailed instructions, students still struggled to identify and correct their own errors. Through a task-based approach, this research showed that errors frequently occur during the checking stage, where students fail to recognize mistakes even when given time to review. This indicates that errors are not solely technical issues but are also related to students' cognitive abilities to assess their work. Andrews and Kaplan (2020), analyzing PISA and TIMSS results, revealed that similar difficulties in understanding and solving linear equations exist in various countries, including Sweden. They emphasize the importance of a systematic approach to teaching,

noting that these challenges involve more than technical obstacles. They reflect broader conceptual relationships in mathematical. This reinforces that, despite various efforts, challenges in understanding linear equations persist. On the other hand, Qetrahi et al. (2021) introduced a new approach to teaching linear equations based on the concept of equivalence. Their study shows that this approach helps students more effectively grasp the relationships between steps in solving equations, both procedurally and conceptually, and addresses common difficulties. Finally, Smith et al. (2022) studied the development of students' symbolic representation in solving systems of linear equations and found that stronger symbolic skills can strengthen students' understanding of linear equations and enable them to tackle more complex problems.

In the Indonesian context, similar learning difficulties have also been extensively documented, particularly in the domains of algebraic reasoning and functional thinking. Fardian et al. (2024) revealed that students frequently encounter learning obstacles arising from the gap between arithmetic and algebraic reasoning, especially when interpreting contextual problems and transforming them into symbolic equations. Their findings further suggest that these obstacles are influenced by both cognitive limitations and didactical factors within classroom practices. Likewise, Utami et al. (2023) highlighted that secondary school students face persistent challenges in identifying, extending, and generalizing patterns, reflecting a limited development of functional thinking as a foundation for algebraic reasoning. Collectively, these studies demonstrate that the nature of students' algebraic difficulties in Indonesia extends beyond procedural shortcomings and encompasses deeper conceptual and contextual dimensions.

Although previous research has identified various errors students make when solving linear equations, most studies focus on technical and cognitive aspects, typically classifying errors as conceptual or computational using quantitative methods (Elkjær & Jankvist, 2021). This approach often overlooks deeper influences, such as how students interpret mathematical problems and the cognitive and psychological factors shaping their problem-solving process. Thus, there is a need for a more holistic approach that considers not only technical errors but also interpretative factors and personal experiences impact problem-solving. Additionally, much of the existing research has not explored how students' subjective experiences affect their understanding of linear equations (Planas et al., 2024). This study adopts a hermeneutic phenomenological approach to investigate the cognitive and interpretative processes involved in solving linear equations. It aims to explore how students' perceptions influence their ability to solve problems and identify the underlying factors contributing to errors. In addition, this research seeks to improve teaching methods by recognizing the factors affecting students' approaches to mathematical problems, helping educators design more effective strategies aligned with their cognitive and interpretative needs.

The main objective of this study is to explore the cognitive and interpretative factors influencing student errors in solving linear equations, focusing on how students understand and interpret mathematical problems. It aims to identify aspects that have been underexplored in prior research, particularly subjective factors and personal experiences that affect problem-solving. Theoretically, this research aims to deepen our understanding of the cognitive and interpretative processes in solving linear equations and contribute to the development of more

holistic mathematical education theories. Practically, it offers valuable insights into designing teaching methods that address both technical errors and conceptual understanding, while incorporating students' personal experiences in mathematical problem-solving.

## 2. METHOD

### 2.1. Research Design

This study adopted a hermeneutic phenomenological approach as outlined by Folgueiras Bertomeu and Sandín Esteban (2023) and Sloan and Bowe (2014) to explore the challenges faced by Indonesian junior high school students in solving linear equations. Hermeneutic phenomenology emphasizes understanding individuals' lived experiences and interpreting the meanings they ascribe to those experiences. This approach is particularly suitable for this research, as it seeks to uncover not only the types of errors students make but also the underlying cognitive, pedagogical, and contextual factors contributing to these errors.

By focusing on students' personal interpretations and experiences, this research aims to provide a deeper qualitative insight into their learning processes. Unlike traditional quantitative approaches, which often focus on statistical trends, hermeneutic phenomenology enables a richer understanding of the nuances in students' mathematical reasoning and their struggles with abstract concepts. This methodological choice aligns with the study's objective to bridge the gap between theoretical understanding and practical application in mathematics education, particularly within the Indonesian context.

### 2.2. Participant and Data Collection

The participants in this study consisted of 37 seventh-grade students from a junior high school located in West Sumatra, Indonesia. The school is a public institution situated in an urban area, characterized by moderate academic achievement and a diverse socioeconomic student population. These students were purposefully selected based on their current engagement with the topic of linear equations in their mathematics curriculum. The selection criteria aimed to capture a diverse range of experiences and challenges encountered by students during their initial exposure to this foundational algebraic concept.

To comprehensively understand the students' experiences and difficulties, the data collection process was conducted in two stages: Students were assigned a test consisting of four problems specifically designed to assess their understanding of linear equations. The test was validated by two experts in mathematics education to ensure its theoretical alignment with the cognitive and developmental levels of middle school students. It was administered over approximately forty minutes, during which students were not allowed to use calculators or collaborate with their peers. This controlled setting ensured that the responses reflected each student's individual understanding and problem-solving abilities.

Following the test, a subset of students participated in semi-structured interviews to gain deeper insights into their thought processes, reasoning strategies, and interpretations of the problems. Seven students were selected for individual interviews based on an initial analysis of their written test answers, which aimed to represent a range of responses, including both correct and incorrect solutions. This selection allowed the researchers to gather detailed

information regarding the reasoning behind each student's answers to the test questions. During the interviews, each student's written responses were presented, and they were encouraged to explain their reasoning in detail. The interviews focused on understanding the students' comprehension of the problems, the steps they took to solve them, and any challenges they encountered. To guide the interviews, a set of general questions was prepared, including: "What do you understand about this question?", "Can you explain the information and question stated in this problem?", "What steps did you take to solve this problem?", and "Did you face any difficulties while solving this problem? If so, what were they?". The semi-structured format allowed for flexibility, enabling the interviewer to ask follow-up questions to clarify students' explanations and reasoning. This process provided a richer understanding of students' thought processes, their conceptual understanding, common errors, and the reasoning behind their approaches to solving linear equation problems.

Prior to data collection, the ethical aspects of this study were carefully reviewed to ensure compliance with research ethics standards. Official permission was obtained from the participating school. All participants provided informed consent in accordance with national ethical standards before taking part in the study. They were clearly informed about the study's purpose, procedures, and their right to withdraw at any time without consequence. Throughout the research process, all participants were treated ethically in compliance with the standards of the American Psychological Association (1992). Participation was entirely voluntary, and confidentiality was maintained throughout the study. By triangulating data from these two sources, test results and interviews, the study aimed to achieve a holistic understanding of the cognitive and pedagogical factors contributing to students' difficulties with linear equations.

