Infinity Journal of Mathematics Education
Volume 15, No. 1, 2026
p-ISSN 2089-6867
e–ISSN 2460-9285
https://doi.org/10.22460/infinity.v15i1.p241-264

Alternative conceptions of university students regarding the concept of the definite integral

Linda Poleth Montiel-Buritica*, **Gerardo Salgado-Beltrán**, **Gustavo Martínez-Sierra**
Facultad de Matemáticas, Universidad Autónoma de Guerrero, Guerrero, Mexico
*Correspondence: 23250800@uagro.mx

Received: Jul 31, 2025 | **Revised**: Dec 23, 2025 | **Accepted**: Dec 28, 2025 | **Published Online**: Feb 23, 2026

Abstract

Understanding the concept of definite integral continues to pose a challenge in university mathematics education, as students often develop alternative conceptions that differ from those accepted by the mathematical community. In this context, this article presents the results of a qualitative case study conducted to identify and characterise these conceptions among university students at a mathematics faculty in the state of Guerrero, Mexico. For data collection, a task-based interview was administered to 10 students, and the thematic analysis method was used to analyse the information obtained. In total, six alternative conceptions of the integral were identified, which are as follows: (1) the integral interpreted as positive area; (2) the derivative and the integral as inverse operations; (3) the indefinite and definite integrals seen as disconnected concepts; (4) the graph of the integral conceived as a graphical transformation of the derivative; (5) the sign of the area under the curve established from the sign of the x-axis on which the graph of the function lies; and (6) the integration constant interpreted as area. These findings invite reflection on the need for future research to promote apprehension of the concept of the integral among university students who will eventually enter the field of mathematics teaching.

Keywords:
Alternative conceptions, Definite integral, Mathematical education, Thematic analysis, Undergraduate students

How to Cite:
Montiel-Buritica, L. P., Salgado-Beltrán, G., & Martínez-Sierra, G. (2026). Alternative conceptions of university students regarding the concept of the definite integral. *Infinity Journal*, *15*(1), 241-264. https://doi.org/10.22460/infinity.v15i1.p241-264

1. INTRODUCTION

The study of the understanding of fundamental concepts of calculus, such as the definite integral, has been the subject of interest in mathematics education research over the last few decades (Jones, 2013, 2015; Martínez-Planell & Trigueros, 2020). Various studies (e.g., Borji et al., 2024; Burgos et al., 2021; Lehmann, 2024) have shown that, in school contexts, meanings that are consistent with formal mathematical approaches are not always constructed, giving rise to alternative conceptions that hinder the learning of mathematical concepts (García, 2018; Serhan, 2015). These conceptions can derive both from everyday experiences and from teaching approaches focused exclusively on algebraic procedures, without adequate articulation with geometric, numerical or contextual meaning (Zolt et al., 2023).

In particular, the definite integral is often interpreted by students as a calculation technique or an operational tool (Sealey, 2014), disconnected from its interpretation as an accumulation of quantities or as an area under a curve. This view has prompted other research to explore the meanings attributed to this concept, from approaches that consider semiotic, cognitive, and epistemological dimensions (Bajracharya et al., 2023; Gulkilik, 2022; McGee & Martinez-Planell, 2014; Wagner, 2018). Among the most studied meanings is the interpretation of the definite integral as an inverse operation of the derivative, that is, as an antiderivative (Brijlall & Ndlazi, 2019). This conceptualisation is closely linked to the Fundamental Theorem of Calculus, which establishes a formal connection between differentiation and integration.

However, the literature has reported that when this conceptualisation is introduced predominantly or exclusively, it can make it difficult for students to understand other fundamental conceptualisations (Maharaj, 2014; Tarr & Maharaj, 2021). Among these is the accumulation function (Thompson & Silverman, 2008), which allows for the interpretation of a dynamic process of totalising variable quantities over an interval, establishing links with phenomena of continuous change and with applications in physical contexts. Similarly, Riemann sums (Park et al., 2013; Sealey, 2014) involve the progressive accumulation of quantities such as displacement, area or volume through successive approximations involving the concept of limits.

These different ways of conceptualising the definite integral not only impact the perspective from which students construct their meanings, but are also reflected in curricular and didactic approaches used in different educational contexts. In addition to the above, as a starting point for examining how they are reflected in curricular proposals and what implications they have for the teaching and learning processes of calculus. For example, Hayes (2024) analyses five textbooks, most of them of American origin, used in university programmes in the United States, revealing a predominant tendency to structure the concept of the definite integral based on Riemann sums. In the Mexican context, emphasis is placed on the primitive, that is, the relationship between derivation and integration as the basis of the Fundamental Theorem of Calculus (Ponce-Campuzano, 2013).

In this regard, Cordero (2005) points out that school textbooks in Mexico often tend to reduce the treatment of the definite integral to an algorithmic approach, favouring a static and formal view. This orientation leaves aside other ways of constructing the concept, such as the use of covariance between variables or geometric interpretation (Aranda & Callejo, 2017). From a didactic perspective, this leads to teaching that hinders students' reconstruction of meanings and obstructs the possibility of establishing connections with real contexts or situations from fields other than strictly mathematical ones.

As a result of this approach, various difficulties associated with learning the concept have been documented in a scattered manner. Previous research has documented common errors made by teachers and students, among which the confusion between the definite integral and the area under the curve stands out (Akkoç & Kurt, 2008; Fuster & Gómez, 1997). In particular, there has been a persistent tendency to interpret the definite integral solely as a positive geometric area, which creates conflicts when attempting to justify negative values (Bajracharya et al., 2023). Furthermore, it leads to errors related to the interpretation of the concept of convergence, as it hinders the understanding of the integral as the limit of a sum of infinitesimal quantities, as proposed in the framework of infinitesimal calculus (Ely, 2017; Tatar & Zengin, 2016).

Given this scenario, it is essential to consider alternative conceptions that are structured around the definite integral. Although these conceptions may be incompatible or partial with the institutionalised meanings of calculus, they should not be considered exclusively as errors (Lucariello et al., 2014; Strike & Posner, 1992). On the contrary, they offer valuable insights into the ways in which teachers and students attempt to attribute meaning to the concept, allowing access to their mental schemas and, in turn, identifying anchor points from which a profound conceptual reconstruction consistent with abstract mathematical knowledge can be promoted.

In line with this need for didactic reconceptualisation, Attorps et al. (2013) warn of a shortage of specific didactic proposals for teaching definite integrals, especially at university level. This is because the concept of the definite integral has received relatively little attention in the educational field (Caglayan, 2016), particularly with regard to the design of instructional sequences that promote deep conceptual understanding beyond the mastery of mechanical procedures.

In relation to the above, identifying the alternative conceptions that students mobilise when confronted with this mathematical object is crucial for the development of more effective instructional designs (Salgado-Beltrán & García-García, 2024). This will allow us to recognise not only conceptual obstacles but also the prior ideas from which students construct meaning. In addition, Balderas Robledo et al. (2014) and Cáceres and Ortega (2021) argue that mathematics teachers should be aware of students' preconceptions and address them during teaching to help them reorganise their knowledge and correct possible errors, thus avoiding the formation of alternative conceptions. This is essential because some alternative conceptions are resistant to change and may persist even after instruction (Bostan Sarioglan, 2016; Chi et al., 2012; Denbel, 2014). Furthermore, some of these conceptions may remain throughout different educational stages and ages (Raviolo et al., 2024). These reasons underpin the importance of studying alternative conceptions, especially in key concepts such as integrals, which serve as the basis for other more advanced mathematical concepts.

This study assumes that investigating university students' alternative conceptions of the concept of integrals is highly relevant, as the information obtained will allow us to infer key aspects about their future teaching practice and the quality of the teaching they will provide to their students on this fundamental concept (Avilés-Canché & Marbán-Prieto, 2023; Zakaryan & Sosa, 2021). Furthermore, given that they are in a formative stage, any difficulties identified in understanding mathematical content could be addressed in a timely manner through teaching, in order to ensure that they achieve an understanding of the concepts they will eventually teach. Therefore, this research aims to answer the following question: What alternative conceptions do university students present when solving tasks related to definite integral?

