

Analyzing student errors in PISA “Change and Relationship” tasks using the AVAEM framework

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Abstract

This study aims to describe students' difficulties in solving PISA problem-solving tasks in the "Change and Relationship" content domain by analyzing errors based on the AVAEM framework, which includes Arithmetic, Variables, Algebraic Expression, Equation Structure, and Mathematical Representation. This study employed a mixed-methods approach with an embedded design. The research participants consisted of 30 eighth-grade students from a junior high school in the city of Bandung. Data were collected through tests, questionnaires, observations, and interviews, and analyzed using the stages of data reduction, data display, and conclusion drawing. The results showed that students' problem-solving abilities were predominantly in the low (50%), medium (27%), and high (23%) categories. Students experienced various difficulties that led to specific types of errors. Arithmetic errors were found across all student categories. Variable errors were mainly observed among students in the frustrated learner category. Algebraic Expression errors were identified in both the instruction learner and frustration learner categories. Meanwhile, no equation errors were identified in any student category, but mathematical errors occurred in all student categories. These findings emphasize the importance of identifying and understanding students' error patterns when solving PISA-type problems as a basis for improving their mathematical problem-solving skills.

Keywords:

AVAEM error category, Change and relationship, Mixed method, PISA, Students' difficulties

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1. INTRODUCTION

Algebraic thinking is an important skill in mathematics that involves understanding patterns, relationships, functions, and the use of mathematical symbols and models to analyze mathematical situations and structures. Algebra invites students to be able to think algebra

(Iilonga & Ogbonnaya, 2023; Nurlaelah et al., 2025; Palinussa, 2020; Wahyuni et al., 2025; Yumiati et al., 2025). Algebraic thinking is not just about manipulating symbols, but also includes generalizations, representations, and reasoning about quantitative relationships and changes (Chimoni et al., 2023; Pitta-Pantazi et al., 2020). In other words, algebra invites students to work with pattern relations and relationships.

In PISA, the essential content of “Change and Relationship,” which falls under the Algebra domain, remains challenging for many students. This is consistent with Stacey (2011), who stated that the change and relationship content is the most difficult question for Indonesian students in the series of PISA questions given. In line with Stacey’s findings, the results of PISA 2018 also show that Indonesian students’ mathematics ability in this content area is far below the OECD average, with an average score of 379 compared to the OECD average of 489 (Afgani & Paradesa, 2021; OECD, 2019; Sistyawati et al., 2023; Wijaya et al., 2024). Meanwhile, the results of the research of Wati and Murtiyasa (2016) showed that students could not solve PISA questions with change and relationship content as much as 55.50% of the total research subjects.

Stephens et al. (2021) also argues the importance of giving students experience working with mathematical structure and relationships as the core of the practice of algebraic thinking from an early age. Working with this mathematical relationship can be done through one of the domains of PISA (Programme for International Student Assessment) questions, namely the content of 'change and relationships'. Change and relationship content is one of the essential content for students to master because it is related to daily life in depicting, modeling and interpreting growth and phenomena (OECD, 2019). However, this is not in line with the fact that students in Indonesia struggle to score high on PISA mathematics assessments (Afgani & Paradesa, 2021; Sistyawati et al., 2023).

Students’ difficulties in solving PISA items within the Change and Relationship content domain indicate that they are not yet able to connect the mathematical concepts they have learned with the contextual problems presented. This reflects the existence of fundamental weaknesses in their mathematical problem-solving abilities. Problem-solving skills are not a generic skill, but rather a human activity that combines concepts and rules that have been previously obtained (Aini et al., 2019; Harahap & Surya, 2017; Yaniawati et al., 2019). The problem-solving skills are the main focus in mathematics learning, so a learning mechanism is needed that can facilitate students to practice consistently in order to be able to solve problems (Azizah et al., 2022; Kurniansyah et al., 2022; Murtafiah et al., 2023; Pramuditya et al., 2022; Sari et al., 2021; Setiawan, 2022). To compile this mechanism, it can first be done through the analysis of students’ mistakes in doing the problem.