### 2.3. Data Analysis

The data analysis in this study was conducted using Atlas.ti version 9 software, a qualitative data analysis software designed to support the systematic organization and interpretation of textual and visual data (Woods et al., 2016). The software was utilized to manage and analyze data collected from students' written tests and semi-structured interviews. The analysis proceeded in the following stages. First, all written test responses and interview transcripts were digitized and imported into Atlas.ti. Interview recordings were transcribed verbatim to ensure the accuracy and completeness of the qualitative data. The transcripts were then reviewed to correct any errors or inconsistencies.

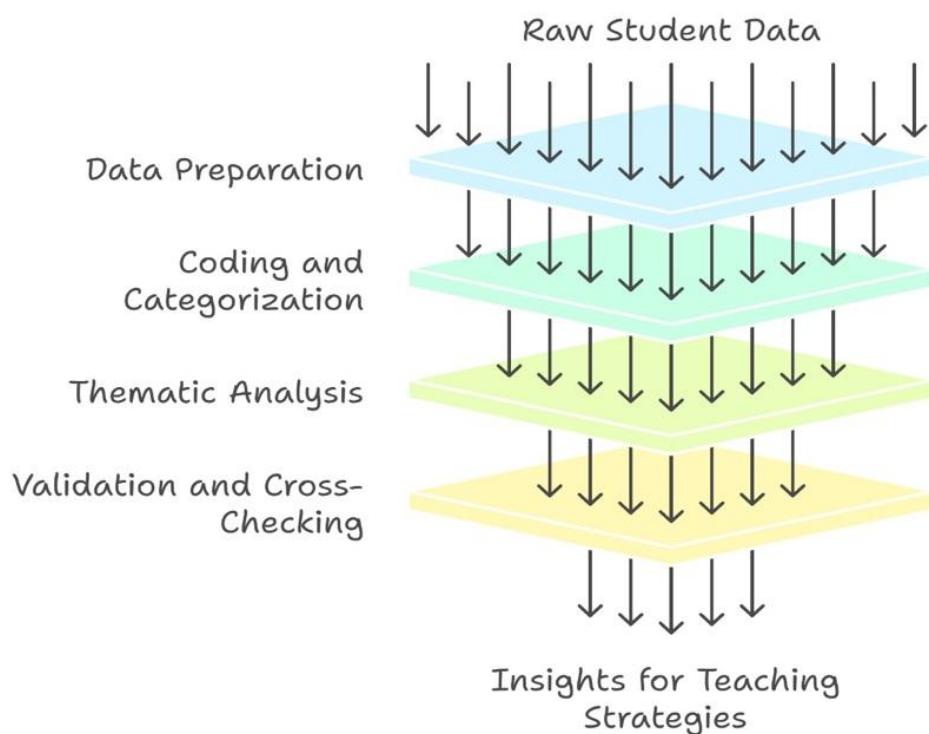
Subsequently, coding and categorization were performed using Atlas.ti. The data were systematically coded to identify recurring patterns, themes, and categories. The analysis focused on three main categories of errors: conceptual errors, procedural errors, and resultant errors. Conceptual errors reflected misunderstandings of mathematical concepts, such as misinterpreting variables or relationships. Procedural errors involved mistakes in solving equations, such as incorrect arithmetic or missing steps. Resultant errors occurred in the final solution, typically stemming from earlier conceptual or procedural mistakes. Initial open coding was performed to identify instances of these errors, which were then grouped into the three predefined categories for further analysis.

After categorization, a thematic analysis was conducted to examine relationships between the three error types and their underlying causes. Atlas.ti's visualization tools,

including network maps and co-occurrence tables, were used to identify patterns and connections among conceptual, procedural, and resultant errors. The identified themes and patterns were then interpreted to provide a comprehensive understanding of the cognitive and pedagogical factors influencing students' performance. Particular attention was given to how conceptual misunderstandings often led to procedural errors, which ultimately resulted in final errors.

To ensure the reliability of the analysis, coding and interpretations were cross-checked by a second researcher. Any discrepancies were discussed and resolved to maintain consistency and accuracy. Organizing errors into these three categories using Atlas.ti enabled a systematic and rigorous approach to identifying the root causes of students' difficulties with linear equations. This approach provided deeper insights into the specific challenges students face and informed strategies for effectively addressing these issues. [Figure 1](#) illustrates an overview of the entire data analysis process.

## **Analyzing Student Errors in Linear Equations**



**Figure 1.** Research process at each stage

### **3. RESULTS AND DISCUSSION**

#### **3.1. Results**

The analysis of students' responses to the linear equation problems highlights several recurring patterns of errors, categorized into conceptual errors, procedural errors, and resultant

errors following the classification proposed by Bernard and Bright (1984) and Carry et al. (1979). These categories were deductively derived from these theoretical frameworks to guide the classification and interpretation of students' responses. These errors reflect the students' struggles to understand mathematical concepts, apply systematic procedures, and verify their solutions. By closely examining their written answers, it becomes clear that students often relied on intuitive or trial-and-error methods, which varied in their levels of accuracy and understanding. To illustrate the context and structure of the instrument used to analyze students' responses, the four test problems developed to assess their understanding of linear equations are shown in **Table 1**.

**Table 1.** Test instrument problems for assessing students' understanding of linear equations

No	Problem Statement
1	When a whole number is multiplied by 2 and then 15 is added to the result, the final answer becomes 27. Determine the whole number!
2	If you do not like eating fruit, drinking fruit juice is a good way to start a healthy lifestyle. By drinking fruit juice regularly, you can meet the daily recommendation for consuming fruits and vegetables, and it may even help prevent certain illnesses.

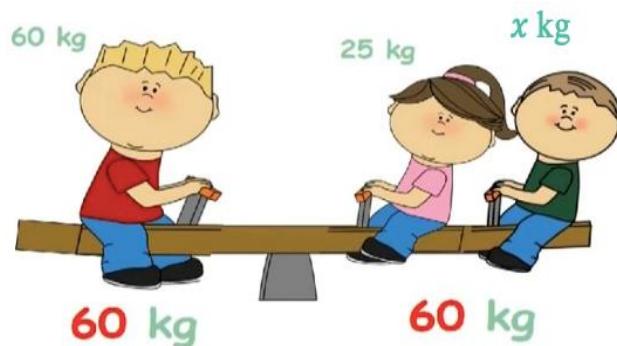


During the school anniversary event, your class set up a fruit juice booth and sold the juice for Rp7,000.00 per glass. The profit earned is equal to the total revenue from selling the juice minus the booth construction cost. The cost of building the booth is Rp90,000.00. Determine the **minimum number of glasses of juice** that must be sold to earn a **profit of Rp400,000.00!**

- 3 There are various kinds of play equipment in a playground such as swings and seesaws. Playgrounds are places where children can gather, socialize, communicate, and reduce dependence on smartphone games. Therefore, taking children to a playground is one way to reduce smartphone addiction. One day, Bela went to the playground with her mother. At the same time, Tomi and Aldi were also visiting the playground. Bela weighs 25 kg and Tomi weighs 60 kg. They sit on opposite sides of a seesaw, but the seesaw is unbalanced.