1.1. Conceptual framework

In the process of teaching and learning mathematics, it is common for students to develop ideas that are incompatible or inconsistent with accepted scientific notions (Kastberg, 2002; Narjaikaew, 2013; Salgado-Beltrán & García-García, 2024). These ideas are ways of thinking that make sense from the student's perspective and are often based on previous experiences, intuitions, or inconsistent generalisations about mathematical concepts (Abouchedid & Nasser, 2000). Far from being random and isolated, these ideas reflect genuine attempts to give meaning and explanations to mathematical concepts, although in many cases they conflict with the formal meanings expected in the classroom; these have been called alternative conceptions (Confrey, 1990).

Evidenced through verbal or written language, alternative conceptions form coherent conceptual frameworks in students' minds and reflect how they conceptualise knowledge, influencing the way in which knowledge is processed and assimilated during learning (Confrey, 1990; Lucariello et al., 2014) and consequently influence the understanding of mathematical concepts (García, 2018). This highlights the need to detect them, find their origin and treat them in order to influence their appearance (Flores, 2004).

Alternative conceptions are generated in school and extracurricular environments (Salgado-Beltrán & García-García, 2024). According to Fujii (2014), they arise when students' conceptions conflict with accepted meanings in mathematics. Therefore, for the purposes of this research, alternative conceptions of the definite integral (hereinafter CAI) are understood to be those pieces of knowledge held by students that are incompatible or inconsistent, even partially, with what the mathematical community recognises as correct. These conceptions are not sporadic, but appear recurrently in students' reasoning when solving tasks involving the concept of integrals, and are manifested through the arguments expressed in their written work (García, 2018; Salgado-Beltrán & García-García, 2024).

1.2. Definite integral

It is an accumulation function, F(x), which represents the sum of small (infinitesimal) quantities, of the form $f(x) \cdot d(x)$, over an interval $I \subset D_f$. In the plane $\mathbb{R}^2$, this accumulation is interpreted geometrically as the area under the curve of f(x) between two points on the x - axis. Depending on the context that f(x) models, this area can represent different physical or real concepts, such as distance travelled, accumulated income, concentration of a drug in the body, among others. Thus, the definite integral not only has a geometric meaning, but also a profound usefulness in the analysis and modelling of dynamic phenomena in different disciplines. In this sense, the accumulation function is expressed by a definite integral with a variable upper limit. That is, $F(x)=\int_a^x f(t)dt$, and under certain conditions of continuity, its derivative coincides with the function that accumulates: $F'(x) = f(x)$, which establishes a fundamental connection between integration and derivative (Larson, 2010; Stewart, 2018).

2. METHOD

This research adopts a qualitative approach and uses a case study as its main methodology. It is oriented towards the exploration of meanings and the interpretation of mathematical concepts from the perspective of the subjects, with an emphasis on description as the final product (Merriam, 2022). In turn, it involves a research process characterised by a detailed, comprehensive and systematic analysis of the phenomenon under study (Rodríguez Gómez et al., 1999). The exploration of CAIs requires an inductive approach, as they will be constructed from the data collected. Therefore, the study focuses on understanding the problem through the ten cases included in the research, which contributes to the construction of a theoretical explanation that supports the phenomenon analysed (Stake, 2013).

2.1. Research Context and Participants

The research was conducted at the Faculty of Mathematics of a public university located in southern Mexico. Ten volunteer students (6 men and 4 women) aged 19-22, enrolled in the 6th semester of the degree programme, participated. The selection criteria were: 1) having passed their calculus courses in previous semesters, 2) being regular students with a minimum grade point average of eight, 3) recognising themselves as competent with regard to the concept of integrals, 4) having basic oral and written communication skills to express their mathematical ideas, and 5) having the time available to participate in interviews or work sessions required by the study. Hereinafter, they will be identified as E1, E2 ... E10 to maintain the confidentiality of their identities.

2.2. Data Collection Procedure

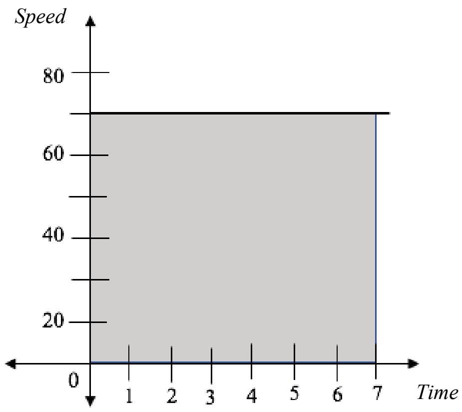
A task-based interview (TBI) was designed and implemented, following the approach proposed by Goldin (2012). This type of interview is characterised by minimal interaction between the interviewer and the participant, mediated by one or more pre-designed tasks, which may be questions, problems or activities aimed at exploring their cognitive processes. This method was chosen because it combines the advantages of an interview, which facilitates the verbalisation of the subject's thoughts, with those of a structured instrument such as a questionnaire, which reveals the procedures and ideas they use when solving proposed tasks. These characteristics make the method ideal for exploring alternative conceptions of integrals in students, as it allows for in-depth investigation of their reasoning.

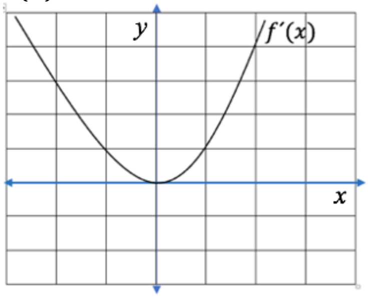
The protocol for the interviewer included basic auxiliary questions common to all tasks, such as: ‘Why did you do it that way?’, ‘Do you know of another solution?’, ‘What do you mean by this term?’ and ‘Why did you use that formula?’ The purpose of these questions was to delve deeper into the student's reasoning, clarify their thinking and encourage reflection on their decisions. In addition, specific questions were incorporated to probe in greater detail the knowledge demonstrated during the resolution of the tasks, as well as to prevent unintentional errors. These questions were asked at key moments: when the participant expressed confusing ideas, did not justify their procedures, or when it was considered relevant to activate possible alternative solutions that they might know.

The tasks were designed in accordance with higher-level differential and integral calculus curricula and programmes, adapting them to the case studies and the research objective. To validate its effectiveness, the initial instrument was applied to two university students. During this stage, the clarity and comprehensibility of the questionnaire instructions were evaluated based on three indicators: (i) explicit comments from participants collected at the end of the application, (ii) questions, requests for clarification, and difficulties observed during the completion of the tasks, and (iii) the consistency of the responses, in order to identify possible misinterpretations of the instructions. It was also verified that the tasks were appropriate for the academic level of the participants.

The validation lasted approximately 80 to 90 minutes and was recorded on video for later analysis. This procedure made it possible to identify and correct inaccuracies in both the wording of the instructions and the formulation of the tasks. Based on the analysis of the results obtained in the pilot test, those tasks that did not adequately respond to the objective of the study were restructured. After making the necessary modifications, it was ensured that the tasks allowed for the identification of various CAIs. As a result of the validation process, the final instrument consisted of seven tasks (see Table 1).

Table 1. Tasks set for students

Task No	Task Description for students
Task 1	What does the concept of integral mean to you? Explain it in your own words.
Task 2	The following graph shows the relationship between *speed* $V(t)$ (in km/h) and *time t* (in hours) during a bus journey. Determine the value of the definite integral: $$\int_0^7 V(t)dt$$ Explain in your own words. What does this integral represent in the context of the graph? 
Task 3	Draw the graph of a function that is integrable.
Task 4	Explain what equality means: $\int f(x)dx = F(x) + C$.
Task 5	Solve the definite integral and explain your procedure. $$\int_{-1}^{1} \frac{1}{x^2} dx$$
Task 6	Explain what happens when we differentiate a function $y = f(x)$ and then integrate the result. What happens to the derivative and the integral in this process?

