Infinity Journal of Mathematics Education
Volume 15, No. 1, 2026
p-ISSN 2089-6867
e–ISSN 2460-9285
https://doi.org/10.22460/infinity.v15i1.p177-200

Local instructional theory STEM: Integrating the context of football into parabola learning to support prospective teachers' flexibility

Tika Dwi Nopriyanti, Zulkardi*, Ratu Ilma Indra Putri, Nyimas Aisyah
Department of Mathematics Education, Universitas Sriwijaya, South Sumatra, Indonesia
*Correspondence: zulkardi@unsri.ac.id

Received: Aug 6, 2025 | **Revised**: Dec 20, 2025 | **Accepted**: Dec 21, 2025 | **Published Online**: Feb 19, 2026

Abstract

Mathematics learning for prospective teachers frequently emphasizes procedural proficiency, while the development of mathematical flexibility, particularly the ability to shift representations and adapt problem-solving strategies, remains underdeveloped. This limitation is often associated with learning designs that insufficiently integrate STEM perspectives and meaningful real-world contexts. To address this issue, this study aims to develop a STEM-based Local Instructional Theory (LIT) using a football context to strengthen the mathematical flexibility of prospective mathematics teachers. The research employed a design research approach, a validation study type, conducted in two main phases: a pilot experiment and a teaching experiment. The learning design was iteratively refined through continuous reflection between the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT). Data were collected through classroom observations, learning videos, students' written work, and documentation of instructional activities, and were analyzed qualitatively. The findings indicate that STEM-based learning grounded in a football context effectively enhances students' representational and strategic flexibility, as evidenced by their ability to move among visual, numerical, and symbolic representations and to reflectively evaluate problem-solving strategies. The integration of digital tools such as Desmos and Kinovea further supported the visualization and validation of mathematical models. This study suggests that the developed LIT provides a contextual and innovative framework for improving mathematics teacher education.

Keywords:
Design research, Football, Local instructional theory, Mathematical flexibility, STEM

How to Cite:
Nopriyanti, T. D., Zulkardi, Z., Putri, R. I. I., & Aisyah, N. (2026). Local instructional theory STEM: Integrating the context of football into parabola learning to support prospective teachers' flexibility. *Infinity Journal*, *15*(1), 177-200. https://doi.org/10.22460/infinity.v15i1.p177-200

1. INTRODUCTION

Geometry is one of the important branches of mathematics that needs to be mastered by students and prospective teachers, as it forms a foundational component of mathematical reasoning and instruction (Aisyah et al., 2023; Maarif et al., 2018). Understanding geometry, particularly parabolic equations, is essential not only for solving problems related to conic sections and function graphs, but also for connecting algebraic and geometric representations, which are central to advanced mathematics learning (Gondin & Sohmer, 1967; Kilner & Farnsworth, 2019; Shriki, 2011; Sudirman et al., 2024). For prospective teachers, this conceptual connection is especially critical, as it directly influences their ability to explain mathematical ideas coherently, select appropriate representations, and design meaningful learning experiences for their future students. Mastery of parabolic concepts therefore does not only support visualization skills, but also underpins the development of logical, analytical, and pedagogical reasoning required in mathematics teaching. Research further indicates that flexible thinking skills play a significant role in deep conceptual understanding and effective problem solving (Hong et al., 2023).

However, in practice, substantial difficulties in mastering parabolic concepts persist among both students and prospective teachers. Parabolas are often perceived as abstract and challenging, particularly when learners struggle to relate algebraic expressions to their corresponding geometric forms. Many learners can interpret verbal or graphical representations separately but fail to understand the underlying connections between them, resulting in fragmented conceptual understanding (Noto et al., 2019; Peralta-García et al., 2020; Sanchez & Glassmeyer, 2016). This issue becomes more critical for prospective teachers, as insufficient conceptual integration may be transferred into their instructional practices, potentially reinforcing students' misconceptions rather than resolving them. Previous studies indicate that limited understanding of parabolas is closely associated with low mathematical flexibility, which constrains learners' ability to employ multiple strategies and solve problems adaptively (Bolat & Arslan, 2024; Dina et al., 2018; Isyrofinnisak et al., 2020; Martinez et al., 2025).

Low flexibility often causes learners to rely on a single solution strategy and experience difficulty when required to shift to more efficient or context-appropriate approaches (Scheibling-Sève et al., 2017). In contrast, mathematical flexibility enables individuals to adapt strategies according to varying problem conditions and instructional goals (Ye et al., 2023). For prospective teachers, such flexibility is indispensable, as they must not only solve problems themselves but also anticipate diverse student responses and guide learning through multiple representations (Star & Rittle-Johnson, 2008).

One promising approach to addressing these challenges is to connect parabola learning with familiar real-world contexts, such as football. The curved trajectory of a kicked ball provides a concrete and observable representation of a parabola, making abstract concepts more accessible and meaningful. This context has been shown to enhance motivation and mathematical problem-solving skills (Permatasari et al., 2018). In this regard, STEM-based learning offers a relevant framework for integrating modeling, technology, and contextual reasoning to foster flexible thinking (Rais et al., 2025; Voica & Singer, 2012).

Within this context, Local Instructional Theory (LIT) provides a powerful framework for systematically designing and refining learning trajectories grounded in students' needs and local contexts. Although LIT has been widely applied in realistic mathematics education, its integration with STEM-based learning, particularly in parabolic instruction using football contexts, remains limited (Gravemeijer & Cobb, 2006). Therefore, the urgency of this research lies in developing and validating a Local Instructional Theory integrated with a football-context-based STEM approach to strengthen prospective teachers' understanding of parabolic concepts and their mathematical flexibility, which are essential competencies for effective mathematics teaching in the 21st century.

2. METHOD

This research employed a qualitative design research approach, adopting the validation study type as proposed by Gravemeijer and Cobb (2006), to develop and validate a STEM-based Local Instructional Theory. This approach was chosen because the main objective of the research is to develop and validate a STEM-based Local Instructional Theory (LIT) that integrates the context of soccer into mathematics learning, specifically for the topic of parabolas. This LIT is designed to support the strengthening of prospective teachers' mathematical flexibility, which is the ability to think adaptively, use various strategies, and switch between mathematical representations in solving problems (Star & Rittle-Johnson, 2008).

Gravemeijer and Cobb (2006) emphasize that Design Research is designed to bridge the gap between theory and practice thru three main goals: (1) generating local instructional theories, (2) developing learning interventions, and (3) understanding the learning process. Within this framework, Local Instructional Theory (LIT) is not merely an end product, but grows dynamically from the design of the Hypothetical Learning Trajectory (HLT), implementation in learning experiments, and reflective analysis of students' learning processes. According to Bakker (2018), Design Research is a methodological strategy that allows researchers to iteratively design and revise learning designs based on empirical data from real classroom practice. In other words, this approach is conjecture-driven, where hypotheses about the expected learning process are compared with the reality of the observed learning process in the classroom.

Furthermore, Plomp and Nieveen (2013) state that design research allows for the development of learning designs that are not only contextual and meaningful, but also theoretical and academically accountable. In the context of mathematics education, using real-world contexts such as sports (in this case, football) has proven effective in increasing the relevance and motivation of students' learning (Queiruga-Dios et al., 2025), as well as strengthening flexibility in modeling and representing mathematical phenomena (Hickendorff et al., 2022). This study focused on the development and validation of a contextual and challenging STEM-based learning design for parabolic concepts. The football contexts of throw-ins and the crossbar challenge were selected for their potential to induce cognitive conflict, stimulate mathematical modeling, and enhance representational flexibility (Zulkardi & Putri, 2020). Thus, tfhe resulting Local Instructional Theory (LIT) was intended to align with curriculum requirements while theoretically strengthening prospective teachers' conceptual understanding of quadratic functions in real-life contexts. This study employed a qualitative design research methodology of the validation study type (Gravemeijer & Cobb, 2006). The research was conducted in three sequential stages: (1) preliminary research, (2) experimental design, and (3) retrospective analysis (Bakker, 2018). Participants were selected using purposive sampling, involving one intact class of prospective mathematics teachers enrolled in a geometry course (46 students). This sampling strategy was chosen to enable an in-depth analysis of the learning process, the refinement of the learning trajectory, and the validation of the developed instructional theory.

2.1. Preliminary

At this stage, the researcher conducted a literature review on the STEM (Science, Technology, Engineering, Mathematics) approach, mathematical flexibility skills, and explored the learning context to be used in teaching the Parabola material (see Table 1). During the preliminary phase, participants were selected using purposive sampling based on their relevance to parabola learning. The participants included prospective mathematics teachers, university mathematics education lecturers, and secondary school mathematics teachers. Lecturers contributed theoretical and curricular perspectives in teacher education, while teachers provided practical classroom insights related to students' learning difficulties and instructional feasibility. Prospective teachers represented learners' perspectives. Data were collected through interviews and guided discussions to identify meaningful contexts for teaching parabolic concepts. The findings indicated that football-related contexts were relevant and cognitively challenging, leading to their selection as the instructional context for this study. As a result, the researcher chose to use the football context in this study for teaching the parabola equation material.