No	Problem Statement
	Then Aldi comes and sits together with Bela, causing the seesaw to become balanced.



Determine **Aldi's weight** so that the seesaw is balanced!

- 4 You are making your own salad dressing. The recipe below is for **100 mL** of dressing:

Ingredient	Amount
Salad Oil	60 mL
Vinegar	30 mL
Soy Sauce	10 mL

How many milliliters of salad oil are needed to make **150 mL** of the dressing?

**Table 2** illustrates the indicators of students' abilities in solving linear equation problems based on the analysis results of the test instrument, which includes four presented problems. Each indicator, category, and code provided in the students' responses displayed in **Table 2** was processed using the Atlas.ti application. The code label "1-C<sub>1</sub>" refers to a problem in Question 1, categorized under conceptual errors (C), and is the first code (1), specifically "Misinterpreting whole numbers".

**Table 2.** Indicators, categories, and codes in students' responses based on test results

Indicator	Category	Code	Code Definition	Number of Students
Understanding the definition of linear equations	Conceptual errors	1-C <sub>1</sub>	Misinterpreting whole numbers	6
		1-C <sub>2</sub>	Lack of understanding of the concept of a linear equation with one variable	20
	Procedural errors	1-P <sub>1</sub>	Operational errors	11
	Resultant errors	1-R <sub>1</sub>	Failure to answer the question	9
		1-R <sub>2</sub>	Misinterpretation of information	12
Modeling mathematical problems into equations	Conceptual errors	2-C <sub>1</sub>	Question 1 Inability to interpret the problem narrative into a mathematical statement	23
		2-C <sub>2</sub>	Misunderstanding of large numbers without units	21
	Procedural errors	2-P <sub>1</sub>	Operational errors	29
		2-R <sub>1</sub>	Failure to answer the question	9
		2-R <sub>2</sub>	Misinterpretation of information	9

Indicator	Category	Code	Code Definition	Number of Students
<b>Question 3</b>				
	Conceptual errors	3-C <sub>1</sub>	Lack of understanding of algebraic methods	24
	Procedural errors	3-P <sub>1</sub>	Operational errors	17
<b>Question 4</b>				
Solving contextual problems using the properties of equations	Conceptual errors	4-C <sub>1</sub>	Failure to understand that the table represents proportional distribution, not fixed amounts	19
		4-C <sub>2</sub>	Lack of understanding of the concept of proportional comparison	23
	Procedural errors	4-P <sub>1</sub>	Failure to explain the relationship between the calculations and the final result	20
	Resultant errors	4-R <sub>1</sub> 4-R <sub>2</sub>	Failure to answer the question Misinterpretation of information	8 15

Based on the analysis presented in [Table 2](#), resultant errors were the least frequent type of error appearing in students' responses. In contrast, conceptual and procedural errors dominated the majority of students' answers in solving single-variable linear equation problems. Overall, students continued to struggle with solving contextual problems by utilizing the properties of equations and algebraic manipulation. Additionally, many students experienced difficulties in translating the given problems into mathematical statements.

To provide a clearer picture of the challenges students faced in solving these problems, [Figure 2](#) presents examples of written responses from two students for Question 1. These examples demonstrate the procedural steps taken by each student and highlight both conceptual and resultant errors. By analyzing these responses, the study reveals deeper insights into the difficulties and misconceptions encountered in solving a basic linear equation. Question 1 serves as a foundation for understanding the types of errors that occurred across all problems.

$  \begin{array}{l}  16 \times 2 = 12 + 15 \\  = 27  \end{array}  $	$  \begin{array}{l}  2x + 15 = 27 \\  2x = 27 - 15 \\  2x = 12 \\  \frac{2x}{2} = \frac{12}{2} \\  x = 7  \end{array}  $
(a)	(b)

**Figure 2.** Example responses from student 1 (a) and student 2 (b) for question 1

Based on [Figure 2](#), which presents the responses of two students to Question 1, several patterns of errors can be observed. These responses highlight the challenges students faced in understanding and solving a simple linear equation. The students' approaches varied, ranging from reliance on trial-and-error methods to more structured procedural attempts. Yet both demonstrated significant gaps in conceptual understanding, procedural accuracy, and result verification.

The analysis of responses to Question 1 reveals a recurring pattern of conceptual errors, in which students relied heavily on trial-and-error methods or incomplete reasoning to arrive at a solution. For instance, one student tested various numbers by multiplying them by 2 and adding 15, stopping only when the correct answer, 6, was reached. This approach reflects a lack of understanding of linear equations as a systematic tool to solve relationships between variables (code 1-C<sub>1</sub> and 1-C<sub>2</sub>). From a phenomenological perspective, this indicates that students struggled to grasp the meaning and utility of equations in simplifying numerical relationships. The inability to directly form the equation  $2x + 15 = 27$  suggests superficial engagement with the problem, focusing more on guessing than conceptualizing the underlying relationships.

From a hermeneutic perspective, this reliance on trial-and-error methods highlights a misinterpretation of the problem's numerical components, likely stemming from weak foundational knowledge of number operations and their roles in equations. For some students, the procedural approach appeared more structured, as they successfully formed the equation  $2x + 15 = 27$  and applied step-by-step procedures to isolate x. However, in certain cases, resultant errors emerged (code 1-R<sub>1</sub> and 1-R<sub>2</sub>). As shown in the provided student responses in [Figure 1](#), while the steps to isolate x were correct, the final answer was incorrect ( $x = 7$  instead of  $x = 6$ ). This suggests a failure to verify the solution by substituting it back into the original equation. From a phenomenological viewpoint, this error may stem from a lack of reflection or confidence in engaging with the problem beyond procedural steps. From a hermeneutic perspective, the narrative of the problem might have been misinterpreted, with the student focusing on mechanical procedures rather than deeply understanding the context or the relationships among the equation's components.

Procedural errors were also observed in cases where students misunderstood basic arithmetic operations, such as addition and multiplication, further complicating their attempts to isolate x (code 1-P<sub>1</sub>). From a phenomenological perspective, this reflects a gap in their lived experiences with arithmetic, hindering their ability to apply basic rules confidently. Hermeneutically, these errors may reflect a misinterpretation of the problem's context, as students struggled to connect the narrative of the question to its mathematical representation. Furthermore, students who failed to provide an answer at all demonstrated a lack of engagement with the problem, possibly due to unclear problem narratives or insufficient foundational mathematical knowledge.

The analysis proceeds with Question 2, which presents new challenges involving proportional reasoning and the contextual application of mathematical operations. [Figure 3](#) provides examples of written responses from two students for Question 2, illustrating their approaches and common errors in solving the problem.