The results of student error analysis can provide information to teachers and help teachers to improve the learning process. This also applies to mistakes made by students in working on PISA questions on change and relationship content, which are in the form of algebra questions. One way that can be used to find out the location of students’ errors in algebra problems is through error analysis with the AVAEM error category. Jupri et al. (2014) categorizes students’ mistakes in doing algebra problems into five categories, namely, (1) Applying arithmetic operations (ARITH), (2) Understanding the notion of variables (VAR),

(3) Understanding algebraic expressions (AE), (4) Understanding the different meanings of the equal sign (EQS), and (5) Mathematization (MATH).

The mistakes made by students are certainly not without cause. The cause of student error is called learning obstacle. Learning obstacles in students are important to study. This is because if these learning obstacles continue, it will have a negative impact on the quality of student learning in the next material (Fadlelmula, 2022). By identifying the root causes of students' difficulties, the findings of this study are expected to inform more targeted and specific recommendations for improving the learning process, particularly in designing didactical situations that can minimize or address these challenges. Enhancing the quality of instruction is closely linked to efforts to strengthen students' problem-solving abilities, which constitute a central focus of the mathematics curriculum.

Problem-solving ability is an intellectual activity to find solutions to solve problems by involving knowledge and experience (Maimunah et al., 2016). Meanwhile, mathematical problem-solving skills are an essential cognitive competency in mathematics (Fisher et al., 2022; Kariadinata et al., 2019; Setiawan et al., 2020; Supianti et al., 2025; Supianti et al., 2022; Tambunan, 2019; Yaniawati et al., 2019; Yusepa et al., 2018b). Through problem solving, according to Nengsих et al. (2019) that a person will be required to think systematically, critically, logically, and have an unyielding attitude to find solutions to the problems faced.

The complex nature of the mathematical relationships and modeling embedded in PISA items necessitates the use of these problem-solving indicators. Change and relationship content is one of the contents of the PISA question which refers to understanding the basic types of change that require mathematical modeling to explain and predict these phenomena. Mathematically, this content deals with functions and algebra, including algebraic expressions, equations and inequalities, as well as creating, interpreting, and translating symbolic and graphic representations of mathematical relationships (Darta et al., 2021; Dewantara, 2019; Putra et al., 2020; Yusepa et al., 2018a). Data interpretation is also an important part of the problem in the category of change and relationships. Mathematical relationships are often expressed in common equations or ratios such as addition, subtraction, and division. The relationship is also expressed in various algebraic symbols, diagrams, geometric shapes, and tables. Because these various forms of representation can be complex and prone to misinterpretation, students are vulnerable to making errors when attempting to comprehend or translate them during the problem-solving process.

The location of the error is defined as part of the problem solving where the deviation occurs. Teachers must re-evaluate the misinformation that students often make when answering questions. This is done so that students do not repeat the same mistakes in the next step. Mistakes made by students in Algebra questions can be categorized by using the AVAEM Error Category. Jupri et al. (2014) divided the categories of AVAEM errors into five types, namely, (1) Applying arithmetic operations (ARITH), (2) Understanding the notion of variables (VAR), (3) Understanding algebraic expressions (AE), (4) Understanding the different meanings of the equal sign (EQS), and (5) Mathematization (MATH). The presentation of the types of errors based on the categories of AVAEM errors is shown in the following Table 1.