The results of the literature review conducted by the researcher on STEM, particularly in Indonesia, indicate that the STEM approach can help students prepare for 21st-century skills and abilities, and assist in developing mathematical thinking skills such as creative thinking, critical thinking, spatial abilities, and learning outcomes (Mandala et al., 2025; Nindiasari et al., 2024; Nopriyanti et al., 2024; Pramasdyahsari et al., 2025). Additionally, the researcher collected initial data from lecturers, teachers, and 8th and 2nd semester students in mathematics eduation in to identify the appropriate context for the study, and the results showed that 42.4% of respondents believed that the context of football could be used in geometry lessons on the topic of parabolas (Nopriyanti et al., 2025).

Table 1. The learning design in parabola topic

Learning Step	Context	Learning Objective	Learning Activities	Thinking and learning process	Parabola content
Parabola Equation Analysis of Arhan's throw-in phenomenon , studying the relationship between throwing angle and distance, making predictions about the	Throw-ins in Football (Arhan's phenomenon)	Students are able to understand the relationship between position, angle, and the trajectory of the ball, analyze and form parabolic equations, develop the ability to predict and validate	Group discussions about Arhan's throwing technique, hands-on experiments with variations in throwing angles and positions, creating graphs and analyzing results using	Students believe graphs can be created from videos or photos by determining specific points, using parameters a, b, and c in the equation $y=ax^2+bx+c$ to adjust the ball's trajectory, demonstrating flexibility in thinking by constructing	Parabola Equation Understanding the quadratic equation y=ax2+bx+ c, analyzing the parabola graph based on the starting, vertex, and ending points, the influence of coefficients on the shape of the parabola

Learning Step	Context	Learning Objective	Learning Activities	Thinking and learning process	Parabola content
parabola graph based on the data obtained, and finding the parabola equation from the analyzed graph.		mathematical models of physical phenomena, and strengthen flexible thinking thru exploring various parameters within the context of parabolas.	Desmos, and determining the parabolic equation from three points, which was then validated thru Desmos graphs.	models from empirical data, evaluating the model's accuracy thru visualization and measurement, and adjusting coefficient values based on changes in the graph as a reflection of conceptual flexibility.	such as $a< 0$ for a parabola opening downwards, and estimating the trajectory of a ball using the Desmos application for prediction and proof purposes.
Tangent Line of a Parabola The learning steps in this lesson include introducing the concept of a tangent point thru everyday phenomena like the Crossbar Challenge, analytically determining the equation of a tangent line, and predicting the equation of a tangent line from a graph using the Desmos application.	The phenomenon of the ball hitting the crossbar (Crossbar Challenge)	Students understand and explain the concept of a tangent point on a parabola, are able to use derivatives to determine the gradient of a tangent line, can find the equation of a tangent line both analytically and visually thru Desmos, and develop critical and flexible thinking skills.	Students watched a Crossbar Challenge video, discussed the ball's trajectory to identify the point of tangency, sketched the trajectory and tangent line using Desmos, performed derivative analysis of the parabolic equation, and visually and analytically predicted and proved that the tangent line only touches the parabola at one point.	Students who initially had the misconception that tangent points only apply to circles, later learned to connect the concepts of derivatives, gradients, graphs, and digital applications in a single activity, demonstrating flexibility in thinking thru the transition between visual and analytical approaches. Discussions and exploration using Desmos helped improve understanding and correct logical errors.	Tangent Line Equation Understanding the general equation of a parabola $y=ax^2+bx+ c$, the concept of the first derivative $y'=2ax+ b$ to determine the slope of the tangent line, the application of derivatives in finding the tangent line at a specific point, and the visualization of graphs as a tool for analytical validation.

2.2. Design Experimen

The experimental design in this study aims to develop, test, and revise the Hypothetical Learning Trajectories (HLT) that were developed in the preliminary stage. The experimental design was conducted thru two cycles: pilot experiment and teaching experiment, with each cycle consisting of nine instructional sessions. 5 meetings were for the content of the parabola equation and 4 meetings were for the content of the tangent line to the parabola.

The pilot experiment stage aims to test the initial feasibility of the learning design, the suitability of the football context with mathematical concepts, and to identify potential misconceptions and initial challenges in implementing the design. This stage was carried out with the involvement of 22 students who have taken the analytical geometry course. Data collection at this stage consists of classroom observation sheets, video recordings of learning, student work documents such as LKM (Student Worksheet) or work done in the Desmos application, and the researcher's field notes or findings during the research. The results of this stage are used to revise the instructional design, including context, sequence of activities, scaffolding, and prompting questions to encourage flexible thinking, thereby enhancing the desired learning activities.

The teaching experiment stage is the implementation of the revised learning design from the pilot experiment stage. The participants consisted of 29 students enrolled in an Analytic Geometry course. The main objectives of this stage are to see how the revised and improved learning design can function in exploring the learning process, the use of the football context in STEM-based learning on the parabola material works well according to learning objectives, and the development of prospective teacher students' thinking flexibility.

2.3. Retrospective Analysis

At this stage, a retrospective analysis was conducted by systematically comparing the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT) identified during the pilot experiment and teaching experiment phases. The analysis followed a qualitative procedure consisting of data organization, data reduction, coding, and interpretative comparison. Data sources included students' written work in activity sheets, classroom video recordings, and the researcher's field notes. First, all data were organized chronologically according to learning sessions and instructional activities. Second, relevant segments related to students' strategies, representations, and interactions were coded to identify patterns of representational and strategic flexibility. Third, the coded data were compared with the anticipated learning processes described in the HLT to identify consistencies, deviations, and emergent learning pathways. Particular attention was given to students' transitions from contextual interpretations of football-related situations (throw-ins and crossbar challenges) to formal mathematical representations of parabola equations and tangent lines, supported by the use of digital tools such as Desmos. The results of this analysis informed the refinement of the Learning Trajectory (LT) and contributed to the development of a validated Local Instructional Theory (LIT) integrating football contexts to support prospective teachers' flexible mathematical thinking.

3. RESULTS AND DISCUSSION

This research develops Local Instructional Theory (LIT) to support the flexibility skills of prospective teachers by integrating the context of soccer into this study, specifically using throw-ins and the crossbar in the game of soccer. Gravemeijer and van Eerde (2009) state that LIT can be designed by involving various LT, according to the complexity of the instructional domain being addressed. Therefore, the LIT developed in this study consists of 2 LT, each discussing the equation of a parabola and the tangent line of a parabola. Each LT is developed based on STEM learning steps contextualized with soccer toward understanding the parabola material. This paper presents the integration of a series of research studies into a broader dissertation, where each research component focuses on a more specific LT and some have been published in reputable journals. Therefore, this paper synthesizes all these findings into LIT to support students' flexibility abilities contextually and relevantly.

3.1. Results

3.1.1. Preliminary Study

The preliminary study aims to develop a hypothetical learning trajectory (HLT) based on student characteristics and local context, making learning meaningful and contextual (Doorman et al., 2016; Prahmana & Kusumah, 2016; van Eerde, 2013). The preliminary studies conducted by the researcher at this stage involved searching for literature reviews on the STEM approach (Goos et al., 2023; Honey et al., 2014), learning using context to promote flexibility skills (Cox et al., 2024; Hasanah & Retnawati, 2022; Hickendorff et al., 2022; Tashtoush et al., 2024; Zulkarnain et al., 2023), and the use of football context in mathematics learning (Andriadi et al., 2021; Cleland, 2015; Karlis et al., 2021; Nopriyanti et al., 2025; Ramulu et al., 2024). The football context used in this study is the throw-in, a phenomenon performed by football player Arhan Pratama, for the parabola equation material, and the crossbar in football for the tangent line material.

At this stage, initial validation was conducted through expert review and a Focus Group Discussion (FGD). The experts involved consisted of mathematics education researchers with expertise in design research and Local Instructional Theory (LIT), curriculum specialists in mathematics education, and instructional design experts. The FGD involved purposively selected participants, including university mathematics education lecturers, prospective mathematics teachers, and secondary school mathematics teachers. In the context of this study, lecturers contributed theoretical, curricular, and pedagogical perspectives related to teacher education and the formal structure of parabolic concepts, while secondary school teachers provided practical classroom-based insights concerning students' learning difficulties, contextual relevance, and instructional feasibility. Prospective teachers participated to represent learners' perspectives within teacher education. The outcomes of this initial validation process informed the refinement of the initial Hypothetical Learning Trajectory (HLT) and the development of the Student Worksheets. Figure 1 illustrates the resulting learning trajectory, highlighting how STEM-based learning in a football context supports the development of mathematical flexibility.