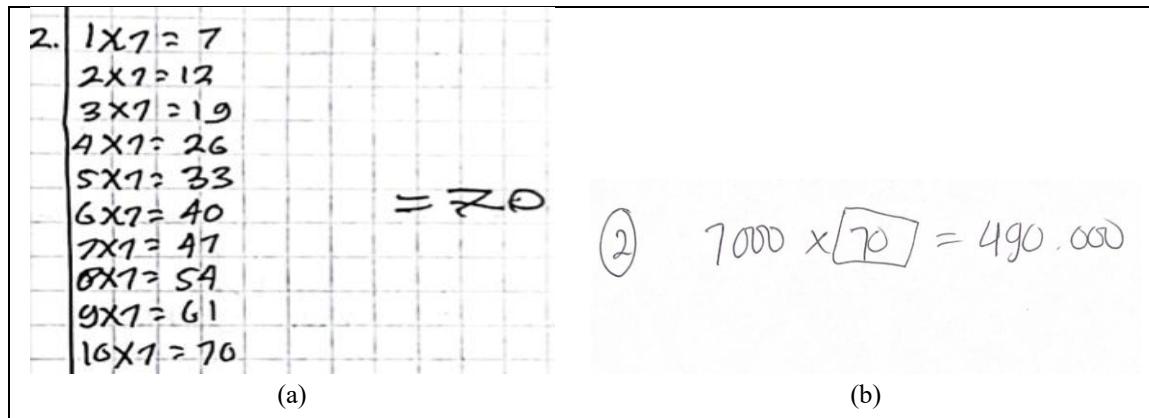


Figure 3. Example responses from student 3 (a) and student 4 (b) for question 2

Question 2 required students to calculate the minimum number of fruit juice glasses needed to achieve a profit of Rp400,000, considering a selling price of Rp7,000 per glass and a setup cost of Rp90,000. This problem introduced a real-world context, requiring students to apply proportional reasoning and basic arithmetic operations. The analysis of students' responses revealed several recurring errors, which were further explored through follow-up interviews.

Based on Figure 3, it can be observed that Student 3 solved the problem using a basic multiplication method, listing multiples of 7 from 1 to 10 to arrive at the final answer of 70. Instead of forming a mathematical model, the student relied on a trial-and-error approach, multiplying various numbers by 7 until approximating the desired revenue (code 2-C<sub>1</sub>). This method often led to errors in calculation and a lack of verification (code 2-R<sub>1</sub> and 2-R<sub>2</sub>). For example, several mistakes occurred in the multiplication results, such as  $2 \times 7 = 12$  (should have been 14) and  $3 \times 7 = 19$  (should have been 21). According to the interview with the student, these errors were caused by carelessness in recording the results rather than a lack of understanding of multiplication concept (code 2-P<sub>1</sub>). The following dialogue illustrates the student's explanation:

*Researcher : I would like to ask about your answer to Question 2. Can you explain how you arrived at your solution?*

*Student 3 : I multiplied the numbers from 1 to 10 by 7. So, I wrote 1 times 7 equals 7, then continued with 2 times 7 equals 14, and so on.*

*Researcher : I see some numbers here. For example, in the second row, you wrote that 2 times 7 equals 12. How did you get this number?*

*Student 3 : Oh, yes. It should have been 14. I made a mistake there.*

*Researcher : Did you realize this mistake when you were working on it?*

*Student 3 : No, I just realized it now.*

This dialogue illustrates the student's reliance on mechanical methods without critically evaluating their calculations. The student did not notice the error during problem-solving, indicating a lack of verification and reflective practice. Additionally, Figure 3 shows that Student 4 did not consider the setup cost when calculating the required profit (code 2-C<sub>1</sub>), leading to a misunderstanding of the relationship between revenue, cost, and profit. The

student initially calculated the total revenue correctly but neglected to subtract the setup cost, resulting in an incomplete understanding of the problem. This issue was clarified during the interview:

*Researcher : Did you consider the cost of setting up the stand in your calculations?*

*Student 4 : Oh, no. I only calculated the total revenue without including the setup cost.*

*Researcher : If we subtract Rp490,000 by the setup cost of Rp90,000, what profit would you get?*

*Student 4 : It should have been Rp400,000, right, Ma'am?*

This exchange highlights a conceptual gap in understanding how costs and revenues interact to determine profit. The student's initial response reflects an incomplete grasp of the problem's requirements, while their reaction during the interview suggests a need for instructional strategies that emphasize connecting mathematical calculations to real-world contexts. This finding also indicates that students' errors were not solely cognitive but were influenced by pedagogical and contextual factors as well. Pedagogically, students' reliance on mechanical procedures stems from earlier learning experiences in elementary and lower secondary classrooms that emphasized procedural fluency over conceptual understanding. Several students explained that they had never encountered similar problem types in their previous schooling, indicating that limited exposure through textbooks, teaching practices, or everyday contexts affected their ability to interpret such problems. Contextual influences also played a role, as unfamiliar problem situations made it difficult for students to connect real-world contexts with algebraic representations. These combined factors illustrate how students' difficulties emerged from broader educational and experiential backgrounds, not just from their immediate reasoning processes.

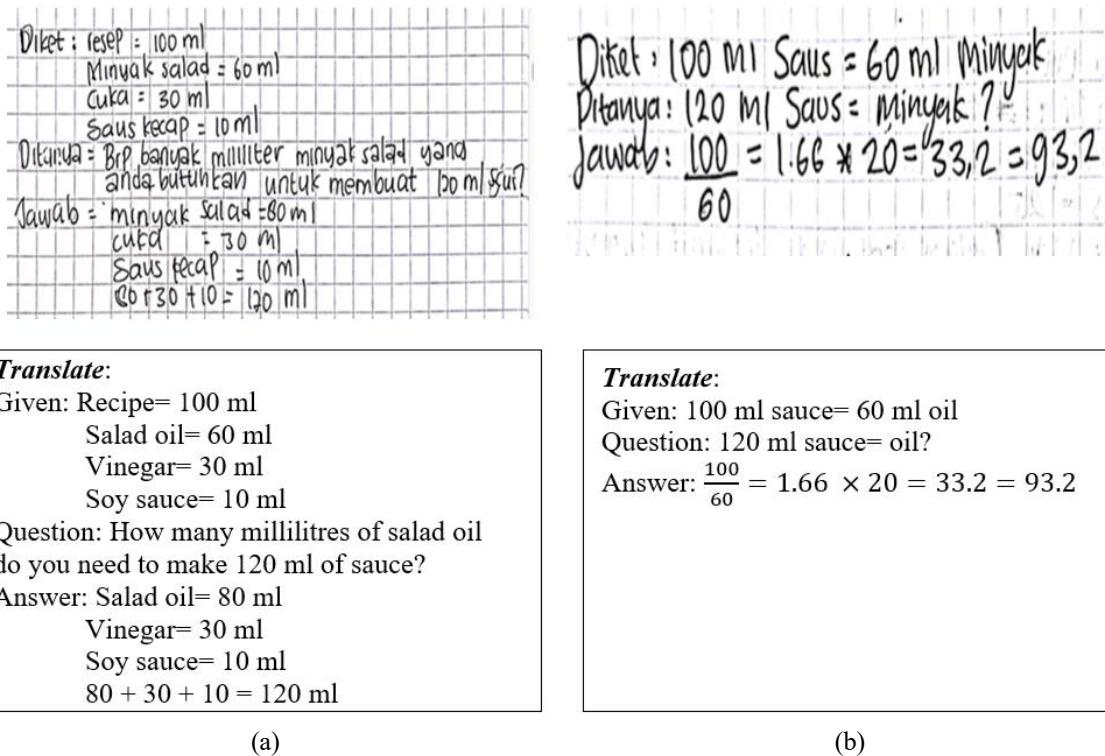
The analysis of Question 3 focuses on students' approaches to solving the equation  $x+25=60$ . This question required students to apply algebraic reasoning to isolate  $x$  and determine its value. However, some students relied on arithmetic methods rather than algebraic manipulation, using trial-and-error to identify the correct answer (code 3-C<sub>1</sub>). [Figure 4](#) presents an example of a student's written response to Question 3, illustrating their approach and the challenges encountered.