Table 1. AVAEM fault categories

Category	Description
<i>Applying arithmetic operations (ARITH)</i>	ARITH is a mistake made by students related to the application of arithmetic operations. The errors in the application of arithmetic operations are divided into three, namely related operation errors, related rule errors, and related property errors.
<i>Understanding the notion of variabel (VAR)</i>	The VAR category is an error related to students' understanding of variables both in algebra and errors in the literal role of symbols.
<i>Understanding algebraic expressions (AE)</i>	The AE category is an error in understanding algebraic expressions. These errors are usually related to errors in the problem decomposition, the expected answer to the problem, the lack of closure of the problem, and the lack of gestalt
<i>Understanding the different meanings of the equal sign (EQS)</i>	The EQS error category is the category of errors in understanding the difference in meaning of the "=" sign. Students don't understand the meaning of the equals of algebraic equivalents, just as students make notation errors as a result of a combination of operations.
<i>Mathematization (MATH)</i>	The category of math errors is a category of errors related to mathizing real problems into the form of algebraic equations. This MATH category error can be divided into two types, namely <i>Horizontal Mathematization</i> and <i>Vertical Mathematization</i> . <i>Horizontal Mathematization</i> is a mistake that cannot translate words or phrases or sentences into mathematical notation and models. Whereas <i>Vertical Mathematization</i> is an error that cannot combine, integrate, or use the information either given in the task or given as a result of calculations in solving symbolic algebraic problems and applying equations.

Understanding these error categories is essential because the types of mistakes identified through AVAEM often reflect deeper learning obstacles experienced by students during the learning process. Learning obstacles are learning obstacles experienced by students naturally (Yusuf et al., 2017). This is in line with the opinion of Nurfadilah et al. (2020) that learning obstacles are situations experienced by students in the form of obstacles during the learning process. The three main factors that cause learning difficulties include Ontogenetic obstacles, epistemological obstacles, and didactical obstacles (Brousseau, 2011). Ontogenetic obstacles are caused by the lack of mental readiness related to students' learning.

Therefore, this study aims to describe students' difficulties in solving PISA problem-solving questions in the "Change and Relationships" content domain by analyzing errors based on the AVAEM framework, which includes Arithmetic, Variables, Algebraic Expressions, Equation Structures, and Mathematical Representations.

2. METHOD

This study employed a mixed-methods approach with an embedded design, in which qualitative inquiry served as the primary strand. The research participants consisted of 30 eighth-grade students from a junior high school in the city of Bandung. From this category, 10 students were purposively selected for in-depth analysis based on their representation of

varying levels of problem-solving ability. The selection was guided by the results of a preliminary test that classified students into three learning categories—Independent learners, instructional learners, and frustration learners. Four students were chosen from the independent learner category, three from the instructional learner category, and three from the frustration learner category. This composition was intentionally structured to ensure diversity of ability levels, thereby enabling a comprehensive examination of error patterns across categories.

The research instruments consisted of both test and non-test instruments. The test instrument comprised PISA items in the “Change and Relationship” content domain, adapted from official OECD releases and other validated sources, and reviewed by two mathematics education experts to ensure content alignment, cognitive appropriateness, and construct validity. Student responses were assessed using a problem-solving rubric and the AVAEM error categories. The non-test instruments included interview guidelines, observation sheets, and documentation protocols, all of which were used to strengthen and triangulate the qualitative findings. Prior to implementation, all instruments underwent readability testing to ensure clarity and suitability for the participants.

3. RESULTS AND DISCUSSION

3.1. Results

The results of the trial on 10 students showed that most of them were not used to working on PISA questions on change and relationship content so that it was difficult to understand the purpose of the questions and only understood them after being guided. The guidance provided consisted of offering clues related to key terms and the contextual elements of the problem, without providing any intervention concerning formulas or solution strategies. Such guidance was necessary to ensure that the difficulties assessed were purely algebraic in nature, rather than resulting from reading barriers or contextual misunderstandings unrelated to the aims of the study. The contextual problems they work on are also not diverse, generally about social arithmetic such as buying and selling and parking and shopping receipts.

Story questions are also still avoided by students because of the difficulty of identifying the information known and asked, the steps to completion, and the formula used. This makes them less enthusiastic when working on contextual questions, even though they actually like the SPLDV material. In addition, students are still very dependent on teachers and rarely relearn the material at home or practice questions independently. The following are the difficulties students in working on PISA problem solving problems for change and relationship content: 1) Identifying known data, data being asked, sufficiency of data for problem solving; 2) Formulate a mathematical problem or compile a mathematical model; 3) Obstacles in the step of compiling a solution plan apply strategies to solve various problems (types and new problems) in or outside mathematics; 4) explain or interpret the results according to the original problem.