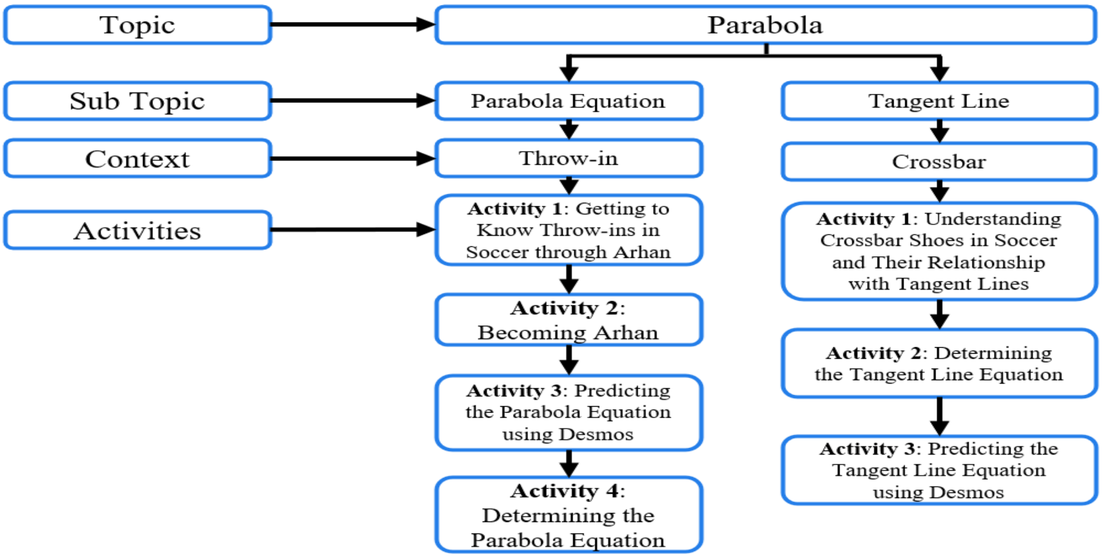


Figure 1. Learning trajectory based on STEM in the context of football to support flexibility skills

Figure 1 shows a STEM-based learning trajectory that integrates the context of soccer to support the flexibility skills of prospective teachers. The learning trajectory, designed using STEM-based learning steps, connects multiple disciplines, in this case sports and physics, so that students understand the relationship or application of mathematical knowledge with other sciences. STEM-based learning for the topic of parabolic equations using the context of a phenomenal throw-in performed by a football player named Arhan. Students will be guided to understand that the type of throw occurring is a parabola, meaning they can find its parabolic equation. At this stage, students will be guided to find the equation of a parabola from the path formed in various ways, thereby encouraging their flexibility. In the second instructional material, the learning context focuses on the crossbar, represented by the event of a ball striking the goalpost. To enhance conceptual understanding, an actual image depicting this situation was presented to students at the beginning of the activity. In the first activity, students were asked to analyze the meaning of the term crossbar and to explore possible reflections of the ball's trajectory (see Figure 2). At this stage, students' mathematical flexibility became apparent through the diversity of their interpretations, representations, and explanations derived from the visual context.

Figure 2. A football hitting the crossbar as a contextual representation for tangent line concepts (Source: youtube)

At this stage, the students' flexibility will be evident in their diverse responses. Students will then sketch and determine the tangent line equation obtained from the crossbar incident in various ways, such as obtaining the tangent line equation from a known vertex or from three different points, thus further developing students' flexibility skills. Context-based learning of parabolas – whether thru digital visualization, physical (real-world) analogies like sports, bridges, or satellite dishes, or collaborative dialog – consistently improves learners' conceptual understanding and representational flexibility in mathematics (Abdu et al., 2022; de Sousa & Alves, 2022; Florio, 2022).

3.1.2. Design Exsperiment

The design experiment stage is conducted after the preliminary stage is completed and two learning trajectories (LTs) have been obtained. The pilot experiment stage is conducted with 22 prospective teacher students with the aim of exploring the sequence of activities in each LT and examining the alignment between the initial design predictions and the students' responses, as well as the flexibility of the prospective teacher students in learning. At this stage, the researcher focuses on observing student engagement in activities within the STEM-based learning process that uses the context of soccer to identify unexpected strategies, different understandings, or misconceptions during learning. The data at this stage uses classroom observation sheets, video recordings of learning, student work documents such as LKM sheets or work in the Desmos application, and notes taken during discussions, such as activities that were still confusing for students in terms of their steps, different understandings in understanding problems that use English, leading to difficulties in completing the given activities, and instructions during field practice that required students to measure the distance of the ball using different measuring tools for each group, thus developing students' flexibility skills in this activity.

Based on these findings, another FGD was conducted with the teaching faculty to re-discuss the results of the pilot experiment. Revisions were made primarily to the language used in the Desmos application, which is in Indonesian, to improve the activity questions in the Desmos application, to provide more detailed instructions in the activity sheets, and to include leading questions in discussions to arrive at the correct solutions and a proper understanding of the activities. After being revised, the activities were implemented in a teaching experiment to validate the improvements and systematically observe student development in learning. Here are the results from each LT, showing how these changes impacted the understanding and learning outcomes of prospective teachers regarding the topic of parabolas.

Learning Trajectory: STEM-Based Learning in the Context of Football for the Parabola Equation Material to Enhance the Flexibility Skills of prospective Teachers

At the pilot stage, the experiment showed that the Throw in context can be integrated into the content of the parabolic equation, especially when students practice the throw in and visualize it in the Desmos application, and determine the parabolic equation formed by the ball's trajectory. Additionally, students will also find that the initial velocity and elevation angle also affect the distance the ball travels. The STEM-based learning implemented can integrate several sciences, in this study, Physics and Sports (Science), the technology used in learning is the Kinovea application to measure elevation angles (Technology), Engineering occurs when prospective teacher students predict the formed parabolic equation, and

Mathematics when finding solutions to problems. Table 2 shows the STEM Elements in Football Contextual Learning Parabola Material.

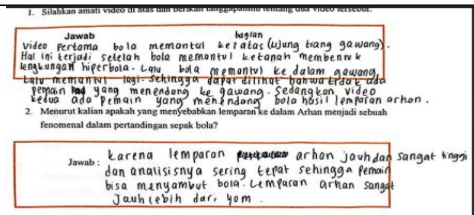
Table 2. STEM elements in football contextual learning parabola material

STEM Components	Implementation In Learning	Goals / Role In Flexibility
Science	Students observe the physical phenomenon of the ball's trajectory during the crossbar challenge and throw-in activities. The discussion covers gravitational force, acceleration, and the laws of projectile motion in a real-world context.	Promoting conceptual understanding of real-world phenomena and building connections between physical representations and mathematical models of trajectories.
Technology	Using video tracker applications like Kinovea to analyze ball movement: measuring throwing angles, initial velocity, and trajectory coordinates; and Desmos to model parabolic functions based on real data.	This enhances dynamic representation and visualization of ball trajectories, allowing students to flexibly compare mathematical models with empirical data.
Engineering	Students design and predict throwing strategies using the principles of angle and throwing speed optimization. They evaluate solutions against the problem context.	Training students to evaluate alternative solutions in problem-solving, integrate concepts from multiple fields, and develop analytical and synthetic abilities for solutions.
Mathematics	Students construct the equation of a parabola from the data obtained, analyze the characteristics of the parabola (vertex, axis of symmetry, tangent line), and use geometric concepts to determine the gradient of the tangent line.	Strengthening students' ability to switch between representations (graphs, algebra, verbal, contextual) and think flexibly in solving open and real-world mathematical problems.

Learning at the experimental pilot stage began with the question, "Do you like playing soccer?" and it turned out that although soccer is often associated with men, there were female students who enjoyed playing it. Next, the lecturer will ask a follow-up question: "Do you know Pratama Arhan?" a phenomenal football player right now, followed by the question, "Why is Arhan's throw so phenomenal?" This question will direct students to focus on the context that will be used, which is throw-ins in football.

To steer the discussion toward students' flexibility skills, the lecturer continues with the question, "In your opinion, what influences a throw?" This question is designed to train students' flexibility skills because it will encourage them to think from various perspectives and provide reasons for each answer, and they will be asked to prove their answers based on the results of the designed activity.

After that, students were given an opening activity sheet asking them to observe and analyze two videos of Pratama Arhan's throw-ins, which served as a strategic starting point for building connections between real-world context, conceptual understanding, and mathematical representation, and for forming and strengthening the flexibility of prospective teacher students in understanding and teaching the concept of parabolas. Real-world context in mathematics learning not only increases motivation but is also highly effective in building mathematical flexibility (Hickendorff et al., 2022; Zulkardi & Putri, 2020). However, observations in the classroom revealed that students had not explicitly compared the two videos or linked the phenomena to mathematical aspects such as angle of elevation, maximum distance, or initial velocity, and the ability to think flexibly in the form of shifting between representations (visual-to-mathematical, verbal-to-graphical) was not strongly evident in either student's responses. Their answers indicate that they are still in the early stages of contextual understanding and have not yet reached the ability to choose or formulate alternative strategies for explaining or mathematically modeling the ball's trajectory. Here are the students' answers to activity 1 (see Figure 3).