Task No	Task Description for students
Task 7	Sketch out the sketch of the graph of $f(x)$ using the graph of its derivative $f'(x)$ as a reference. 

In order to establish a clear correspondence between the proposed tasks and the alternative concepts, each task was designed with the aim of exploring specific conceptual aspects associated with the definite integral. For example, task 1 allowed us to investigate the general conceptions that students have about the meaning of the integral and the definite integral, as well as their connection or disconnection. Task 2, contextualised on the basis of the speed-time relationship, was aimed at exploring alternative conceptions related to the interpretation of the definite integral as the area under the curve, the meaning of the sign of the integral and its link to accumulated physical quantities. Task 3 focused on examining the notion of integrability and students' ideas about the conditions that a function must satisfy in order to be integrable.

Task 4 allowed us to identify alternative conceptions associated with the relationship between differentiation and integration, particularly the understanding of the role of the integration constant. Task 5 was used to analyse procedural conceptions of the definite integral, including the tendency to favour algorithmic calculation over conceptual interpretation. Finally, tasks 6 and 7 were designed to explore the understanding of the inverse relationship between derivative and integral and the graphical interpretation of this relationship, as well as possible difficulties in coordinating algebraic and graphical representations.

2.3. Data Analysis

The interviews provided both written and verbal evidence from the participants, recorded on worksheets and in video recordings. To analyse this information, Braun and Clarke's (2012) thematic analysis method was used, complemented by researcher triangulation, in order to ensure the consistency, credibility and rigour of the results.

In line with this framework, the analysis was developed from a mixed analytical stance (inductive–deductive): on the one hand, it was based on previous theoretical categories related to alternative conceptions of the definite integral, derived from the conceptual framework of the study; on the other hand, the empirical data were allowed to give rise to the identification of emerging codes that were not initially anticipated. This approach made it possible to identify, organise, and interpret patterns of meaning (themes) based on the participants' responses to the proposed tasks, maintaining a balance between theory and empirical evidence. Thematic analysis is structured in six phases, which are described below.

Phase 1. Familiarisation with the data

At this stage, the interviews were transcribed and the participants' written work was digitised. Next, the collected material was read through in its entirety in order to obtain an overview of the responses, identify initial ideas about the CAIs evidenced by the case studies, and begin to familiarise ourselves with their language and forms of expression.

Phase 2. Generation of initial codes

Words, expressions, or mathematical procedures, including calculations used by participants to refer to the concept of definite integral, were identified that were incompatible or inconsistent with the conceptions accepted and recognised as correct by the mathematical community. In this phase, 52 initial codes were identified, associated with the different ideas that the students mentioned or used in relation to the concept of integral. In some cases, the participants referred to more than two ideas, which led to differentiated coding, even though only one of them was used as the main strategy to solve the task.

Subsequently, the initial codes were compared and contrasted, taking into account their conceptual similarities, the type of conceptual conflict they revealed, and the role they played in completing the task. As a result of this process, the codes were grouped into sub-themes representing recurring patterns of interpretation of the concept of definite integral. The formation of these sub-themes was guided by three criteria: (i) internal conceptual coherence among the grouped codes, (ii) recurrence of the pattern in the productions of different participants, and (iii) relevance of the pattern to the study objectives and the theoretical framework adopted. This coding process was carried out based on the productions of the ten participants.

For example, in the excerpt from E2 for task 1, although the objective of this task was to investigate the meaning of the concept of integral in general terms, the excerpt focused on the definite integral. However, during his explanation, the student stated that, for him, the integral corresponded to a general concept and that the definite integral was one of its particular forms, even pointing out the existence of different types of integrals. In his speech, E2 stated that the integral could be understood as a function, and clarified that what he was specifically referring to in his answer was the definite integral. In line with this interpretation, the key phrases that allowed two initial codes to be generated were highlighted in italics: “the definite integral means the area under the graph of the function and is always positive” and “the definite integral means the area under the curve and its nature is to be positive”.

E2 : *Well, for me, the definite integral means the area under the graph of the function.*
Researcher : *Can you explain what the sign for that area is?*
E2 : *..., Well, positive.*
Researcher : *Why?*
E2 : *It's a professional area, the area is always positive.*
Researcher : *Do you always assume that?*
E2 : *That is what the definite integral means, the area under the curve. If you ask me, the area must be positive by its very nature.*
Researcher : *Could it be negative?*
E2 : *Ummm, but it's an area.*

Furthermore, in order to minimise the introduction of leading or trick questions, the interviewer's interventions were limited exclusively to requests for clarification, further explanation or examples of the ideas expressed by the students themselves, avoiding at all times the incorporation of mathematical terminology not previously mentioned by the participant. On the other hand, during the analysis process, this possible influence was taken into account through a critical review of the discursive segments, giving priority to spontaneous expressions and reasoning initiated by the students, and discarding those interpretations that could be attributed mainly to the interviewer's mediation. This methodological reflection is incorporated as a quality control mechanism for the analysis and contributes to strengthening the credibility of the reported results.

Phase 3. Search for themes and subthemes

The initial codes were compared in order to group those that shared a common meaning or presented similar response patterns, which allowed subthemes to be formed. These were then associated with a more general theme corresponding to a CAI. For example, from the sub-theme ‘the area under the curve is a positive quantity,’ the theme ‘the definite integral interpreted as positive area’ was constructed, representing a CAI.

Phase 4. Review of themes and subthemes

Two levels of review were carried out, as proposed by Braun and Clarke (2006). At the first level, the codes generated within each theme were examined to ensure that they formed a coherent pattern. At the second level, the review was extended to the entire data set, evaluating the consistency of the themes in relation to the entire corpus. In addition, at both levels, a peer review (conducted by the study researchers) was implemented with the aim of strengthening the quality and validity of the analysis. This process allowed for the refinement and more precise delimitation of the alternative conceptions identified.

Phase 5. Definition and naming of topics and subtopics

Each CAI was defined, identified, and named based on the analysis of the data obtained during the working sessions. These concepts are based on the evidence gathered from the participants' written and verbal contributions.

Phase 6. Report preparation

A report was prepared that organises the defined subtopics, grouping them around general themes that correspond to the CAIs. To meet the research objectives, the results are presented structured according to each alternative conception identified.

3. RESULTS AND DISCUSSION

3.1. Results

The analysis of the participants' written and verbal productions allowed us to construct six sub-themes associated with the concept of definite integral, which correspond to six alternative conceptions identified in the process of solving the proposed tasks (see Table 2).

Table 2. Alternative conceptions identified among participants

Alternative conceptions	Definition	Subtopics	Frequency
The integral interpreted as a positive area	It refers to the integral as the area under the curve, considering that its value is always positive.	The area under the curve is a positive quantity.	25
The derivative and the integral are inverse operations	This refers to the fact that, when applying the derivative and the integral successively in a continuous process, both operations cancel each other out, given that they are inverse operations.	The derivative and integral are inverse operations.	21
The indefinite and definite integrals viewed as disconnected concepts	It refers to interpreting indefinite and definite integrals as concepts of a different nature.	The indefinite integral is a function, and the definite integral represents the area under the curve.	19
The graph of the integral is a graphical transformation of the derivative	It refers to the interpretation of the integral graph as a graphical reflection of the derivative on the cartesian plane.	The graph of the integral is the reflection of the graph of $f'(x)$ with respect to the x-axis.	16
The sign of the area under the curve is determined by the sign of the x-axis on which the graph of the function lies	It establishes that the sign of the area under the curve depends on the location of the graph of the function $f(x)$ in the cartesian plane, mainly on the sign of the x-axis where it is located.	The area under the curve of $f(x)$ takes a positive value around the positive x-axis or a negative value around the negative x-axis.	9
The integration constant interpreted as area	It refers to interpreting the integration constant as the amount of area missing to complete the total under the curve.	The integration constant represents a missing area.	8

It is important to note that, although the total number of interviewees was ten participants, the frequencies indicated correspond to the number of appearances of each alternative conception throughout the data corpus. Thus, the same conception could be expressed repeatedly in the responses, explanations, and dialogues of the same participant, as well as at different moments during the interview or in the solution of the tasks. For this reason, the frequencies associated with some conceptions may exceed the value of ten. The use of the term frequency is therefore justified as an indicator of both the discursive and cognitive recurrence of these alternative conceptions.