Analysis of Student Errors

Based on the findings in the test, the researchers summarized them in [Table 2](#) which shows the percentage of student AVAEM errors for each category.

Table 2. Number of student errors

Student Mistakes	Question						Sum	Percentage
	1a	1b	2	3	4a	4b		
ARITH	2	3	21	1	0	2	29	15.18%
VAR	10	0	0	0	0	0	10	5.24%
AE	9	0	0	0	0	0	9	4.71%
EQS	0	0	0	0	0	0	0	0%
MATH	7	12	29	31	37	27	143	74.87%
Total							191	100%

ARITH Category Errors

An ARITH category error occurs in arithmetic calculation operations, i.e. errors in decimal calculation operations. This results in inaccuracy in concluding the answers that have been worked on. The error occurred in question number 2 with the student's answer as [Figure 1](#).

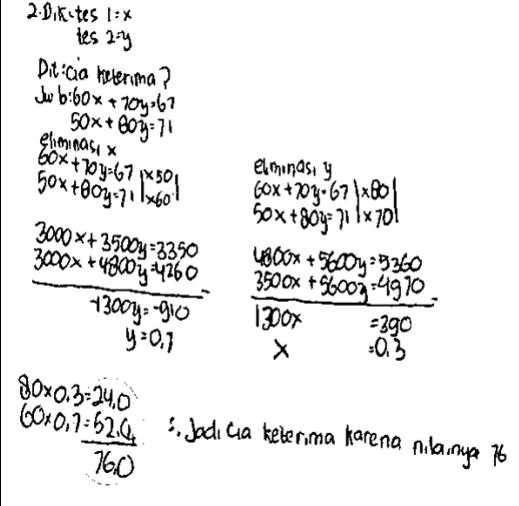
	<p>Translate:</p> <p>Known : Test 1 = x Test 2 = y Asked : Is Cia accepted to work? Solution : Elimination x $\begin{aligned} 60x + 70y &= 67 \quad \times 50 \quad 3000x + 3500y = 3350 \\ 50x + 80y &= 71 \quad \times 60 \quad 3000x + 4800y = 4260 \\ &\hline -1300y &= -910 \\ y &= 0.7 \end{aligned}$ Elimination y $\begin{aligned} 60x + 70y &= 67 \quad \times 80 \quad 4800x + 5600y = 5360 \\ 50x + 80y &= 71 \quad \times 70 \quad 3500x + 5600y = 4970 \\ &\hline 1300x &= 3910 \\ x &= 0.3 \end{aligned}$ $\begin{aligned} 80 \times 0.3 &= 24.0 \\ 60 \times 0.7 &= 42.0 \\ &\hline 76.0 \end{aligned}$ <p>\therefore Jadi Cia diterima karena nilainya 76</p> <p>Cia was accepted to work because she got a score of 76</p> </p>
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Figure 1. ARITH category error 1

Based on the [Figure 1](#), it can be seen that the student writes the multiplication result between 60 and 0.7 is 52.0 where the multiplication result should be 42.0. This causes the score summing results to be inaccurate. In addition, students also give wrong conclusions. The correct conclusion should have been that Cia was not accepted because Cia's score did not meet the minimum limit of 70. Another mistake from the ARITH category error is related to the related rules that students do in question number 2. Students give an incorrect x-value when determining a set of completions such as [Figure 2](#).