1. Silahkan amati video di atas dan berikan tanggapanmu tentang dua video tersebut.

Jawab

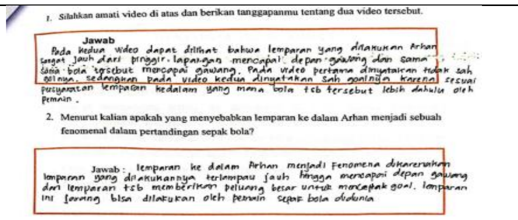
video pertama bola memantul ke atas (ujung tiang gawang). Hal ini terjadi setelah bola memantul ketanah membentuk lengkungan hiperbola. Lalu bola memantul ke dalam gawang, Lalu memantul lagi. sehingga dapat dilihat bahwa tidak ada pemain yang menendang ke gawang. Sedangkan video kedua ada pemain yang menendang bola hasil lemparan arhan.

2. Menurut kalian apakah yang menyebabkan lemparan ke dalam Arhan menjadi sebuah fenomenal dalam pertandingan sepak bola?

Jawab : karena lemparan arhan jauh dan sangat tinggi dan analisisnya sering tepat sehingga pemain bisa menyambut bola. Lemparan arhan sangat jauh lebih dari yom.

English Version1

1. Watch the video above and write your opinion about it.
   Answer:
   The first video shows the ball hitting the crossbar. This happened after the ball formed a parabolic trajectory. In the second video, the ball was thrown straight into the goal. Based on this, Arhan's throw is very strong and directed. The ball could reach far, and from his movement, the ball seemed to spin forward due to the angle and technique of the throw.
2. Why do you think Arhan's throw-in has become a phenomenon in football?
   Answer:
   Because Arhan's throw-in distance is very far and his accuracy is often precise, making the throw very difficult to anticipate—even the goalkeeper is very far away.



1. Silahkan amati video di atas dan berikan tanggapanmu tentang dua video tersebut.

Jawab

Pada kedua video dapat dilihat bahwa lemparan yang dilakukan Arhan sangat jauh dari pinggir lapangan mencapai depan gawang dan sama sama bola tersebut mencapai gawang. Pada video pertama dinyatakan tidak sah golnya. sedangkan pada video kedua dinyatakan sah golnya karena sesuai persyaratan lemparan kedalam yang mana bola tersebut lebih dahulu oleh pemain.

2. Menurut kalian apakah yang menyebabkan lemparan ke dalam Arhan menjadi sebuah fenomenal dalam pertandingan sepak bola?

Jawab : lemparan ke dalam Arhan menjadi Fenomena dikarenakan lemparan yang dilakukannya terlampau jauh hingga mencapai depan gawang dan lemparan tsb memberikan peluang besar untuk mencetak goal. lemparan ini jarang bisa dilakukan oleh pemain sepak bola didunia

English Version2

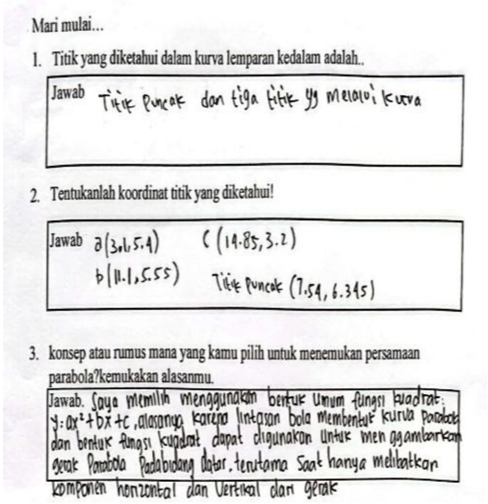
1. Watch the video above and write your opinion about it.
   Answer:
   In both videos, it can be seen that the throw-ins performed by Arhan are very far. The parabolic path reached the goal or the crossbar. The throw-in technique is excellent, and the power is very strong. This is clearly visible from how far the ball was thrown. Even if no one touched the ball, it could still head straight to the goal.
2. Why do you think Arhan's throw-in has become a phenomenon in football?
   Answer:
   Because Arhan's throw-in is a rare phenomenon. His ability to throw so far and precisely makes this technique difficult for the opposing team to anticipate and is not commonly seen in football matches.

Figure 3. Student responses to activity 1

The early learning pathway is designed based on the characteristics of STEM-based learning, which include: (1) Integration of Disciplines; (2) Meaningful Real-World Context; (3) Active and Inquiry-Based Learning; (4) Use of Technology and Tools; (5) Engineering Design Process; (6) Collaboration and Communication; (7) Development of 21st-Century Skills; (8) Project-Based and Authentic Contextual Learning (Bybee, 2013; Chen & Chang, 2021; Holmlund et al., 2018; Larkin & Lowrie, 2022). The designed learning path has guided students to explore and practice their flexibility skills, however, field findings indicate that students still experience difficulty in determining the throwing angle in the video during the learning process in activity 2, and have misconceptions in activities 3 and 4.

Based on these findings, changes were made to the teaching experiment process for the activity sheets. In activity 2, the changes included providing instructions for determining the throwing angle using the Kinovea application, using Indonesian language in the Desmos activities, and adding questions in activity 4. The changes made still focus on the students' flexibility in solving parabolic problems within the context of a throw-in in soccer.

In the fourth activity, students will determine the equation of the parabola formed by the ball's trajectory. During the learning process, students will discuss with their respective groups to determine their individual methods for finding the equation of a parabola. The findings in the student class showed that some students were finding the equation of a parabola from three known points, while others were using one known point and the vertex.


Mari mulai...

1. Titik yang diketahui dalam kurva lemparan kedalam adalah..

Jawab: Titik Puncak dan tiga titik yg melalui kurva

2. Tentukanlah koordinat titik yang diketahui!

Jawab: a (3.1, 5.4) c (19.85, 3.2) b (11.1, 5.55) Titik Puncak (7.54, 6.345)

3. konsep atau rumus mana yang kamu pilih untuk menemukan persamaan parabola?kemukakan alasanmu.

Jawab. Saya memilih menggunakan bentuk Umum fungsi kuadrat: y = ax²+bx+c, alasannya karena lintasan bola membentuk kurva parabola dan bentuk fungsi kuadrat dapat digunakan untuk menggambarkan gerak parabola pada bidang datar, terutama saat hanya melibatkan komponen horizontal dan vertikal dari gerak

English version:

1. The known points on the ball throw curve are?
   Answer: The vertex and three points that lie on the curve.
2. Determine the coordinates of the known points!
   Answer:
   a (3.1, 5.4)
   b (11.1, 5.55)
   c (19.85, 3.2)
   Vertex Point: (7.54, 6.345)
3. Which concept or formula do you choose to find the equation of the parabola? Explain your reason.
   Answer: I chose to use the general form of the quadratic function: y = ax^2 + bx + c. The reason is that the ball's trajectory forms a parabolic curve, and the quadratic function can be used to represent parabolic motion on a flat plane, especially when considering only the horizontal and vertical components of the motion.

Figure 4. Student responses to activity 2

Questions 1 to 3 show that students are able to move from visual representations (ball trajectory images) to numerical representations (coordinate points), and then to algebraic representations (quadratic functions), demonstrating representational flexibility (see Figure 4). The use of symbolic visual representations and the construction of a representational flexibility model involving transformations between coordinate graphs, verbal, symbolic, and numerical representations strengthens students' algebraic thinking—showing the relationship between visual and symbolic representations in improving mathematical reasoning (Deliyianni et al., 2016; Ünal et al., 2023). Next, the students will perform mathematical calculations to find the equation of the parabola.

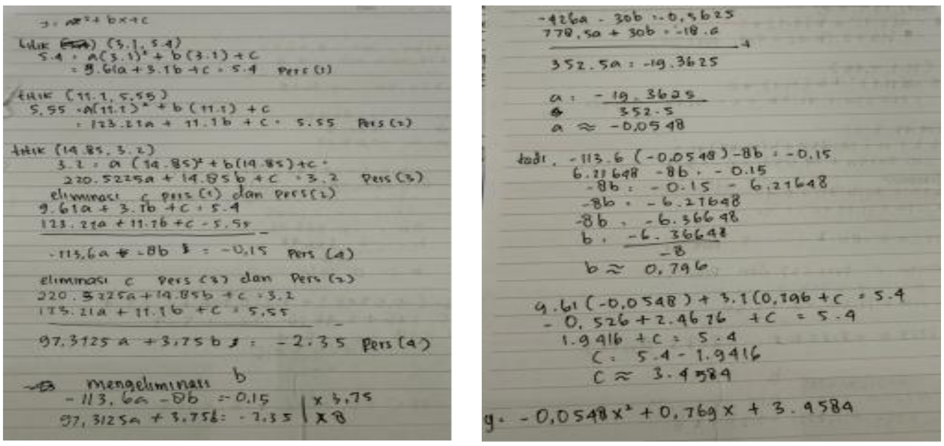


Figure 5. Student calculations in determining the equation of a parabola

Based on the Figure 5, it can be seen that the student successfully formed a system of 3 equations based on those points and arranged the elimination systematically and logically. This demonstrates algebraic ability and understanding of the relationships between variables in quadratic functions, as well as flexibility in solving systems of equations, not being limited to just one method. After obtaining the equation, the student verified the algebraic results ($y = -0.0548x^2 + 0.796x + 3.4584$) on the Desmos application, which showed a curve that matched the trajectory of the ball. This reflects an understanding of the relationship between representations: from algebra to visual graphics (converting real-world paths into mathematical models). Figure 6 shows the students' work in the Desmos application.