3.1.1. The integral interpreted as a positive area

This alternative conception interprets the integral as the area under the curve, always considering it positive. This idea was present in the responses of all participants, except in the cases of students E8 and E10, and was manifested in the reasoning used to answer tasks 1, 2, and 4. The frequency with which it appeared varied according to the case study and the corresponding task; however, this CAI was evident at different times throughout the EBT. It is presented in greater detail below, supported by written and verbal evidence.The Results should include the rationale or design of the experiments as well as the results of the experiments. Results can be presented in figures, tables, and text.

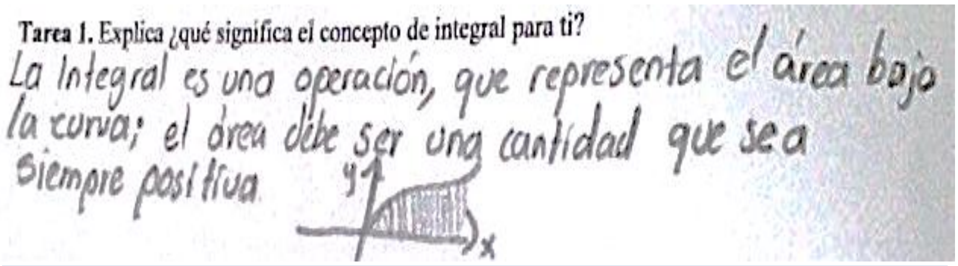
The area under the curve is a positive quantity

One of the ideas that predominated in the students' explanations was to understand the integral, from their perspective, as the area under the curve that graphically represents a function f(x) on the cartesian plane, with the particularity that this area is always considered a positive quantity. This shows that the geometric interpretation of the integral (as a measure of area) prevails over the algebraic or analytical notion, which makes it possible to conceptualise that the integral can take negative values depending on the behaviour of the function.

For example, when answering what the concept of integral means, students who expressed this CAI limited themselves to describing it as the area under the curve. During follow-up questions in the TBI (Task-Based Interview), their reasoning showed that, from a geometric standpoint, they considered that the nature of the area is always positive; therefore, they assumed that the integral, when interpreted as an area, must necessarily be positive. This idea was evident in expressions such as: *"The integral represents the area under the curve, which must be positive because it is an area", "the area is always positive", "no matter the context, the area cannot be negative"*, or *"the integral is the area under the curve, which must necessarily be positive"*. These interpretations reflect a partial and limited understanding of the concept of integrals. Even in task 2, where the use of integrals was contextualised, students who demonstrated this CAI calculated the integral but, when interpreting the result, resorted to the notion of area unrelated to the context, emphasising its value as positive. To illustrate some of the verbal evidence that contributed to the construction of this CAI, an excerpt from the dialogue with E1 for task 1 and Figure 1 are presented.

E1 : *Well, formally, the integral is the area under the curve. It's like an operation that always gives us the area under the curve sketched on the cartesian plane.*
Researcher : *Is the integral an operation?*
E1 : *That's how I see it. To do this, we use integration formulas that always give us a result.*
Researcher : *Is it always possible to arrive at a result?*
E1 : *Yes, professor.*
Researcher : *Explain a little more about the nature of that area you point out.*
E1 : *Yes [draws an xy plane on which he plots a graph in the first quadrant and highlights the region under the curve], look, the area is positive, that's why we add square units.*
Researcher : *Tell me about the sign of that area.*
E1 : *It is positive; in fact, the area is always positive.*
Researcher : *Are you looking at it from the integral?*

E1 : *..., the integral is area and the area is positive, the integral must always be positive area, that's how I see it.*



Translate:
The integral is an operation that represents the area under the curve; the area must be a quantity that is always positive.

Figure 1. The integral interpreted as the area under the curve of positive sign

3.1.2. The derivative and the integral are inverse operations

This alternative conception holds that, by successively applying the derivative and the integral in a continuous process, the two operations cancel each other out as inverses. All the case studies expressed this idea, which was particularly evident in the responses to task 6. The frequency with which this conception appeared varied from case to case. It is set out in more detail below, accompanied by evidence collected during the TBI.

The derivative and integral are inverse operations

One of the most recurrent notions in the students' reasoning was the idea that the derivative and integral are inverse operations. This conception arises in the context of teaching and learning Differential and Integral Calculus and can lead to incorrect generalisations if not internalised accurately. It is likely that this interpretation originates from the way the Fundamental Theorem of Calculus is introduced in the school context where the inverse relationship between derivation and integration is emphasised. However, this presentation may lead students to assume that both operations always cancel each other out, an idea that is not consistent with mathematical rigour and is only valid under specific conditions.

The elaboration of the CAI started when students explained that, when deriving a function f(x) and subsequently integrating the result, both operations cancel out, since one returns to the original function, assuming that the constant of integration is zero. This idea was reflected in expressions raised by the case studies, such as: *"If both are applied consecutively, they are cancelled", "they are inverse operations, so they cancel", "they cancel because they are inverse operations, assuming C=0" or "consider* $y' = 2x$*, then* y' *= 2x and* $\int 2x\, dx = x^2$*"*. These statements were collected and grouped together to form the alternative conception involved in interpreting the derivative and integral as inverse operations (see excerpt from dialogue with E3 in task 6 and Figure 2).

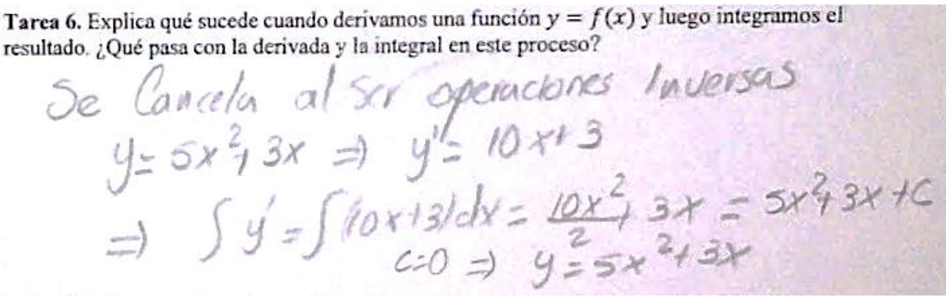
E3 : *If I take a function and derive it and then take the integral from that result, they will cancel each other out as they are inverse operations.*

Researcher : *Why do you think so?*

E3 : *Well, I will consider a specific case [write* $y = 5x^2 + 3x \Longrightarrow y' = 10x + 3 \Longrightarrow \int y' = \int (10x + 3)dx = \frac{10}{2}x^2 + 3x + c = 5x^2 + 3x + c,\ c = 0 \Rightarrow \int (10x + 3)dx = 5x^2 + 3x$ *] which is the function from which i started.*

Researcher : *Explain, should the value of c always be zero?*

E3 : *Yes, to return to the original function.*


Tarea 6. Explica qué sucede cuando derivamos una función $y = f(x)$ y luego integramos el resultado. ¿Qué pasa con la derivada y la integral en este proceso?