**Figure 4.** Example responses from student 5 for question 3

Question 3 required students to solve the equation  $x + 25 = 60$  by identifying the value of  $x$  that satisfies this equation. The correct solution involves isolating  $x$  by subtracting 25

from both sides, yielding  $x = 35$ . However, the student's response, as shown in Figure 4, reveals that they did not use algebraic methods to solve the problem. Instead, the student relied on arithmetic reasoning and a trial-and-error approach to determine the number that, when added to 25, equals 60. From the response, it is clear that the student calculated  $60 - 25 = 35$  directly, without explicitly expressing this operation as part of an algebraic process (code 3-P<sub>1</sub>). Rather than isolating  $x$  using algebraic manipulation, the student appeared to have guessed possible values for  $x$  and verified them by adding to 25. While the final answer of 35 is correct, this approach demonstrates a lack of understanding of algebra as a systematic tool for solving equations.

After analyzing the responses to Question 3, which revealed students' reliance on arithmetic reasoning rather than algebraic methods, the focus now shifts to Question 4. This problem required students to apply proportional reasoning to adjust ingredient quantities in a recipe as the total amount changed. Figure 5 illustrates a student's response to Question 4, providing insight into their approach and the challenges encountered.



**Figure 5.** Example responses from Student 6 (a) and Student 7 (b) for Question 4

Student 6 solved the problem using an arithmetic approach. They began by calculating the difference between the 120 mL sauce recipe and the original 100 mL version, identifying a difference of 20 mL. This difference was then added to the initial amount of salad oil, which was 60 mL, resulting in an adjusted quantity of 80 mL. Finally, the student combined all adjusted ingredient quantities, including vinegar and soy sauce, to ensure the total matched the required 120 mL.

Another student approached the problem using a proportional method, but with a slightly different calculation process. They started by dividing the original quantity of 100 mL sauce by the 60 mL salad oil, resulting in a ratio of approximately 1.66. The ratio was then

multiplied by the additional 20 mL difference (i.e., 120 mL - 100 mL), yielding an adjusted value of 33.2 mL. Adding this to the original 60 mL salad oil, the student calculated the final amount as 93.2 mL.

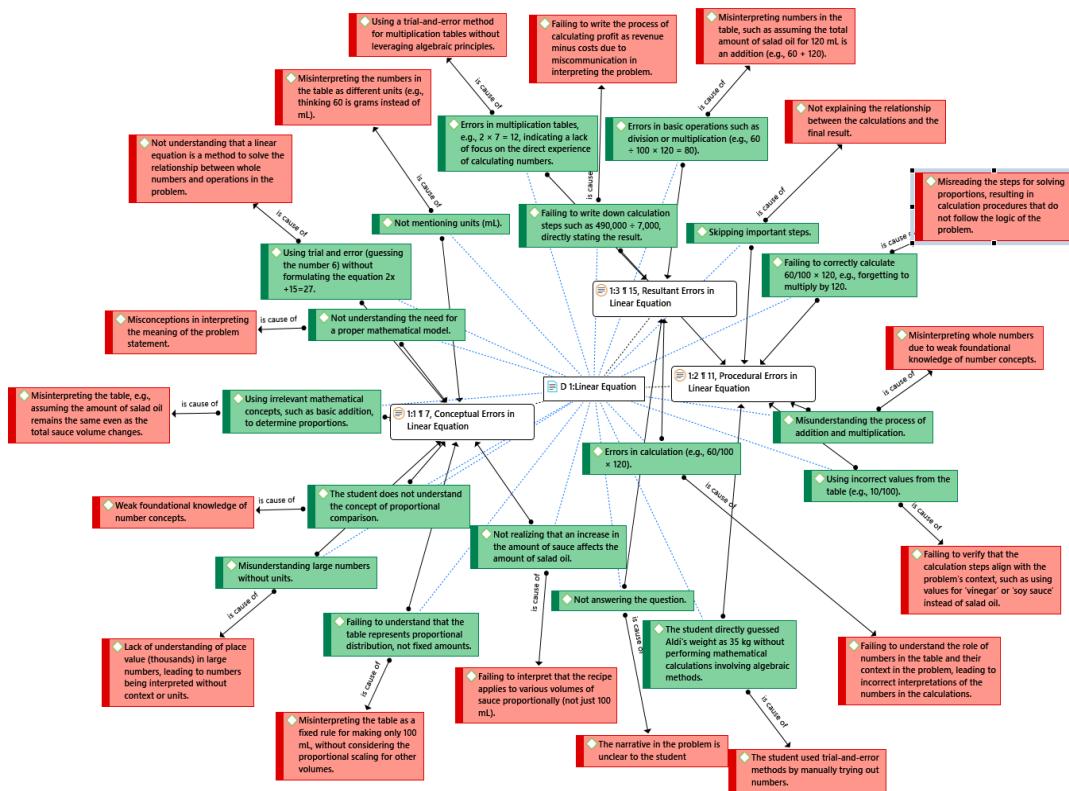
Based on interviews conducted with students, the challenges they encountered in solving mathematical problems were not solely due to difficulties in understanding specific concepts. Their reliance on habitual problem-solving methods also contributed. For instance, one student admitted to preferring trial-and-error approaches when working on algebraic equations, as they found it easier than systematically applying algebraic rules. This reliance on informal strategies highlights a gap in their ability to generalize problem-solving methods to different contexts.

Additionally, interviews revealed that some students misinterpreted the narratives of contextual problems, particularly those involving proportional reasoning (code 4-C<sub>1</sub> and 4-C<sub>2</sub>). One student explained their struggle to connect mathematical operations to the real-world context, stating, "I just calculated the numbers, but I didn't think about how they relate to the question" (code 4-P<sub>1</sub>). This response indicates a tendency among students to focus on numerical operations without fully grasping the broader relationships between variables.