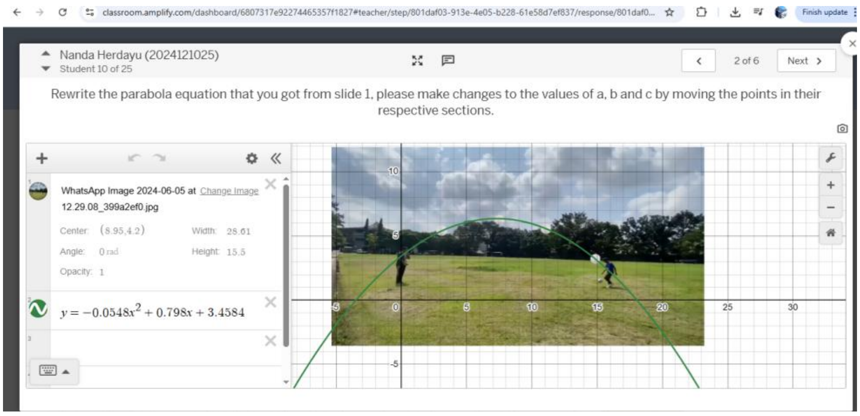


Figure 6. Student responses in activity desmos

Learning Trajectory: STEM-based Learning in the Context of Football for the Tangent Line of a Parabola Material to Enhance prospective Teachers' Flexibility Skills

This second learning trajectory presents the results of an experimental design focused on the second learning trajectory, namely STEM-based learning with the context of the crossbar challenge in soccer games for the topic of tangent lines to parabolas. This context was chosen because it naturally represents a situation where the ball's trajectory touches the top of the goalpost, which can be mathematically modeled as the point of tangency between a parabolic trajectory and a horizontal line. This context provides an authentic and meaningful learning experience for prospective teachers, and explicitly bridges the transition from visual representation to algebra. The main goal of this trajectory is to develop students' mathematical flexibility, particularly in interpreting contextual situations, choosing appropriate strategies, and flexibly transitioning between mathematical representations (visual, numerical, symbolic). Learning outcomes are analyzed based on students' thinking processes in constructing parabolic equations and tangent lines, as well as their ability to reflect on the interconnectedness between context, concepts, and problem-solving strategies. Table 3 shows the STEM elements in the context of football learning: Tangent lines and flexibility skills.

Table 3. STEM elements in the context of football learning: Tangent lines and flexibility skills

STEM Aspect	Implementation in Learning	Goal/Role Flexibility
Science	Understanding the parabolic trajectory as a representation of projectile motion in a ball kick (the physics of projectile motion).	Visually connecting physics concepts with the shape of the trajectory enhances representational understanding.

STEM Aspect	Implementation in Learning	Goal/Role Flexibility
Technology	Using the Desmos application to model and verify the trajectory of the ball and the tangent line.	Training the transition from visual to symbolic thru exploration and digital graphic simulation.
Engineering	Designing a kicking strategy to get the ball over the crossbar and analyzing the possible outcomes of the trajectory.	Encouraging students to consider various possible solutions and contextual approaches.
Mathematics	Determining the equation of a parabola, the point of tangency, and the equation of the tangent line using algebraic and derivative approaches.	Improving the ability to switch between numerical, graphical, and symbolic representations in problem-solving.

Learning will begin with a video about crossbars in football matches. "What can you conclude from the video that was shown?" Students will be encouraged to think and analyze from their respective perspectives to enhance the flexibility of future teacher candidates. After that, the lecturer will ask a follow-up question: "Why can a crossbar incident happen in a football match?" From this question, the students' answer was: "The ball hit the top edge of the goal." Next, to focus on the tangent line material, the lecturer asked, "In mathematics, what is the intersection between the ball and the crossbar called?" This question will initiate the activities to be carried out on the parabola tangent line material.

In Activity 1, students are invited to understand the concept of a tangent point in real life thru the phenomenon of a ball's trajectory touching the goalpost crossbar in the context of a crossbar challenge. The activity began with an observation of a ball-kicking challenge video, where students were asked to describe the ball's trajectory and identify the moment when the ball touched the crossbar. Without being given a formal definition, students are guided to intuitively explore the meaning of "touching" as the event when the trajectory curve intersects with the horizontal line (goalpost). This process encourages the transition from visual to geometric representation and supports the early development of representational flexibility, as students begin to build connections between physical phenomena and more abstract mathematical concepts.

In Activity 2, learning is focused on the more formal and mathematical application of the concept of tangent lines, with the aim that students are able to determine and sketch the equation of a parabola's tangent line using both numerical and differential approaches. In this activity, students continue the crossbar challenge context by modeling the ball's trajectory as a quadratic function and the goalpost as a horizontal line. Students are asked to determine the point where the ball touches the crossbar, then construct the equation of the tangent line using two different methods: first, by using a system of equations for two functions (parabola and line); second, by using derivatives to determine the gradient at the point of tangency. This activity requires students to move between different representations – from numerical (coordinates), to visual (graphs in Desmos), to symbolic (functions and their derivatives) – as well as compare two different solution strategies. This process strengthens students' mathematical flexibility in both representational and strategic aspects, as they not only understand concepts procedurally but are also able to select and adapt approaches based on the problem context.

In Activity 3, students are guided to independently explore the concept of a parabola's tangent line thru the use of digital technology, specifically the Desmos application. The purpose of this activity is for students to be able to predict the equation of the tangent line of a parabola graph and a given specific point. Students were asked to build their own parabola graphs in Desmos, choose points on the curve, and then discuss in groups how to determine the gradient and derive the equation of the tangent line analytically. After performing the calculations, students verified their prediction results using Desmos' visualization features. Group discussions continued when the prediction results did not match the graph, prompting them to revise their strategies and calculations. This activity provides ample opportunity to strengthen mathematical flexibility, particularly in the ability to switch between visual (graphical), symbolic (equations), and numerical (coordinates) representations, as well as in comparing analytical and visual results. Students are also challenged to reflect on the influence of parameters in the parabolic equation on the properties of the tangent line, which reinforces flexibility in dynamically understanding the relationships between concepts.

These three activities were initially given to the experimental pilots, and findings were obtained during the learning process, leading to revisions or improvements. For example, the structure of the steps and questions in the first activity was improved. The problems presented in the second activity, which were the same two problems in the second experimental pilot (i.e., the ball did not hit the crossbar), were revised in the teaching experiment stage to one problem where the ball touched the crossbar and one where it did not. As for the third activity, there were no changes or improvements from the experimental pilot to the teaching experiment. Here are some responses or answers from students in completing the given activity.

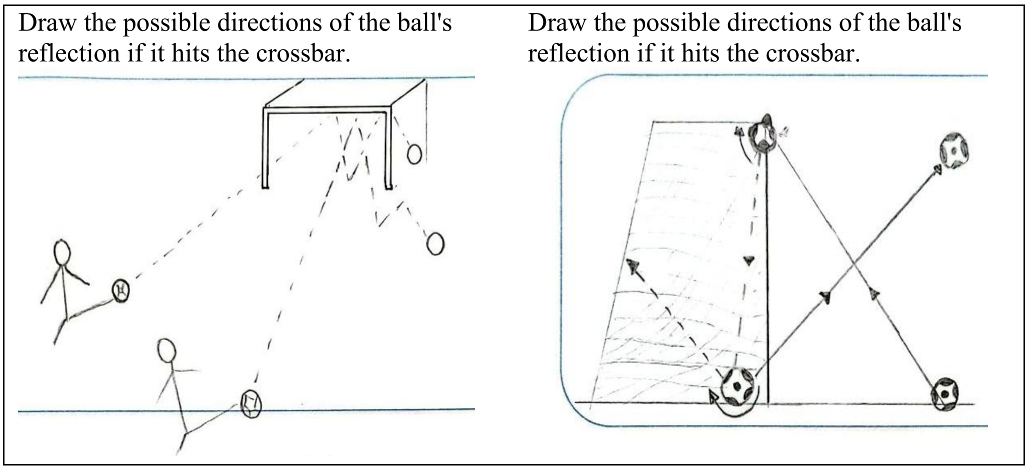
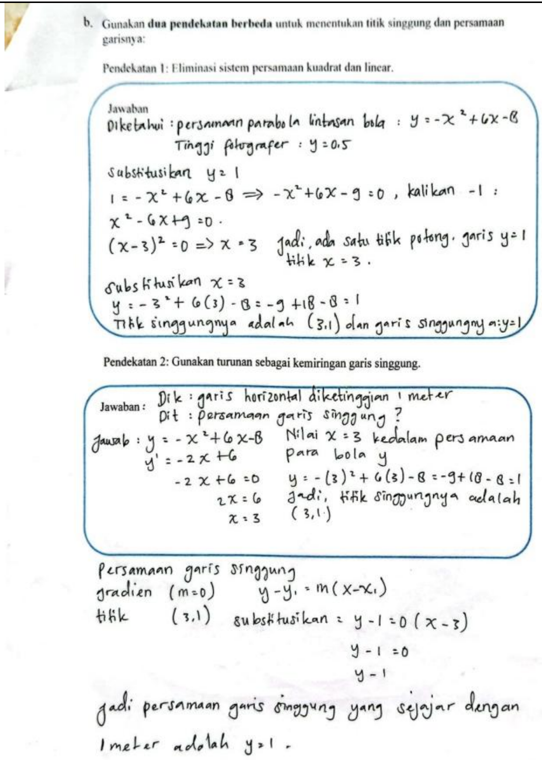