Se Cancela al ser operaciones Inversas
$y = 5x^2 + 3x \Rightarrow y' = 10x + 3$
$\Rightarrow \int y' = \int (10x+3)dx = \frac{10x^2}{2} + 3x = 5x^2 + 3x + C$
$C = 0 \Rightarrow y = 5x^2 + 3x$

Figure 2. The derivative and integral interpreted as inverse operations

3.1.3. The indefinite and definite integral seen as disconnected concepts

This alternative conception consists of understanding the indefinite and definite integral as concepts that are different in nature. This idea was present in almost all the case studies, with the exception of case E6, and was most clearly manifested in the responses to tasks 1 and 4. The frequency with which this conception appeared varied from case to case. A detailed analysis of how this conception is evidenced in the data obtained through the TBI is presented below.

The indefinite integral is a function and the definite integral represents the area under the curve.

One of the most salient ideas in the reasoning of the case studies was the distinction between the indefinite integral and the definite integral, which were evidently understood as distinct mathematical concepts. In explaining the meaning of equality $\int f(x)dx = F(x) + C$, participants who evidenced this CAI established that the result of calculating an indefinite integral is a function, while the definite integral is mainly related to the idea of area under the curve. By contrasting these ideas through the auxiliary questions in the TBI, it was corroborated that the case studies conceptualised two different notions.

The construction of the CAI was based on the observation of initial codes present in the written and oral productions of the case studies. Below are some expressions that contributed to this construction: *"The indefinite integral is a function while the definite one is the area under the curve", "the indefinite and definite integral denote different things", "the indefinite integral of a function is not a number like the definite one is"* or *"a function is not a number"*. These statements were collected and grouped together to shape the alternative conception that implies interpreting the derivative and the integral as disconnected concepts (see extract from the dialogue with E5 in task 4).

E5 : *The equality represents the integral of the function f(x). The letter C is the constant of integration.*

Researcher : *Is it a definite or an indefinite integral?*

E5 : *It is undefined, because the limits of integration, i.e. the numbers that appear in the sign of the integral, are not indicated.*

Researcher : *What is the relationship between the definite and indefinite integral?*

E5 : *... Well, the indefinite integral, when solved, gives a function, while the definite integral gives the area under the curve, expressed as a number.*

Researcher : *Can you complement your reflection? How would you describe both?*

E5 : *Well, they are different, teach. A function is not a number. So, that makes me think that each type of integral represents different things.*

Researcher : *Is this how you were taught?*

E5 : *I'll be honest, I don't remember seeing it that way in class. Rather, we focused on the formal definition, how to solve them, and then we looked at the theorems.*

3.1.4. The graph of the integral is a graphical transformation of the derivative

This alternative conception refers to the interpretation of the graph of an integral as a graphical reflection of the derivative in the cartesian plane. This idea was expressed by most of the case studies, with the exception of cases E7, E9 and E10, and was mainly evident in the answers to task 7. The frequency with which this conception appeared varied between participants. A more detailed analysis of this conception is presented below, accompanied by the evidence collected during the TBI.

The graph of the integral is the reflection of the graph of f'(x) with respect to the x-axis

The construction of this CAI started when the case studies attempted to sketch the graph of the primitive from the graph of the derivative (see Figure 3). In this process, they resorted as a first strategy to reflect the graph of the derivative with respect to the x-axis, in order to obtain the graph of the primitive. This idea was explicitly expressed through traces and statements such as: *"The graph of the primitive must be reflected on the x-axis"*, *"the integral must be located in quadrants I and IV"*, or *"the x-axis is a mirror to obtain the graph of the integral"*. These expressions were collected and grouped together to characterise the alternative conception that consists of interpreting the graphs of the derivative and the integral as symmetrical reflections of each other with respect to the x-axis. This interpretation is exemplified in the dialogue extract with case E2 during task 7.

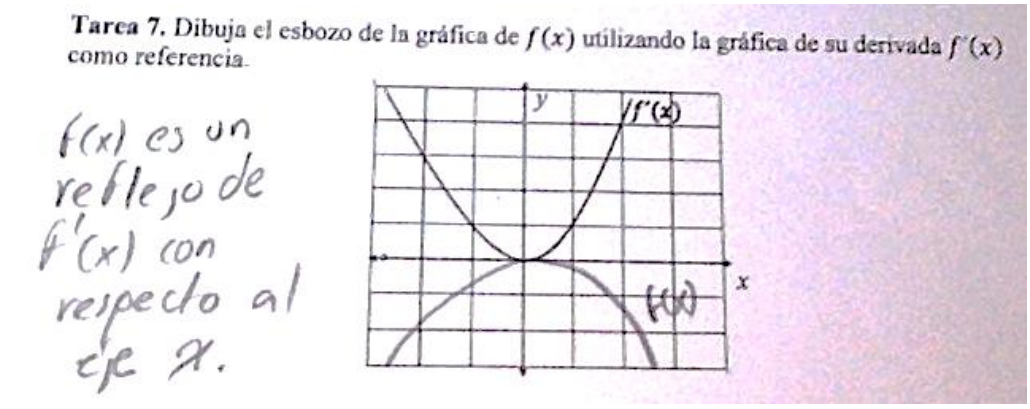
E2 : *I don't have the formula for the function, and that complicates it a bit, but i know that, graphically, i have to reflect it.*

Researcher : *Could you explain that graphical relationship you are using?*

E2 : *Professor, that's what I'm thinking of here. Look: the derivative represents the slope of the tangent line to a curve, and the integral represents the area under the curve. So, maybe that's why, graphically, i assume that one is like the reflection of the other.*

Researcher : *Explain that graphical relationship you use.*

E2 : *Professor, suddenly that occurs to me here, look the derivative is the slope of the tangent line to the curve, and the integral is the area under the curve, suddenly graphically i assume that it must be inverted.*



Translate:
f(x) is a reflection of f'(x) with respect to the x-axis.

Figure 3. The graph of the primitive interpreted as a reflection of the graph of the derivative

3.1.5. The sign of the area under the curve is determined by the sign of the x-axis on which the graph of the function lies

Less frequently than the previous conceptions, this alternative conception refers to the interpretation that the sign of the area under a curve depends on the location of the graph of the function f(x) in the cartesian plane, particularly on the sign of the semi-axis x on which the semi-plane where it is represented is located. This idea was expressed in case studies E1, E7 and E9, and was evident in the reasoning expressed during task 1.When interpreting the integral as the area under the curve, a positive or negative sign was attributed to it depending on the sign of the x-axis. The frequency with which this conception emerged varied between the three cases. A more detailed analysis of this conception is presented below, accompanied by the evidence collected during the TBI.

The area under the curve of f(x) takes a positive value around the positive half-axis x or a negative value around the negative half-axis x

The construction of this CAI originated from the three aforementioned case studies, who, in meaning the concept of integral from their own perspective, pointed out that the sign of the area under the curve is determined by the sign of the x-axis on which the graph of the function is represented. This idea was expressed through expressions such as: "The sign of the area is given by the x-axis", *"the area under the curve is positive if the graph is in quadrant I", "the area under the curve is negative if the graph is on the negative x-axis"* or *"the area is positive if the curve is projected on the positive x-axis"*. These statements were collected and grouped together to characterise the alternative conception presented here (see excerpt from the dialogue with E9 in task 1).