The findings also suggest that students' challenges were compounded by a lack of confidence in verifying their solutions (code 4-R<sub>1</sub> and 4-R<sub>2</sub>). A student reflected, "I didn't check my answer because I wasn't sure how to confirm if it was correct." This lack of verification demonstrates the need for instructional strategies that emphasize reflective practice and encourage students to critically evaluate their answers. The interviews provide valuable insights into the cognitive and interpretative barriers faced by students, underscoring the importance of integrating contextual examples and encouraging critical reflection to bridge the gap between procedural fluency and conceptual understanding.

### **3.2. Discussion**

The findings of this study underscore the multifaceted challenges students face in solving problems involving linear equations and proportional reasoning. These challenges are categorized into conceptual errors, procedural errors, and resultant errors, as illustrated in [Figure 6](#), which maps the interconnections between these types of errors and their underlying causes.



**Figure 6.** Errors experienced by students in linear equations

As shown in Figure 6, these errors are closely interrelated: conceptual misunderstandings often lead to procedural errors, which in turn result in incorrect answers. The analysis highlights not only gaps in students' mathematical knowledge but also challenges in interpreting and applying mathematical concepts to real-world or abstract problems. In Figure 6, green nodes represent phenomenological themes, focusing on students' lived experiences and conceptual understanding, while red nodes represent hermeneutic themes, emphasizing interpretative reasoning and procedural missteps. The connections between nodes illustrate how phenomenological and hermeneutic challenges are interconnected and contribute to resultant errors.

From a phenomenological perspective, students' reliance on arithmetic reasoning demonstrates their preference for familiar numerical operations over algebraic abstraction. This suggests that students often perceive algebraic problems as straightforward arithmetic challenges rather than as opportunities to apply algebraic methods. Such perceptions are influenced by various factors, including the developmental progression of algebraic thinking, the role of representations in problem-solving, and instructional approaches (Jupri et al., 2014; Putri, Juandi, Turmudi, et al., 2025). In the framework proposed by Brousseau (2002), errors that occur due to limitations in the initial context or situation when students learn a concept are categorized as epistemological obstacles. These epistemological obstacles become evident when students' understanding is effective within the context of arithmetic methods but cannot be applied flexibly within the context of algebraic methods.

The developmental progression of algebraic thinking shows that students typically transition from concrete arithmetic reasoning to abstract algebraic reasoning. According to

Sun et al. (2023), students initially engage with manageable arithmetic tasks, which leads them to view algebraic problems through an arithmetic lens. This tendency is supported by Bye et al. (2022), who emphasize the critical role of arithmetic representations in algebra problem-solving strategies, often limiting students' flexibility in engaging with algebraic concepts. Ünal et al. (2023) also highlights that while students may develop visual and symbolic representations in algebra, the transition from arithmetic to algebraic reasoning is not straightforward, which can prevent students from fully appreciating the distinct nature of algebraic tasks. According to Brousseau (2002), errors arising from a misalignment between the level of instruction and students' cognitive readiness can potentially result in ontogenetic obstacles. Suryadi (2019) further classifies this phenomenon as instrumental ontogenetic obstacles, referring to obstacles stemming from technical limitations. These obstacles manifest when students interpret algebraic problems through an arithmetic rather than an algebraic framework, thereby hindering their ability to engage effectively with the learning process.

Instructional strategies play a pivotal role in helping students shift their perceptions of algebra from an extension of arithmetic to a unique mathematical domain. Cai and Hwang (2022) stress the importance of embedding algebraic ideas within arithmetic contexts to deepen students' understanding of algebraic relationships. Effective teaching practices that emphasize the conceptual understanding of the equals sign and the relational nature of algebra can also bridge the cognitive gap between arithmetic and algebra (Baiduri, 2015). For instance, Kieran and Martínez-Hernández (2022) discuss the necessity of coordinating computational and structural approaches to enhance students' understanding of equivalence, a fundamental aspect of algebraic reasoning.

Additionally, integrating visual aids and diagrams into algebra instruction has been shown to support students in connecting arithmetic and algebraic concepts. Research suggests that students who construct diagrams can use informal arithmetic strategies to solve algebraic problems, making these problems more accessible (Chu et al., 2017). This aligns with Nathan and Koellner (2007), who argue that a solid understanding of arithmetic provides a strong foundation for algebraic reasoning.

From a conceptual standpoint, many students struggled to grasp fundamental ideas such as variable representation, proportional reasoning, and the principle of equality in equations. Jupri and Drijvers (2016) emphasize that these difficulties often arise when students attempt to mathematize word problems. Their study revealed that students frequently failed to construct accurate mathematical models from real-world contexts, reflecting a gap in both horizontal and vertical mathematical understanding. This issue was evident in Question 4, where students misinterpreted proportional relationships and miscalculated adjustments for varying ingredient amounts. These findings are consistent with Jupri and Drijvers' conclusions regarding students' struggles to bridge contextual understanding and symbolic algebra. Based on Brousseau's (2002) framework, these difficulties can be categorized as epistemological obstacles. Such obstacles arise when students' prior knowledge or conceptual frameworks are insufficient or inappropriate for understanding new mathematical concepts. In this context, students' inability to represent variables, apply proportional reasoning, and interpret the principle of equality reflects limitations in their existing knowledge structures, which hinder

their ability to construct accurate mathematical models and transition effectively from contextual understanding to symbolic algebraic reasoning.

The hermeneutic perspective sheds light on the interpretative processes students use to make sense of mathematical problems. Many errors arose from misinterpretations of problem narratives or instructions, leading to incorrect assumptions about the relationships between quantities. For example, in Question 2, some students miscalculated profit by failing to subtract setup costs from revenue, reflecting a misunderstanding of the problem's requirements. Mengistie (2020) similarly emphasizes the importance of clear problem narratives to support structured reasoning. Within Brousseau's (2002) framework, errors that emerge as a consequence of the design of the instructional process are categorized as didactical obstacles. These obstacles occur when the teaching methods, materials, or explanations provided by the teacher fail to adequately support students' conceptual understanding or problem-solving processes (Fardian, Suryadi, & Prabawanto, 2025). As a result, students struggle to construct meaningful connections between the given information and the underlying mathematical concepts, ultimately impeding effective learning.

Procedural missteps, such as skipping essential steps or relying on trial-and-error methods, were prevalent and indicate gaps in students' structured mathematical reasoning. As shown in Figure 5, these procedural errors often stemmed from incomplete conceptual understanding. For instance, students frequently made errors in proportional calculations, such as failing to multiply by the total volume in scaling problems. These findings align with Kwakye (2020), who recommends using alternative strategies, such as the flag diagram and least common multiple (LCM) methods, to build procedural fluency and improve accuracy in solving proportional problems. In Brousseau's (2002) framework, errors that occur due to a lack of prerequisite knowledge, such as skipping essential steps or relying on trial-and-error methods, can lead to ontogenetic obstacles. Suryadi (2019) further classifies this type of obstacle as a conceptual ontogenetic obstacle, which arises when the instructional design does not correspond to students' prior experiences or background knowledge.