Figure 7. The student sketches the ball's reflection if it hits the crossbar

Based on Figure 7, it shows different levels of geometric understanding and mathematical flexibility. The first student simply described the event of the ball bouncing off the crossbar with one downward bounce scenario, reflecting a basic understanding of the point where the ball's trajectory meets the crossbar, but not yet demonstrating any exploration or variation in possible bounce outcomes. The visual representation used is still limited, without a transition to symbolic forms or consideration of reflection angles, resulting in low flexibility. Conversely, the second student presented a more complex image with multiple possible reflection directions, demonstrating a deeper understanding of geometry as well as stronger potential for representational flexibility. This student is considering variations in trajectory based on the direction and strength of the kick, and has indirectly established an initial connection to the concept of tangent lines or the angle of incidence and reflection, although it has not yet been expressed symbolically. This indicates that the second student is more reflective in connecting real-world contexts with diverse mathematical representations.

Next, in the second activity, students were given a problem and asked to use two different approaches or methods to solve it. Here are the results of the students' responses or work.


b. Gunakan **dua pendekatan berbeda** untuk menentukan titik singgung dan persamaan garisnya:

Pendekatan 1: Eliminasi sistem persamaan kuadrat dan linear.

Jawaban

Diketahui : persamaan parabola lintasan bola : $y = -x^2+6x-8$

Tinggi fotografer : $y = 0.5$

Substitusikan $y = 1$

$1 = -x^2+6x-8 \Rightarrow -x^2+6x-9 = 0$, kalikan -1 :

$x^2-6x+9 = 0$.

$(x-3)^2 = 0 \Rightarrow x = 3$ Jadi, ada satu titik potong. garis $y=1$ titik $x = 3$.

Substitusikan $x = 3$

$y = -3^2+6(3)-8 = -9+18-8 = 1$

Titik singgungnya adalah (3,1) dan garis singgungnya: $y=1$

Pendekatan 2: Gunakan turunan sebagai kemiringan garis singgung.

Jawaban : Dik : garis horizontal diketinggian 1 meter

Dit : persamaan garis singgung ?

Jawab : $y = -x^2+6x-8$ Nilai $x = 3$ kedalam persamaan parabola y

$y' = -2x+6$ $y = -(3)^2+6(3)-8 = -9+18-8 = 1$

$-2x+6 = 0$ Jadi, titik singgungnya adalah (3,1)

$2x = 6$

$x = 3$

Persamaan garis singgung

gradien ($m=0$) $y-y_1 = m(x-x_1)$

titik (3,1) substitusikan : $y-1 = 0(x-3)$

$y-1 = 0$

$y-1$

Jadi persamaan garis singgung yang sejajar dengan 1 meter adalah $y=1$.

English version:

b. Use two different approaches to determine the point of tangency and the equation of the tangent line.

Approach 1: Eliminating the system of quadratic and linear equations

Solution

Given:

The equation of the parabolic trajectory:

$$y = -x^2+6x-8$$

Height of the photographer: $y = 1$

Substitute $y= 1$ into the parabola equation:

$1 = -x^2+6x-8 \rightarrow x^2 + 6x -9 = 0$

$(x-3)^2= 0$, $x = 3$

Thus, there is one point of intersection. The line $y=1$ touches the parabola at $x=3$

Substitute $x=3$ into the parabola equation:

Approach 2: Using derivatives as the slope of the tangent line

Solution

Given:

A horizontal line at a height of 1 meter

Find:

The equation of the tangent line

The parabola equation: $y = -x^2+6x-8$

First derivative $y'= -2x + 6$, $y'=0$

$-2x+6 = 0$, $x = 3$

Substitute $x =3$ into parabola equation:

$y = -(3)^2+6(3)-8 = 1$, thus the point of tangency is (3,1)

Equation of the Tangent Line

Slope: $m = 0$

Point: (3,1)

Using the point–slope form:

$y-y_1=m(x-x_1)$

$y-1=0(x-3) \rightarrow y - 1 = 0$,

Therefore, the equation of the tangent line parallel to the 1-meter height line is: $y = 1$

Figure 8. Students determine the equation of a tangent line in two ways

Based on the answers on the worksheet (see Figure 8), it is evident that students are able to solve the problem of finding the point of tangency and the equation of the tangent line to a parabola using two different approaches. In Approach 1 (eliminating the system of quadratic and linear equations), students substitute the value y=1 (height of the crossbar) into the parabola equation y=−x2+6x−8. From the calculation, we obtain the solution x = 3, so the point of tangency is (3,1). Next, the students determine the equation of the tangent line parallel to the goalpost, which is y=1. This process demonstrates the students' ability to model contextual problems into mathematical equation systems and solve them using algebraic procedures.

In Approach 2 (using derivatives to determine the slope of the tangent line), students differentiate the equation of the parabola to obtain the gradient $y' = -2x + 6$. By substituting $x = 3$, the gradient $m = 0$ is obtained, which means the tangent line is horizontal. Thus, the equation of the tangent line is y=1. This step demonstrates that students are able to connect the concept of derivatives with the geometry of parabolas to obtain the equation of the tangent line.

Both approaches yielded the same result: the tangent line y=1. This indicates that students not only mastered one method but were also able to switch between methods and demonstrate the consistency of their results. Thus, it can be concluded that the students have demonstrated mathematical thinking flexibility, which is the ability to use various different representations and problem-solving strategies to reach the same solution.

Overall, the work illustrates mathematical flexibility, as the student successfully shifted between different solution strategies, integrated multiple representations (symbolic, numerical, and graphical), and confirmed consistency across methods. This indicates a deeper conceptual understanding and aligns with the goals of STEM-based mathematics learning that emphasizes flexible problem-solving skills.

3.1.3. Restrospective Analysis

The retrospective analysis stage in this study was conducted by comparing the previously designed hypothetical learning trajectory (HLT) with the actual learning trajectory (ALT) observed during the pilot experiment and teaching experiment. The purpose of this analysis is to understand how STEM-based learning with a football context influences students' learning processes, specifically in strengthening mathematical flexibility skills. The data used consists of student work documents, video recordings of learning, classroom observations, and field notes from the researcher.

The first finding indicates that the context of football, such as the throw-in phenomenon of Pratama Arhan and the crossbar challenge, successfully builds a concrete bridge between the real world and abstract mathematical concepts. Students find it easier to understand the concepts of parabolas and tangents because they can directly relate them to known physical phenomena. Visualizing the ball's trajectory using the Desmos application and measuring the throwing angle with the Kinovea application makes it easier for students to model quadratic functions and determine the tangent line of a parabola, leading to a natural transition from concrete experience to conceptual understanding.

Additionally, there was a significant increase in students' representational flexibility. They are able to move between representations – from visual (video and trajectory graphs), to numerical (coordinate points), and then to symbolic (quadratic functions and derivatives). Students began constructing a mathematical model of the ball's trajectory, solving the system of equations from the known points, and visually verifying the results in Desmos. This ability demonstrates that students not only understand concepts procedurally, but also flexibly grasp the connections between diverse representations.

Strategic flexibility capabilities also develop gradually. In the learning process, students are not fixated on a single approach, but rather try various problem-solving strategies. For example, in the parabola material, some students used three points to set up a system of equations, while others chose the vertex and one additional point. In the context of tangents, students are asked to solve the problem using two different methods: using a system of equations and using derivatives. The results show that students can evaluate the strengths and weaknesses of each approach and choose the strategy most suitable for the problem context, demonstrating more reflective thinking flexibility.

However, in the initial stages of implementation, it was found that students still had difficulty interpreting the context mathematically. Their responses to the football video were still narrative and descriptive, without connecting the phenomena occurring to concepts such as elevation angle, maximum range, or point of tangency. The representations used tend to be limited to visuals, and there has not yet been an explicit connection made with symbolic or algebraic forms. This highlights the importance of more directed scaffolding and the design of prompting questions to enable students to transition from intuitive understanding to formal mathematical understanding. Revisions were made to the activity sheets, including the use of Indonesian in Desmos, instructions for using the Kinovea application, and the addition of reflective questions in activities 2 and 4. This revision has proven capable of improving the connection between context, concepts, and problem-solving strategies.