<p>2) 2 tes berbeda berbeda dit: kegagalan (cara ditentukan)</p> <p>dit: 2 tes berbeda tes 1 x = bobot tes 1 y = bobot tes 2</p> <p>Jwb: $60x + 70y = 67$ $60x + 70y = 67 \times 80 \Rightarrow 4800x + 5600y = 5360$ $50x + 80y = 71$ $50x + 80y = 71 \times 70 \Rightarrow 3500x + 5600y = 4970$ $60x + 70y =$ $1300x = 390$ $y = 3$</p> <p>$60x + 70y = 67 \times 80 \Rightarrow 3000x + 3500y = 3350$ $50x + 80y = 71 \times 60 \Rightarrow 3000x + 4800y = 4260$ $1300y = -910$ $y = 1.42$</p> <p>$\therefore x = 3$ $y = 1.42$</p>	<p>Translate: Two tests with different scores Known x = test score 1 y = test score 2 Asked Why was CIA not accepted to work? Solution $60x + 70y = 67 \Rightarrow 60x + 70y = 67$ $50x + 80y = 71 \quad 50x + 80y = 71$ $60x + 70y = 67 \times 80 \Rightarrow 4800x + 5600y = 5360$ $50x + 80y = 71 \times 70 \quad 3500x + 5600y = 4970$ $1300x = 390$ $x = 3$ $60x + 70y = 67 \times 50 \Rightarrow 3000x + 3500y = 3350$ $50x + 80y = 71 \times 60 \quad 3000x + 4800y = 4260$ $1300y = -910$ $y = 1.42$</p>
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Figure 2. ARITH 2 category errors

Based on the Figure 2, it can be seen that students are not precise in determining the values of x and y. The value of x should be obtained from 390 divided by 1300 and the value of y obtained from -910 divided by -1300. However, the inverse student performs division, where to determine the set of completions (looking for the values x and y) can use the inverse multiplication rule.

VAR Category Errors

The mistake that students make in this category is that students do not interpret variables correctly. Students define silver with the variable "k" but when modeling the two equations, Shiva uses the variable "p" as a symbol of silver like in the blue circle. Another mistake made by students is the mistake in question no. 1b (attachment item 1) as indicated by the red line. It can be seen that students are the result of multiplying algebraic expressions $3k \times 3$. Students determine the result of the operation on the algebraic expression with a different variable, namely "p". The two errors are as Figure 3.

<p>Dik: 6 susu, 4 Silver King, pembayaran 100.000 Dit: harga susu dan Silver King, kembalian 2</p> <p>Jwb: susu = s silver = k</p> <p>$4s + 3k = 118.000$ $\times 3 12y + 2p = 39.000$ $3s + p = 12.000$ $\times 4 12s + 4p = 36.000$</p> <p>$4p = 3.000$ $p = 1.500$</p> <p>$3s + p = 9.000$ $3s + 1.500 = 9.000$ $3b = 7.500$ $b = 2.500$</p>	<p>Translate: Known = 6 milk, 4 Silver King Total amount 100,000 Asked = The price of milk and silver King, change Solution Milk = s Silver King = k $4s + 3k = 118,000 \quad \times 3 \quad 12y + 2p = 39,000$ $3s + p = 12,000 \quad \times 4 \quad 12s + 4p = 36,000$ $4p = 3,000$ $p = 1,500$</p> <p>$3s + p = 9,000$ $3s + 1,500 = 9,000$ $3b = 7,500$ $b = 2,500$</p>
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Figure 3. VAR category errors

The mistake made by students in this category is that students do not interpret variables correctly and determine the results of algebraic operations with different variables. These two errors show that students' understanding of variables is still lacking and students have not understood that one variable means an object or something else. Another mistake made by students in this category is the mistake in question number 4 (attachment item 4) where students are not correct in using variables during modeling as shown in the [Figure 4](#).

<p>tempat Kolam renang A $\Rightarrow y = 362 + 12x$ $B \Rightarrow x = 248 + 18x$</p> <p>a. model: $2y + 2x = 610.000$</p>	<p>Translate: Swimming pool A $\Rightarrow y: 362 + 12y$ $B \Rightarrow x: 248 + 18x$</p> <p>Model: $2y + 2x = 610,000$</p>
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Figure 4. VAR category errors 2

The correct modeling is $y = 362 + 12x$ and $y = 248 + 18x$. This shows that students do not understand the concept of consistency in the use of variables in mathematical models. In the context of problems involving the same object or variable, it is important to use the same symbols or letters so that the meaning and relationships between variables remain clear and not confusing. Inconsistencies in the use of variables can indicate that students have not fully understood that in a mathematical model, each variable has a specific meaning that must be consistent across the model. These mistakes show that students' understanding of variables is still lacking. In line with the error in the ARTH category, the error shows that students have not fully understood the concept of algebraic operations.