A notable observation from the analysis is the limited reflective thinking among students, which hindered their ability to recognize and correct mistakes. For instance, students who miscalculated did not verify their solutions by substituting the answers back into the equations, leading to repeated errors. This lack of verification reflects a broader issue: students' low confidence in critically engaging with their solutions. Fardian et al. (2024) further note that students' reliance on mechanical procedures, rather than interpretative reasoning, prevents them from fully understanding problem contexts.

#### 4. CONCLUSION

Based on the research findings, students face significant challenges in solving linear equations, primarily due to conceptual errors, procedural errors, and resultant errors. These errors are often interconnected: conceptual errors frequently lead to procedural mistakes, which then result in incorrect final answers. The study highlights the importance of addressing not only the technical aspects of error correction but also the cognitive and interpretative factors that influence students' problem-solving abilities. The findings underscore the need for instructional strategies that foster a deeper conceptual understanding and encourage reflective

practices. By bridging the gap between procedural fluency and conceptual comprehension, the study provides valuable insights for educators to design more effective teaching methods, tailored to the cognitive needs and learning experiences. These insights can help enhance students' overall problem-solving skills and their understanding of linear equations in algebra.

Based on the conclusion of this study, it is recommended that educators incorporate more holistic teaching strategies that target both the procedural and conceptual dimensions of linear equations. Additionally, fostering a learning environment where students are encouraged to reflect on their solutions and think critically about their approach will further develop their problem-solving abilities. Furthermore, educational interventions should address the cognitive barriers that prevent students from applying systematic methods when solving algebraic problems.

Beyond its empirical findings, this study highlights the unique methodological contribution of the hermeneutic phenomenological approach in mathematics education research. Unlike conventional error analyses that primarily focus on categorizing or quantifying students' mistakes, this approach enables a deeper interpretation of how students experience and make sense of their problem-solving processes. By emphasizing meaning and context, it reveals the cognitive and experiential dimensions underlying mathematical errors, offering richer insights into the complexities of learning algebra and informing future qualitative studies in mathematics education.

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## Declarations

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## REFERENCES

- American Psychological Association, E. C. (1992). Ethical principles of psychologists and code of conduct. *American psychologist*, 47(12), 1597–1611. <https://doi.org/10.1037/0003-066x.47.12.1597>
- Andrews, P., & Kaplan, K. P. (2020). PISA, TIMSS and Swedish students' knowledge of linear equations: A 'telling' case of a system fixing something that may not be broken. *Annales Universitatis Paedagogicae Cracoviensis | Studia ad Didacticam Mathematicae Pertinentia*, 12, 5–29. <https://doi.org/10.24917/20809751.12.1>
- Azizah, T., Fauzan, A., & Harisman, Y. (2022). "Flipped classroom type peer instruction-based learning" based on a website to improve student's problem solving. *Infinity Journal*, 11(2), 325–348. <https://doi.org/10.22460/infinity.v11i2.p325-348>
- Baiduri, B. (2015). Mathematics education students' understanding of equal sign and equivalent equation. *Asian Social Science*, 11(25), 15–24. <https://doi.org/10.5539/ass.v11n25p15>
- Bernard, J. E., & Bright, G. W. (1984). Student performance in solving linear equations†. *International Journal of Mathematical Education in Science and Technology*, 15(4), 399–421. <https://doi.org/10.1080/0020739840150401>
- Brousseau, G. (2002). Epistemological obstacles, problems, and didactical engineering. In N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield (Eds.), *Theory of Didactical Situations in Mathematics: Didactique des Mathématiques, 1970–1990* (pp. 79–117). Springer Netherlands. [https://doi.org/10.1007/0-306-47211-2\\_6](https://doi.org/10.1007/0-306-47211-2_6)
- Bye, J. K., Harsch, R. M., & Varma, S. (2022). Decoding fact fluency and strategy flexibility in solving one-step algebra problems: An individual differences analysis. *Journal of Numerical Cognition*, 8(2), 281–294. <https://doi.org/10.5964/jnc.7093>
- Cai, J., & Hwang, S. (2022). Seeing algebra in arithmetic through mathematical problem posing. *The Korean Society of Educational Studies in Mathematics - Journal of Educational Research in Mathematics*, 32(3), 309–329. <https://doi.org/10.29275/jerm.2022.32.3.309>
- Carry, L. R., Lewis, C., & Bernard, J. E. (1979). *Psychology of equation solving: An information processing study*. The University of Texas.
- Chu, J., Rittle-Johnson, B., & Fyfe, E. R. (2017). Diagrams benefit symbolic problem-solving. *British Journal of Educational Psychology*, 87(2), 273–287. <https://doi.org/10.1111/bjep.12149>
- del Carmen, A. L., del Carmen, W. L., García-García, J., & Salgado-Beltrán, G. (2024). Mathematical connections made by preservice mathematics teachers when solving tasks about systems of linear equations. *International Electronic Journal of Mathematics Education*, 19(4), em0799. <https://doi.org/10.29333/iejme/15590>
- Elkjær, M., & Jankvist, U. T. (2021). Designing tasks for a dynamic online environment: Applying research into students' difficulties with linear equations. *Mathematics*, 9(5), 557. <https://doi.org/10.3390/math9050557>
- Fardian, D., Suryadi, D., & Prabawanto, S. (2024). Students' learning obstacles in solving mathematical proficiency tasks: A hermeneutic phenomenological study focused on Algebra. *Kreano, Jurnal Matematika Kreatif-Inovatif*, 15(2), 593–618. <https://doi.org/10.15294/36aygs91>