Researcher : *How do you determine the sign of the area under the curve?*
E9 : *For me, the sign is determined by the x-axis.*
Researcher : *Could you elaborate on that?*
E9 : *Yes. Yes, if the graph of the function is on the positive x-axis, then the area is positive; if it is on the negative x-axis, the area is negative.*

Researcher : *What makes you think that way?*

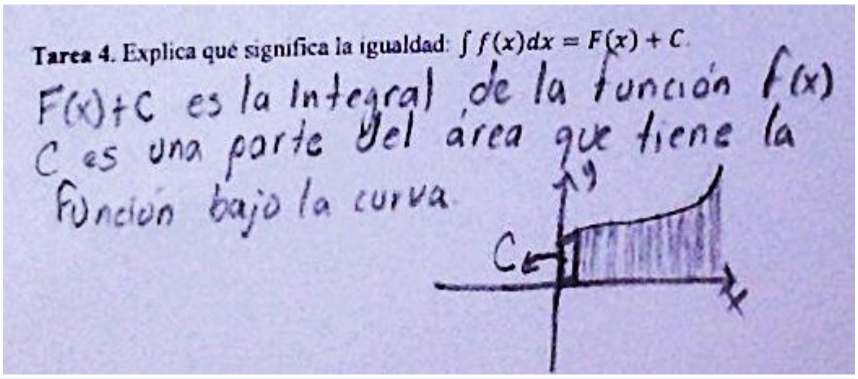
E9 : *I assume so because i see that when the curve is to the right of the y-axis, i.e. on the positive part of the x-axis, the area appears to be positive, and when it is to the left, on the negative part of the x-axis, then the area becomes negative. To me, the x-axis makes that difference in the sign of the area under the curve.*

3.1.6. The constant of integration interpreted as area

This alternative conception implies interpreting the constant of integration as an amount of missing area to complete the total under the curve. This idea appeared in two case studies, E1 and E4, and was most clearly manifested in the responses to task 4. The frequency with which this conception appeared varied between the two cases. The following is a detailed analysis of the evidence for this conception from the data obtained using the TBI.

The integration constant represents a missing area

The formulation of this CAI emerged in the case studies, when explaining what equality means $\int f(x)\,dx = F(x) + C$, referred to the constant of integration as a real number representing a portion of area that completes the area under the curve of f $f(x)$. This idea was reflected in expressions such as: *"C represents the missing area", "C completes the area under the curve"* or *"C fully integrates the area under the curve"*. These statements were collected and grouped together to form the alternative conception linked to the interpretation of the constant of integration as the missing area under the curve (see Figure 4).



Translate:
$F(x) + C$ is the integral of the fuction $f(x)$, C is a portion of the area that the fuction has under the curve.

Figure 4. The constant of integration represented as a missing area

3.2. Discussion

The results of this research show that, although the students had previously been exposed to the concept of the integral at previous educational levels, various alternative conceptions persist in their understanding. These conceptions emerge both from everyday experiences and from school practices centred on algorithmic approaches and little articulated with conceptual meanings, which coincides with what Salgado-Beltrán and García-García (2024) pointed out. From the analysis, the following is identified:

The alternative conception of the definite integral as the area under the curve —understood exclusively as a positive magnitude— was recurrent in the procedures and arguments of eight students, which places it as the most prominent in the study. However, a conceptual disconnection was observed between this geometric interpretation and the definition of the indefinite integral. When questioned, several students stated that the definite integral corresponded to a function or, failing that, to a family of functions derived from the additive constant, indicating a confusion between the operational and conceptual meanings of both notions (Bajracharya et al., 2023; Sealey, 2014). This situation reinforces previous studies that show how students tend to interpret the definite integral in a partial way, either from a purely geometric or algorithmic viewpoint, without establishing solid links with other institutionalised meanings, such as the accumulation function or the antiderivative (Martínez-Planell & Trigueros, 2020; Thompson & Silverman, 2008).

In particular, the case of E6 is significant, since his representation of the concept of integral was limited to conceiving it only as "area under the curve", without distinguishing between the various types of integral (definite and indefinite) or between its various functional, numerical or geometric meanings. This stance evidences a limited conceptual image and a partial understanding of the mathematical object, which significantly reduces their ability to articulate ideas within integral calculus (Mateus-Nieves & Font Moll, 2021).

However, the alternative conception according to which the derivative and the integral are inverse operations that cancel each other out was expressed in all the case studies, although it was not always accompanied by a consistent formal justification. This conception, although based on the Fundamental Theorem of Calculus, was generalised without taking into account the necessary and sufficient conditions for the relation between derivative and integral to be valid. The risk of this interpretation lies in an oversimplification of the integration process, where students tend to omit essential elements such as the constant of integration, the domain of the function and differentiability, reflecting a partial understanding of the Fundamental Theorem of Calculus and its superficial applicability (Ergene & Özdemir, 2022; Sealey, 2014). In some cases, the constant of integration was arbitrarily restricted to zero, which limited its understanding as a family of solutions. Moreover, an emphasis on the algorithmic was evident, in which students resorted to mechanical procedures to "undo" the derivation, without questioning the meaning of the operational reversal. This restricted conception of the concept is not only evident at the symbolic and procedural level, but is also transferred to the graphical level, giving rise to new forms of incorrect conceptualisation.

An alternative conception was also identified in which students interpret the graph of the integral as a symmetrical reflection of the derivative with respect to the horizontal axis. This type of visual reasoning reflects an attempt to establish graphical links between related concepts, although it lacks rigorous mathematical foundations. Such an interpretation highlights the need to strengthen the development of conceptual notions through the articulated use of different semiotic registers —graphical, algebraic and verbal— as stated by Duval (2006), in order to avoid erroneous visual analogies that distort the mathematical meaning of the integral.

In this same line of graphical interpretations leading to alternative conceptions, the idea that the sign of the area under the curve depends on the semi-axis (positive or negative) on which the graph of the function lies was observed, as expressed by students in the first task (cases E1, E7, E9). This conception reflects a disconnection with the institutionalised mathematical meanings of the concept of the definite integral, as it omits that the sign of the value of the integral is determined by the sign of the values of the function integrating f(x), and not by the location of the curve with respect to the axes of the cartesian plane. This idea is consistent with the findings of Sealey (2014), who notes that students tend to attribute only positive values to the area under the curve, and that the move to recognising that an integral can be negative is conceptually complex for them. Likewise, Bajracharya et al. (2023) report that several students do not achieve a coherent meshing between the function, its graph and the value of the integral, especially with regard to the determination of the sign.

Finally, another alternative conception identified was to interpret the constant of integration as a "*missing area*" (cases E1 and E4), which is a metaphorical interpretation, although it may have heuristic value, in the sense of facilitating an initial intuitive understanding of the integration process, it does not conform to the formal definition of Integral Calculus. This conception seems to arise from an incomplete analogy between the definite integral as an accumulation of area under a curve and the indefinite integral as a set of primitive functions. In making this association, some students mistakenly conceive of the constant $+C$ as a quantity of area "*necessary*" to complete the integral, revealing a difficulty in moving from an operational approach (sum of areas) to a structural approach (family of functions). This confusion, as pointed out by Martínez-Planell and Trigueros (2020); Ely (2017) is common at university levels, where students have not yet internalised the constant as an abstract component of the general solution set of an antiderivative.

On the other hand, a central contribution of this research lies not only in identifying alternative conceptions of the definite integral. Although some of these conceptions have been previously documented in the literature, others had not been described. More importantly, the value of the study lies in showing how these conceptions operate in an interrelated and recurrent manner in students' reasoning, giving rise to coherent patterns of thought. These patterns are evident through the recurrence of similar arguments, expressions, and representations in different data extracts, which appear consistently in students' verbal explanations, symbolic procedures, and graphic productions throughout the tasks and interviews.

In this sense, the innovative contribution of the study lies in the characterisation of the recurrence and transfer of these conceptions through the different meanings of the definite integral (geometric, algebraic and functional). This approach allows us to understand how a local interpretation of the definite integral delimits other interpretations. Therefore, this integrative reading provides empirical evidence that conceptual difficulties in integral calculus are not explained solely by the absence of formal knowledge, but by the lack of connection between mathematical conceptions accepted and validated by the academic community, thus offering a more solid framework to guide future teaching interventions focused on the construction of conceptual connections.

4. CONCLUSION

Therefore, the results show that future mathematics graduates, most of whom will work as pre-university teachers, have multiple alternative conceptions regarding the concept of definite integral, many of which are persistent and resistant to change. This shows that the difficulties are not limited to concepts of differential calculus, such as the derivative (Salgado-Beltrán & García-García, 2024), but also extend to integral calculus. This situation is concerning, given that teachers' limited understanding can hinder the creation of meaningful opportunities for their students to build a solid knowledge of the definite integral (Byerley & Thompson, 2017).