The overall findings from this stage indicate that the Local Instructional Theory (LIT) developed evolved iteratively and responsively to empirical data. Revisions to the activity structure, language use, and learning aids resulted in a more effective learning trajectory for developing mathematical flexibility. The revalidation process thru teaching experiments shows that students are more capable of integrating cross-disciplinary knowledge (science, technology, engineering, and mathematics) and applying various representations and strategies in meaningful real-world contexts. Thus, the developed LIT is not only a theoretical framework but also an effective and contextual instructional one for enhancing the mathematical thinking flexibility of prospective teachers.

3.2. Discussion

The results of this study show that integrating STEM-based learning with a football context in the development of Local Instructional Theory (LIT) is effective in fostering mathematical flexibility among prospective teachers. Retrospective analysis indicates that this approach enhances student engagement and strengthens their ability to shift between representations and solve problems reflectively.

First, the effectiveness of real-world contexts in building conceptual bridges between phenomena and mathematical representations supports previous findings on context-based learning (Hickendorff et al., 2022; Queiruga-Dios et al., 2025). In this study, the contexts of throw-ins and crossbar challenges generated meaningful cognitive conflict that encouraged students to relate the ball's trajectory to parabolic shapes. The use of digital tools such as Kinovea and Desmos functioned as visual–analytical scaffolding, enabling students to validate their informal observations through graphical and symbolic representations. This aligns with Goos et al. (2023) and Tashtoush et al. (2024), who emphasize the role of technology in supporting spatial reasoning and cognitive flexibility.

Second, students’ flexibility developed through a sequence of scaffolding strategies. These included (1) contextual prompting, in which students were asked to describe observable features of the football phenomena; (2) guided questioning, directing attention from descriptive language toward invariant mathematical features (such as symmetry, maximum points, and slope); and (3) representational scaffolding, where students were encouraged to translate visual interpretations into symbolic equations and numerical models using technological tools. Through this progression, students moved from initial descriptive responses (e.g., “the ball curves” or “the ball bounces back”) to more formal mathematical interpretations involving parabolic equations and tangent lines. This transition reflects increasing representational and strategic flexibility (Deliyianni et al., 2016; Star & Rittle-Johnson, 2008; Ünal et al., 2023).

However, the analysis also shows that flexibility did not emerge instantly. In early learning stages, many students focused on a single representation without explicitly connecting the context to formal mathematical models. Iterative scaffolding, reflective discussion, and carefully sequenced tasks were essential in supporting this transition, reinforcing the importance of design-based learning cycles as proposed by Gravemeijer and Cobb (2006) and Bakker (2018).

Overall, these findings confirm that STEM-based LIT with a football context not only improves prospective teachers’ understanding of parabolas and tangents, but also systematically supports the development of mathematical flexibility through well-designed scaffolding and progressive mathematization.

4. CONCLUSION

This research has successfully developed a STEM-based Local Instructional Theory (LIT) by integrating the context of soccer into mathematics learning, specifically for the topics of parabolic equations and tangents. The research findings indicate that using real-world contexts such as throw-ins and the crossbar challenge in soccer can increase student engagement, build connections between the real world and mathematical concepts, and significantly strengthen the mathematical flexibility skills of prospective teachers.

STEM-based learning processes supported by technology such as Kinovea and Desmos encourage students to move between representations (visual, symbolic, numerical) and reflectively evaluate various problem-solving strategies. Additionally, this approach facilitates the development of critical, adaptive, and collaborative thinking skills, which are essential in 21st-century mathematics learning.

Findings from the retrospective analysis phase show that students' representational and strategic flexibility develops thru active interaction with context, technology, and group discussions. Despite initial challenges in transforming context into mathematical models, revisions to instructional design and appropriate scaffolding were able to guide students toward a more conceptual and flexible understanding.

Thus, the STEM-based LIT developed in this study can be used as an innovative and applicable alternative learning model to improve the quality of mathematics learning. This study recommends that similar approaches be further developed on other mathematics topics, as well as integrated into the systematic training of prospective teachers to equip them with pedagogical competencies that are contextual, reflective, and relevant to students' real-life experiences.

Acknowledgments

The author gratefully acknowledges the research team of Universitas PGRI Palembang for their cooperation and support during the data collection process. The author also thanks the Pusat Pembiayaan dan Asesmen Pendidikan Tinggi (PPAPT), Ministry of Higher Education, Science, and Technology of Indonesia (Kemendiktisaintek), for awarding the Beasiswa Penyelesaian Studi Program Doktor Tahun 2025, which supported the publication of this article. Appreciation is also extended to the reviewers and editors for their constructive feedback.

Declarations

Author Contribution	: TDN: Conceptualization, Visualization, Writing - original draft, and Writing - review & editing; Z: Formal analysis, Methodology, and Writing - review & editing; RIIP: Formal analysis, Methodology, and Writing - review & editing; NA: Supervision, and Validation.
Funding Statement	: This research is part of a dissertation funded by the Pusat Pembiayaan dan Asesmen Pendidikan Tinggi (PPAPT), Ministry of Education, Culture, Research, and Technology of Indonesia (Kemendiktisaintek), through the 2025 Doctoral Program Completion Scholarship.
Conflict of Interest	: The authors declare no conflict of interest.
Additional Information	: Additional information is available for this paper.

REFERENCES

Abdu, R., Olsher, S., & Yerushalmy, M. (2022). Correction to: Pedagogical considerations for designing automated grouping systems: The case of the parabola. *Digital Experiences in Mathematics Education*, *8*(1), 99–124. https://doi.org/10.1007/s40751-021-00097-5

Aisyah, N., Susanti, E., Meryansumayeka, M., Siswono, T. Y. E., & Maat, S. M. (2023). Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning. *Infinity Journal*, *12*(2), 377–392. https://doi.org/10.22460/infinity.v12i2.p377-392

Andriadi, A., Suningsih, A., & Putra, Y. Y. (2021). Webinar pembelajaran daring matematika dalam sepakbola [Online learning webinar on mathematics in football]. *Batoboh*, *6*(1), 66–74. https://doi.org/10.26887/bt.v6i1.1319

Bakker, A. (2018). *Design research in education*. Routledge. https://doi.org/10.4324/9780203701010

Bolat, R. C., & Arslan, Ç. (2024). Examination of mathematics teachers' strategic flexibility in solving mathematical problems. *Thinking Skills and Creativity*, *54*, 101679. https://doi.org/10.1016/j.tsc.2024.101679

Bybee, R. W. (2013). *The case for STEM education: Challenges and opportunities*. NSTA Press.

Chen, Y. H., & Chang, C. Y. (2021). Exploring the effects of STEM-oriented project-based learning on students' scientific creativity and 21st-century competencies. *International journal of STEM education*, *8*(1), 1–8.

Cleland, J. (2015). *A sociology of football in a global context*. Routledge. https://doi.org/10.4324/9780203735114

Cox, S. K., Burns, M. K., Hughes, E. M., Wade, T., & Brown, M. (2024). Defining, measuring, and teaching mathematical flexibility. *The elementary school journal*, *124*(3), 479–498. https://doi.org/10.1086/728591

de Sousa, R. T., & Alves, F. R. V. (2022). Didactic engineering and learning objects: A proposal for teaching parabolas in analytical geometry. *Indonesian Journal of Science and Mathematics Education*, *5*(1), 1–16. https://doi.org/10.24042/ijsme.v5i1.11108

Deliyianni, E., Gagatsis, A., Elia, I., & Panaoura, A. (2016). Representational flexibility and problem-solving ability in fraction and decimal number addition: A structural model. *International Journal of Science and Mathematics Education*, *14*(S2), 397–417. https://doi.org/10.1007/s10763-015-9625-6

Dina, N. A., Amin, S. M., & Masriyah, M. (2018). Flexibility in mathematics problem solving based on adversity quotient. *Journal of Physics: Conference Series*, *947*(1), 012025. https://doi.org/10.1088/1742-6596/947/1/012025

Doorman, M., Bakker, A., Drijvers, P., & Wijaya, A. (2016). Design-based research in mathematics education. In *Sriwijaya University Learning and Education International Conference*, (Vol. 2, pp. 21–46).