- Fardian, D., Suryadi, D., & Prabawanto, S. (2025). A praxeological analysis of linear equations in Indonesian mathematics textbooks: Focusing on systemic and epistemic aspect. *Journal on Mathematics Education*, 16(1), 225–254. <https://doi.org/10.22342/jme.v16i1.pp225-254>
- Fardian, D., Suryadi, D., Prabawanto, S., & Jupri, A. (2025). (Re)conceptualizing linear equations: A snapshot from teaching and learning in Indonesia. *Jurnal Ilmiah Ilmu Terapan Universitas Jambi*, 9(3), 1205–1226. <https://doi.org/10.22437/jiituj.v9i3.37450>
- Folgueiras Bertomeu, P., & Sandín Esteban, M. P. (2023). The research question in hermeneutic phenomenology and grounded theory research. *The Qualitative Report*, 28(5), 1452–1472. <https://doi.org/10.46743/2160-3715/2023.5715>
- Gryaznov, S., Ovcharov, S., & Riazanov, A. (2024). Resolution over linear equations: combinatorial games for tree-like size and space. *ACM Transactions on Computation Theory*, 16(3), 1–15. <https://doi.org/10.1145/3675415>
- Hu, Q., Son, J.-W., & Hodge, L. (2022). Algebra teachers' interpretation and responses to student errors in solving quadratic equations. *International Journal of Science and Mathematics Education*, 20(3), 637–657. <https://doi.org/10.1007/s10763-021-10166-1>
- Izsák, A., & Beckmann, S. (2022). Future teachers' use of co-occurrence and same extent perspectives when generating linear equations. *The Journal of Mathematical Behavior*, 68, 101011. <https://doi.org/10.1016/j.jmathb.2022.101011>
- Johari, P. M. A. R. P., & Shahrill, M. (2020). The common errors in the learning of the simultaneous equations. *Infinity Journal*, 9(2), 263–274. <https://doi.org/10.22460/infinity.v9i2.p263-274>
- Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(9), 2481–2502. <https://doi.org/10.12973/eurasia.2016.1299a>
- Jupri, A., Drijvers, P., & Heuvel-Panhuizen, M. v. d. (2014). Student difficulties in solving equations from an operational and a structural perspective. *International Electronic Journal of Mathematics Education*, 9(1), 39–55. <https://doi.org/10.29333/iejme/280>
- Kieran, C., & Martínez-Hernández, C. (2022). Coordinating invisible and visible sameness within equivalence transformations of numerical equalities by 10- to 12-year-olds in their movement from computational to structural approaches. *ZDM – Mathematics Education*, 54(6), 1215–1227. <https://doi.org/10.1007/s11858-022-01355-5>
- Kwakye, D. O. (2020). Assisting pupils to solve linear equations involving the variable. *International Journal of Innovative Research and Development*, 9(8), 205–213. <https://doi.org/10.24940/ijird/2020/v9/i8/MAY20044>
- Mengistie, S. M. (2020). Enhancing students' understanding of linear equation with one variable through teaching. *International Journal of Trends in Mathematics Education Research*, 3(2), 69–80. <https://doi.org/10.33122/ijtmer.v3i2.148>
- Nathan, M. J., & Koellner, K. (2007). A framework for understanding and cultivating the transition from arithmetic to algebraic reasoning. *Mathematical thinking and learning*, 9(3), 179–192. <https://doi.org/10.1080/10986060701360852>

- Planas, N., Alfonso, J. M., Arnal-Bailera, A., & Martín-Molina, V. (2024). Mathematical naming and explaining in teaching talk: Noticing work with two groups of mathematics teachers. *ZDM – Mathematics Education*, 56(6), 1211–1222. <https://doi.org/10.1007/s11858-024-01576-w>
- Putri, A. D., Juandi, D., Herman, T., & Turmudi, T. (2025). Investigating pedagogical strategies in mathematics education with flipbook integration: A qualitative analysis. *AIP Conference Proceedings*, 3316, 120010. <https://doi.org/10.1063/5.0290040>
- Putri, A. D., Juandi, D., Turmudi, & Suparman. (2025). Blended learning and math achievement: A meta-analytic review highlighting the effectiveness and heterogeneity. *Electronic Journal of e-Learning*, 23(1), 113–128. <https://doi.org/10.34190/ejel.23.1.3781>
- Putri, A. D., Juandi, D., & Turmudi, T. (2024). Realistic mathematics education and mathematical literacy: a meta-analysis conducted on studies in Indonesia. *Journal of Education and Learning (EduLearn)*, 18(4), 1468–1476. <https://doi.org/10.11591/edulearn.v18i4.21650>
- Qetrani, S., Ouailal, S., & Achtaich, N. (2021). Enhancing students' conceptual and procedural knowledge using a new teaching approach of linear equations based on the equivalence concept. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(7), em1978. <https://doi.org/10.29333/ejmste/10938>
- Sandoval, I., García-Campos, M., & Sosa, L. (2023). Providing support and examples for teaching linear equations in secondary school: The role of knowledge of mathematics teaching. *International Journal of Science and Mathematics Education*, 21(4), 1265–1287. <https://doi.org/10.1007/s10763-022-10283-5>
- Santos, M. L. (2022). Exploring the meaning-making process in conceptual mathematics: The case of linguistic markers in algebraic equations. *Journal of Higher Education Theory and Practice*, 22(5), 138–150. <https://doi.org/10.33423/jhetp.v22i5.5209>
- Seage, S. J., & Türegün, M. (2020). The effects of blended learning on STEM achievement of elementary school students. *International Journal of Research in Education and Science*, 6(1), 133–140.
- Siregar, G. M. A., Wahyudin, W., & Herman, T. (2025). Case study in a grounded theory perspective: Students' reasoning abilities in Lithner's framework across self-regulated. *Infinity Journal*, 14(1), 259–282. <https://doi.org/10.22460/infinity.v14i1.p259-282>
- Sloan, A., & Bowe, B. (2014). Phenomenology and hermeneutic phenomenology: the philosophy, the methodologies, and using hermeneutic phenomenology to investigate lecturers' experiences of curriculum design. *Quality & Quantity*, 48(3), 1291–1303. <https://doi.org/10.1007/s11135-013-9835-3>
- Smith, J., Lee, I., Zandieh, M., & Andrews-Larson, C. (2022). Una progresión de la simbolización de estudiantes: Soluciones de un sistema de ecuaciones lineales [A progression of student symbolizing: Solutions to systems of linear equations]. *Avances de investigación en Educación Matemática*(21), 45–64. <https://doi.org/10.35763/aiem21.4237>
- Sun, S., Sun, D., & Xu, T. (2023). The developmental progression of early algebraic thinking of elementary school students. *Journal of Intelligence*, 11(12), 222. <https://doi.org/10.3390/jintelligence11120222>

- Supianti, I. I., Yaniawati, P., Osman, S. Z. M., Al-Tamar, J., & Lestari, N. (2022). Development of teaching materials for e-learning-based statistics materials oriented towards the mathematical literacy ability of vocational high school students. *Infinity Journal*, 11(2), 237–254. <https://doi.org/10.22460/infinity.v11i2.p237-254>
- Suryadi, D. (2019). *Landasan filosofis penelitian desain didaktis (DDR) [The philosophical foundation of Didactical Design Research (DDR)]*. Pusat Pengembangan DDR Indonesia.
- Tatira, B. (2023). Undergraduate students' conceptualization of elementary row operations in solving systems of linear equations. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(11), em2349. <https://doi.org/10.29333/ejmste/13679>
- Ünal, Z. E., Ala, A. M., Kartal, G., Özal, S., & Geary, D. C. (2023). Visual and symbolic representations as components of algebraic reasoning. *Journal of Numerical Cognition*, 9(2), 327–345. <https://doi.org/10.5964/jnc.11151>
- Utami, N. S., Prabawanto, S., & Suryadi, D. (2023). How students generate patterns in learning algebra? A focus on functional thinking in secondary school students. *European Journal of Educational Research*, 12(2), 913–925. <https://doi.org/10.12973/eu-jer.12.2.913>
- Woods, M., Paulus, T., Atkins, D. P., & Macklin, R. (2016). Advancing qualitative research using qualitative data analysis software (QDAS)? Reviewing potential versus practice in published studies using ATLAS.ti and NVivo, 1994–2013. *Social Science Computer Review*, 34(5), 597–617. <https://doi.org/10.1177/0894439315596311>