In relation to the research question, it was evident that the interpretation of the definite integral as positive area continues to predominate; however, this coexists with alternative conceptions that are more inconsistent from an epistemological point of view. Despite numerous studies conducted in recent years, these persistent conceptions reflect the difficulty students face in integrating the multiple conceptualisations of the integral.

Given this scenario, it is recommended to rethink strategies for teaching integral calculus, promoting tasks that require students to argue and justify their interpretations from multiple semiotic registers. Likewise, it is recommended to incorporate theoretical frameworks such as APOS, mathematical connections to structure activities that support the development of more robust mental schemas, facilitating the internalisation of the concept of the integral in its semantic complexity. These strategies can reduce to bridge the gap between students' recurring misconceptions and a more integrated understanding of the concept, creating scenarios that are conducive not only to cognitive development but also to strengthening the teaching skills of future mathematics teachers.

As a limitation, it should be noted that the study focused on a small sample of ten students, which restricts the generalisation of the results. Future studies could expand the sample, include different educational contexts, and explore the effectiveness of teaching interventions designed to strengthen the learning of the mathematical object present in this study.

Acknowledgments

The authors would like to thank the university students of the Universidad Autonoma of Guerrero (Mexico) Chilpancingo node who contributed to this work.

Declarations

Author Contribution : LPM-B: Investigation, Methodology, Project administration, Visualization, Writing - original draft, and Writing - review & editing; GS-B: Investigation, Formal analysis, Investigation, Methodology, Resources, Validation, Writing - original draft, and Writing - review & editing; GM-S: Resources, Validation, and Writing - review & editing.

Funding Statement : The author discloses no receipt of the following financial support for this article's research, authorship, and publication.

Conflict of Interest : The authors declare no conflict of interest.
Additional Information : Additional information is available for this paper.

REFERENCES

Abouchedid, K., & Nasser, R. (2000). *The role of presentation and response format in understanding, preconceptions and alternative concepts in algebra problems*. United States Department of Education.

Akkoç, H., & Kurt, S. (2008). İntegral kavramına ilişkin oğrenme zorlukları ve integral oğretimi [The learning difficulty about integral and teaching integral]. In M. F. Özmantar, E. Bingöl-bali, & H. Akkoç (Eds.), *Matematiksel kavram yanılgıları ve çözüm önerileri içinde [Mathematical misconceptions and solution proposals]* (pp. 257–290). Pegem Academy.

Aranda, C., & Callejo, M. L. (2017). Construcción de la función integral y razonamiento covariacional: Dos estudios de casos [Integral function construction and covariational reasoning: Two case studies]. *Bolema: Boletim de Educação Matemática*, *31*(58), 777–798. https://doi.org/10.1590/1980-4415v31n58a13

Attorps, I., Björk, K., Radic, M., & Tossavainen, T. (2013). Varied ways to teach the definite integral concept. *International Electronic Journal of Mathematics Education*, *8*(2-3), 81–99. https://doi.org/10.29333/iejme/275

Avilés-Canché, K., & Marbán-Prieto, J. M. (2023). Perfiles de autoeficacia docente y conocimiento especializado para la enseñanza de Las matemáticas [Teacher self-efficacy profiles and specialized knowledge for teaching mathematics]. *Revista Electrónica Interuniversitaria de Formación del Profesorado*, *26*(2), 57–85. https://doi.org/10.6018/reifop.559321

Bajracharya, R. R., Sealey, V. L., & Thompson, J. R. (2023). Student understanding of the sign of negative definite integrals in mathematics and physics. *International Journal of Research in Undergraduate Mathematics Education*, *9*(1), 62–91. https://doi.org/10.1007/s40753-022-00202-y

Balderas Robledo, R. G., Block Sevilla, D., & Guerra Ramos, M. T. (2014). "Sé cómo se hace, pero no por qué". Fortalezas y debilidades de los saberes sobre la proporcionalidad de maestros de secundaria ["I know how it's done, but not why"". Strengths and weaknesses of knowledge about the proportionality of secondary school teachers]. *Educación matemática*, *26*(2), 7–32. https://doi.org/10.24844/EM2602.01

Borji, V., Martínez-Planell, R., & Trigueros, M. (2024). Students' understanding of riemann sums and double integrals: The case of task design in APOS theory. *International Journal of Research in Undergraduate Mathematics Education*. https://doi.org/10.1007/s40753-024-00250-6

Bostan Sarioglan, A. (2016). Conceptual level of understanding about sound concept: Sample of fifth grade students. *Online Submission*, *7*(1), 87–97.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., & Clarke, V. (2012). Thematic analysis. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology, Vol 2: Research designs: Quantitative, qualitative, neuropsychological,* *and biological.* (pp. 57–71). American Psychological Association. https://doi.org/10.1037/13620-004

Brijlall, D., & Ndlazi, N. J. (2019). Analysing engineering students' understanding of integration to propose a genetic decomposition. *The Journal of Mathematical Behavior*, *55*, 100690. https://doi.org/10.1016/j.jmathb.2019.01.006

Burgos, M., Bueno, S., Pérez, O., & Godino, J. D. (2021). Onto-semiotic complexity of the definite integral. *Journal of Research in Mathematics Education*, *10*(1), 4–40. https://doi.org/10.17583/redimat.2021.6778

Byerley, C., & Thompson, P. W. (2017). Secondary mathematics teachers' meanings for measure, slope, and rate of change. *The Journal of Mathematical Behavior*, *48*, 168–193. https://doi.org/10.1016/j.jmathb.2017.09.003

Cáceres, L., & Ortega, J. (2021). Debilidades conceptuales encontradas en estudiantes de precálculo i en la UPRM [Conceptual weaknesses found in precalculus I students at UPRM]. *Espacio Matemático*, *2*(2), 98–112.

Caglayan, G. (2016). Teaching ideas and activities for classroom: Integrating technology into the pedagogy of integral calculus and the approximation of definite integrals. *International Journal of Mathematical Education in Science and Technology*, *47*(8), 1261–1279. https://doi.org/10.1080/0020739X.2016.1176261

Chi, M. T. H., Roscoe, R. D., Slotta, J. D., Roy, M., & Chase, C. C. (2012). Misconceived causal explanations for emergent processes. *Cognitive Science*, *36*(1), 1–61. https://doi.org/10.1111/j.1551-6709.2011.01207.x

Confrey, J. (1990). A review of the research on student conceptions in mathematics, science, and programming. *Review of Research in Education*, *16*(1), 3–56. https://doi.org/10.3102/0091732x016001003

Cordero, F. (2005). El rol de algunas categorías del conocimiento matemático en educación superior: Una socioepistemología de la integral [The role of some categories of mathematical knowledge in higher education: A socio-epistemology of the integral]. *RELIME: Revista latinoamericana de investigación en matemática educativa*, *8*(3), 265–286.

Denbel, D. G. (2014). Students' misconceptions of the limit concept in a first calculus course. *Journal of Education and Practice*, *5*(34), 24–40.

Duval, R. (2006). Un tema crucial en la educación matemática: La habilidad para cambiar el registro de representación [A crucial topic in mathematics education: The ability to change the register of representation]. *La Gaceta de la Real Sociedad Matemática Española*, *9*(1), 143–168.

Ely, R. (2017). Definite integral registers using infinitesimals. *The Journal of Mathematical Behavior*, *48*, 152–167. https://doi.org/10.1016/j.jmathb.2017.10.002

Ergene, Ö., & Özdemir, A. Ş. (2022). Understanding the definite integral with the help of Riemann sums. *Participatory Educational Research*, *9*(3), 445–465. https://doi.org/10.17275/per.22.75.9.3

Flores, C. D. (2004). Acerca del análisis de funciones a través de sus gráficas: Concepciones alternativas de estudiantes de bachillerato [About the analysis of functions through their graphs: Alternative conceptions of high school students]. *RELIME: Revista latinoamericana de investigación en matemática educativa*, *7*(3), 195–218.