Florio, E. (2022). The parabola: Section of a cone or locus of points of a plane? Tips for teaching of geometry from some writings by mydorge and wallis. *Mathematics*, *10*(6), 974. https://doi.org/10.3390/math10060974

Gondin, W. R., & Sohmer, B. (1967). *Intermediate algebra & analytic geometry: Made simple*. Elsevier. https://doi.org/10.1016/C2013-0-12592-5

Goos, M., Carreira, S., & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: Recent developments and future directions. *ZDM – Mathematics Education*, *55*(7), 1199–1217. https://doi.org/10.1007/s11858-023-01533-z

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 29–63). Routledge. https://doi.org/10.4324/9780203088364-12

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The elementary school journal*, *109*(5), 510–524. https://doi.org/10.1086/596999

Hasanah, S., & Retnawati, H. (2022). Assessment of contextual learning in mathematics education. *AIP Conference Proceedings*, *2575*, 040018. https://doi.org/10.1063/5.0111142

Hickendorff, M., McMullen, J., & Verschaffel, L. (2022). Mathematical flexibility: Theoretical, methodological, and educational considerations. *Journal of Numerical Cognition*, *8*(3), 326–334. https://doi.org/10.5964/jnc.10085

Holmlund, T. D., Lesseig, K., & Slavit, D. (2018). Making sense of "STEM education" in K-12 contexts. *International journal of STEM education*, *5*(1), 32. https://doi.org/10.1186/S40594-018-0127-2

Honey, M., Pearson, G., & Schweingruber, H. (2014). *STEM integration in K-12 education: Status, prospects, and an agenda for research*. National Academies Press. https://doi.org/10.17226/18612

Hong, W., Star, J. R., Liu, R.-D., Jiang, R., & Fu, X. (2023). A systematic review of mathematical flexibility: Concepts, measurements, and related research. *Educational Psychology Review*, *35*(4), 104. https://doi.org/10.1007/s10648-023-09825-2

Isyrofinnisak, F., Kusmayadi, T. A., & Fitriana, L. (2020). Flexibility in solving open-ended mathematics problems based on students' thinking styles. *Journal of Physics: Conference Series*, *1613*(1), 012015. https://doi.org/10.1088/1742-6596/1613/1/012015

Karlis, D., Ntzoufras, I., & Repoussis, P. (2021). Mathematics meet sports. *IMA Journal of Management Mathematics*, *32*(4), 381–383. https://doi.org/10.1093/imaman/dpab028

Kilner, S. J., & Farnsworth, D. L. (2019). Characterisations of the parabola. *The Mathematical Gazette*, *103*(558), 416–430. https://doi.org/10.1017/mag.2019.102

Larkin, K., & Lowrie, T. (2022). *STEM education in the early years: Thinking about tomorrow*. Springer. https://doi.org/10.1007/978-981-19-2810-9

Maarif, S., Wahyudin, W., Noto, M. S., Hidayat, W., & Mulyono, H. (2018). Geometry exploration activities assisted with dynamic geometry software (DGS) in a teacher education classroom. *Infinity Journal*, *7*(2). https://doi.org/10.22460/infinity.v7i2.p133-146

Mandala, A. S., Anwar, L., Sa'dijah, C., & Zulnaidi, H. (2025). Development of mobile augmented reality-based geometry learning games to facilitate spatial reasoning. *Infinity Journal*, *14*(2), 323–348. https://doi.org/10.22460/infinity.v14i2.p323-348

Martinez, L. J., Paloma, L. L., & Ruiz, F. J. (2025). Learning the parabola mediated by problem-solving, semiotic representations, and applets in geogebra. *International Journal of Mathematics and Computer Science*, *20*(1), 101–108. https://doi.org/10.69793/ijmcs/01.2025/hernandez

Nindiasari, H., Pranata, M. F., Sukirwan, S., Sugiman, S., Fathurrohman, M., Ruhimat, A., & Yuhana, Y. (2024). The use of augmented reality to improve students' geometry concept problem-solving skills through the STEAM approach. *Infinity Journal*, *13*(1), 119–138. https://doi.org/10.22460/infinity.v13i1.p119-138

Nopriyanti, T. D., Zulkardi, Z., Putri, R. I. I., & Aisyah, N. (2024). STEM in Indonesian mathematics education. In *Proceedings of International Conference on Education*, (Vol. 2, pp. 175–183). https://doi.org/10.32672/pice.v2i1.1335

Nopriyanti, T. D., Zulkardi, Z., Putri, R. I. I., & Aisyah, N. (2025). The design of learning trajectory for parabola equation in geometry STEM-based learning for flexibility skills. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *9*(4), 1271–1286. https://doi.org/10.31764/jtam.v9i4.32560

Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of preservice mathematics teachers on transformation geometry. *Journal on Mathematics Education*, *10*(1), 117–126. https://doi.org/10.22342/jme.10.1.5379.117-126

Peralta-García, J. X., Encinas-Pablos, F. J., & Cuevas-Salazar, O. (2020). Diagnóstico de conocimientos previos sobre la parábola en estudiantes universitarios [Diagnosing prior knowledge about parables in university students]. *Revista de Educación Superior*, *3*(8), 1–11. https://doi.org/10.35429/JHS.2019.8.3.1.11

Permatasari, R., Putri, R. I. I., & Zulkardi, Z. (2018). PISA-like: Football context in Asian games. *Journal on Mathematics Education*, *9*(2), 271–280. https://doi.org/10.22342/jme.9.2.5251.271-280

Plomp, T., & Nieveen, N. (2013). *An introduction to educational design research*. Netherlands Institute for Curriculum Development (SLO).

Prahmana, R. C. I., & Kusumah, Y. S. (2016). The hypothetical learning trajectory on research in mathematics education using research-based learning. *Pedagogika*, *123*(3), 42–54. https://doi.org/10.15823/p.2016.32

Pramasdyahsari, A. S., Rubowo, M. R., Nindita, V., Astutik, I. D., Pant, B. P., Dahal, N., & Luitel, B. C. (2025). Developing engaging STEAM-geometry activities: Fostering mathematical creativity through the engineering design process using Indonesian cuisine context. *Infinity Journal*, *14*(1), 213–234. https://doi.org/10.22460/infinity.v14i1.p213-234

Queiruga-Dios, M. Á., Vázquez Dorrío, J. B., Sáiz-Manzanares, M. C., López-Iñesta, E., & Diez-Ojeda, M. (2025). STEM approach using soccer: improving academic performance in Physics and Mathematics in a real-world context. *Frontiers in psychology*, *16*, 1503397. https://doi.org/10.3389/fpsyg.2025.1503397

Rais, H., Ramadhani, R., & Yassin, A. (2025). The effect of STEM learning approach on students' mathematical problem-solving ability. *Vocational: Journal of Educational Technology*, *1*(2), 74–84. https://doi.org/10.58740/vocational.v1i2.351

Ramulu, P., Yadav, E. R., & Sattemma, D. (2024). A review of mathematical concepts in sports and games: Examining the intersection of mathematics and sports. *International Journal of Scientific Research in Engineering and Management*, *8*(12), 1–7. https://doi.org/10.55041/IJSREM39949

Sanchez, W. B., & Glassmeyer, D. M. (2016). Connecting parabolas and quadratic functions. *The Mathematics Teacher*, *110*(5), 380–386. https://doi.org/10.5951/mathteacher.110.5.0380

Scheibling-Sève, C., Sander, E., & Pasquinelli, E. (2017). Developing cognitive flexibility in solving arithmetic word problems. In *Proceedings of the Annual Meeting of the Cognitive Science Society*, (Vol. 39, pp. 3076–3081).

Shriki, A. (2011). Parabolas: Connection between algebraic and geometrical representations. *Australian Senior Mathematics Journal*, *25*(2), 38–42.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, *18*(6), 565–579. https://doi.org/10.1016/j.learninstruc.2007.09.018

Sudirman, S., García-García, J., Rodríguez-Nieto, C. A., & Son, A. L. (2024). Exploring junior high school students' geometry self-efficacy in solving 3D geometry problems through 5E instructional model intervention: A grounded theory study. *Infinity Journal*, *13*(1), 215–232. https://doi.org/10.22460/infinity.v13i1.p215-232

Tashtoush, M. A., Al-Qasimi, A. B., Shirawia, N. A., & Rasheed, N. M. (2024). The impact of STEM approach to developing mathematical thinking for calculus students among Sohar University. *European Journal of STEM Education*, *9*(1), 13. https://doi.org/10.20897/ejsteme/15205

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and symbolic representations as components of algebraic reasoning. *Journal of Numerical Cognition*, *9*(2), 327–345. https://doi.org/10.5964/jnc.11151

van Eerde, H. A. A. (2013). Design research: Looking into the heart of mathematics education. In *Proceedings 1st South-East Asian Design Research Conference*, (pp. 1–11).

Voica, C., & Singer, F. M. (2012). Creative contexts as ways to strengthen mathematics learning. *Procedia - Social and Behavioral Sciences*, *33*, 538–542. https://doi.org/10.1016/j.sbspro.2012.01.179

Ye, H., Ng, O.-L., & Cui, Z. (2023). A thematic analysis exploring flexibility in programming-based mathematical problem solving. In *The 31st International Conference on Computers in Education*, (Vol. 1, pp. 183–188). https://doi.org/10.58459/icce.2023.967

Zulkardi, Z., & Putri, R. I. I. (2020). Supporting mathematics teachers to develop jumping task Using PISA framework (JUMPISA). *Jurnal Pendidikan Matematika*, *14*(2), 199–210. https://doi.org/10.22342/jpm.14.2.12115.199-210

Zulkarnain, R., Kurniawan, A., & Lisarani, V. (2023). Cognitive flexibility in learning mathematics senior high school. *JPMI (Jurnal Pendidikan Matematika Indonesia)*, *8*(2), 81–89. https://doi.org/10.26737/jpmi.v8i2.4301