Fujii, T. (2014). Misconceptions and alternative conceptions in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 625–627). Springer.

Fuster, J. L. L., & Gómez, F. J. S. (1997). Una interpretación de las dificultades en el aprendizaje del concepto de integral [An interpretation of the difficulties in learning the concept of integral]. *Divulgaciones Matemáticas*, *5*(1/2), 61–76.

García, J. (2018). *Conexiones matemáticas y concepciones alternativas asociadas a la derivada ya la integral en estudiantes del preuniversitario [Mathematical connections and alternative conceptions associated with the derivative and the integral in pre-university students]*. Universidad Autónoma de Guerrero. https://doi.org/10.13140/RG.2.2.16095.15522

Goldin, G. A. (2012). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517–545). Routledge.

Gulkilik, H. (2022). Representational fluency in calculating volume: An investigation of students' conceptions of the definite integral. *International Journal of Mathematical Education in Science and Technology*, *53*(11), 3092–3114. https://doi.org/10.1080/0020739X.2021.1958943

Hayes, A. H. (2024). The story of definite integrals: A calculus textbook narrative analysis. *International Journal of Education in Mathematics, Science and Technology*, *12*(1), 139–177. https://doi.org/10.46328/ijemst.3045

Jones, S. R. (2013). Understanding the integral: Students' symbolic forms. *The Journal of Mathematical Behavior*, *32*(2), 122–141. https://doi.org/10.1016/j.jmathb.2012.12.004

Jones, S. R. (2015). The prevalence of area-under-a-curve and anti-derivative conceptions over riemann sum-based conceptions in students' explanations of definite integrals. *International Journal of Mathematical Education in Science and Technology*, *46*(5), 721–736. https://doi.org/10.1080/0020739X.2014.1001454

Kastberg, S. E. (2002). *Understanding mathematical concepts: The case of the logarithmic function*. University of Georgia. Retrieved from https://jwilson.coe.uga.edu/Pers/Dissertations/kastberg_signe_e_200205_phd.pdf

Larson, R. (2010). *Cálculo de una variable [Calculus of a variable]*. Mc Graw Hill. https://ingindustrial869624637.wordpress.com/wp-content/uploads/2019/02/calculo-1-ron-larson-.pdf

Lehmann, T. H. (2024). Sustaining quantitatively-grounded meanings for definite integrals in high school calculus. *Mathematical thinking and learning*, *28*(1), 23–52. https://doi.org/10.1080/10986065.2024.2384689

Lucariello, J., Tine, M. T., & Ganley, C. M. (2014). A formative assessment of students' algebraic variable misconceptions. *The Journal of Mathematical Behavior*, *33*, 30–41. https://doi.org/10.1016/j.jmathb.2013.09.001

Maharaj, A. (2014). An APOS analysis of natural science students' understanding of integration. *Journal of Research in Mathematics Education*, *3*(1), 54–73. https://doi.org/10.4471/redimat.2014.40

Martínez-Planell, R., & Trigueros, M. (2020). Students' understanding of riemann sums for integrals of functions of two variables. *The Journal of Mathematical Behavior*, *59*, 100791. https://doi.org/10.1016/j.jmathb.2020.100791

Mateus-Nieves, E., & Font Moll, V. (2021). Epistemic complexity of the mathematical object “integral”. *Mathematics*, *9*(19), 2453. https://doi.org/10.3390/math9192453

McGee, D. L., & Martinez-Planell, R. (2014). A study of semiotic registers in the development of the definite integral of functions of two and three variables. *International Journal of Science and Mathematics Education*, *12*(4), 883–916. https://doi.org/10.1007/s10763-013-9437-5

Merriam, S. B. (2022). Qualitative research: Designing, implementing, and publishing a study. In V. Wang (Ed.), *Handbook of research on educational leadership and research methodology* (pp. 346–360). IGI Global Scientific Publishing. https://doi.org/10.4018/978-1-6684-4144-2.ch019

Narjaikaew, P. (2013). Alternative conceptions of primary school teachers of science about force and motion. *Procedia - Social and Behavioral Sciences*, *88*, 250–257. https://doi.org/10.1016/j.sbspro.2013.08.503

Park, J., Park, M. S., Park, M., Cho, J., & Lee, K. H. (2013). Mathematical modelling as a facilitator to conceptualization of the derivative and the integral in a spreadsheet environment. *Teaching Mathematics and its Applications*, *32*(3), 123–139. https://doi.org/10.1093/teamat/hrt012

Ponce-Campuzano, J. C. (2013). Developing prospective mathematics teachers in Mexico: A lesson on the relationship between integration and differentiation. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 996–1006. https://doi.org/10.1080/0020739X.2013.826386

Raviolo, A., Ekkert, T., & Carballo, C. (2024). La comprensión de estudiantes de primer año de universidad sobre el concepto de densidad. Análisis de entrevistas [First-year university students' understanding of the concept of density: An analysis of interviews]. *Educación Química*, *35*(3), 113–124. https://doi.org/10.22201/fq.18708404e.2024.3.86553

Rodríguez Gómez, G., Gil Flores, J., & García Jiménez, E. (1999). *Metodología de la investigación cualitativa [Qualitative research methodology]*. Ediciones Aljibe.

Salgado-Beltrán, G., & García-García, J. (2024). Concepciones alternativas sobre el concepto de pendiente en estudiantes de nivel medio superior de una zona rural [Alternative conceptions of the concept of slope among upper secondary school students in a rural area]. *IE Revista de Investigación Educativa de la REDIECH*, *15*, e1942. https://doi.org/10.33010/ie_rie_rediech.v15i0.1942

Sealey, V. (2014). A framework for characterizing student understanding of riemann sums and definite integrals. *The Journal of Mathematical Behavior*, *33*, 230–245. https://doi.org/10.1016/j.jmathb.2013.12.002

Serhan, D. (2015). Students' understanding of the definite integral concept. *International Journal of Research in Education and Science*, *1*(1), 84–88.

Stake, R. E. (2013). *Multiple case study analysis*. Guilford press.

Stewart, J. (2018). *Cálculo de una variable: Trascendentes tempranas [Single-variable calculus: Early transcendentals]*. Cengage Learning.

Strike, K. A., & Posner, G. J. (1992). A revisionist theory of conceptual change. In R. Duschl & R. Hamilton (Eds.), *Philosophy of science, cognitive psychology, and educational theory and practice* (pp. 147–176). SUNY.

Tarr, H. L., & Maharaj, A. (2021). A preliminary genetic decomposition for conceptual understanding of the indefinite integral. *The Journal of Mathematical Behavior*, *63*, 100891. https://doi.org/10.1016/j.jmathb.2021.100891

Tatar, E., & Zengin, Y. (2016). Conceptual understanding of definite integral with geogebra. *Computers in the Schools*, *33*(2), 120–132. https://doi.org/10.1080/07380569.2016.1177480

Thompson, P., & Silverman, J. (2008). The concept of accumulation in calculus. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 43–52). The Mathematical Association of America. https://doi.org/10.5948/UPO9780883859759.005

Wagner, J. F. (2018). Students' obstacles to using riemann sum interpretations of the definite integral. *International Journal of Research in Undergraduate Mathematics Education*, *4*(3), 327–356. https://doi.org/10.1007/s40753-017-0060-7

Zakaryan, D., & Sosa, L. (2021). Conocimiento del profesor de secundaria de la práctica matemática en clases de geometría [Secondary school teacher's knowledge of mathematical practice in geometry classes]. *Educación matemática*, *33*(1), 71–97. https://doi.org/10.24844/EM3301.03

Zolt, H., Wrightsman, E., Ford, L., & Patterson, C. L. (2023). Believing in infinity: Exploring students' conceptions of improper integrals. *PRIMUS*, *33*(5), 502–516. https://doi.org/10.1080/10511970.2022.2082615