AE Category Errors

The AE category error occurs because students ignore the importance of negative signs of algebraic forms (the lack of gestalt view) as shown in the [Figure 5](#).

<p>model mat: $60x + 70y = 67$ $50x + 80y = 71$</p> <p>Dir: bobot 1: x 2: y</p> <p>Dit: bobot masing-masing test</p> <p>Jwb: $60x + 70y = 67$ $50x + 80y = 71$</p>	<p>Translate: Mathematical Models (I) $60x + 70y = 67$ (II) $50x + 80y = 71$</p> <p>Known = Score 1: x Score 2: y</p> <p>Asked: Score each test</p> <p>Solution: $60x + 70y = 67 \mid \times 50 \mid 300x + 350y = 3,350$ $50x + 80y = 60 \mid \times 60 \mid 300x + 4,800y = 4,260$ $4,445y = 910$</p>
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Figure 5. AE category errors

Based on [Figure 5](#), the students' mistakes in question number 2. It should be if the first number is smaller than the second, then the result of the operation is a negative number. However, as the [Figure 5](#) shows, students assign a positive number to the result of the algebraic operation. This shows that students ignore negative signs in algebraic operations.

EQS Category Errors

The percentage of errors in the EQS category was 0%, indicating that none of the students across all ability categories made errors related to understanding the meaning of the equal sign. This result suggests that, in general, students had already mastered the relational meaning of the equal sign (=), interpreting it as a symbol representing equivalence or balance. This finding indicates that obstacles related to the acquisition of the basic concept of algebraic equivalence have likely been addressed. It further clarifies that the primary conceptual challenges faced by students do not lie in their fundamental understanding of the equality symbol but rather in more complex stages of the problem-solving process.

MATH Category Errors

MATH category errors are the category of errors with the highest number of cases committed by students. The first mistake is that students only answer wrong and correct grades without including a reason. These errors are included in the category of MATH with the Vertical Mathematics indicator. This shows that students are not yet able to combine, integrate, or use the information either given in the assignment or given as a result of calculations in solving symbolic algebraic problems and applying solving equations when simplifying algebraic expressions. Question number 3 asks students to state the value of truth along with the reason of the statement given, but students make mistakes in determining the value of truth and reason as shown in [Figure 6](#).

Persamaan	Nilai kebenaran		Alasan
	Benar	Salah	
$s = w + 0,375$	✓		Karena $w = 0,625$ jika $+ 0,375 = 1,000$
$w = s - 0,375s$	✓		Karena $1,000 - 0,375 = 9a$
$s = 1,375w$		✓	—
$w = 0,625s$	✓		Karena $u = 575\% = 0,375$ $w = 62,5\% - 0,375$

Translate:			
Equation	Truth value		Reason
	True	False	
$s = w + 0,375$	✓		Because $w = 0,625$ if $+ 0,375 = 1,000$
$w = s - 0,375s$	✓		Because $1,000 - 0,375 = 9a$
$s = 1,375w$		✓	—
$w = 0,625s$	✓		Because $u = 575\% = 0,375$ $w = 62,5\% - 0,375$

Figure 6. Errors in the MATH category (vertical mathematics)

The correct statement of truth based on the question (see [Figure 6](#)) is false, false, true, false. However, no truth value is answered correctly by the students. The reasons given by the students were also not right. In the first and second statements, students have not understood the concept of profit percentage. This can be seen from the reasons given by the students that justify the profit obtained from $0,375s$, where the profit should have been obtained from $0,375w$. Meanwhile, the reasons given by students in the third and fourth statements reinforce

the indication that students do not understand the concept of social arithmetic and are not able to use the information provided properly. Another mistake made by this category is not completing the settlement to the end. This is shown in Figure 7.

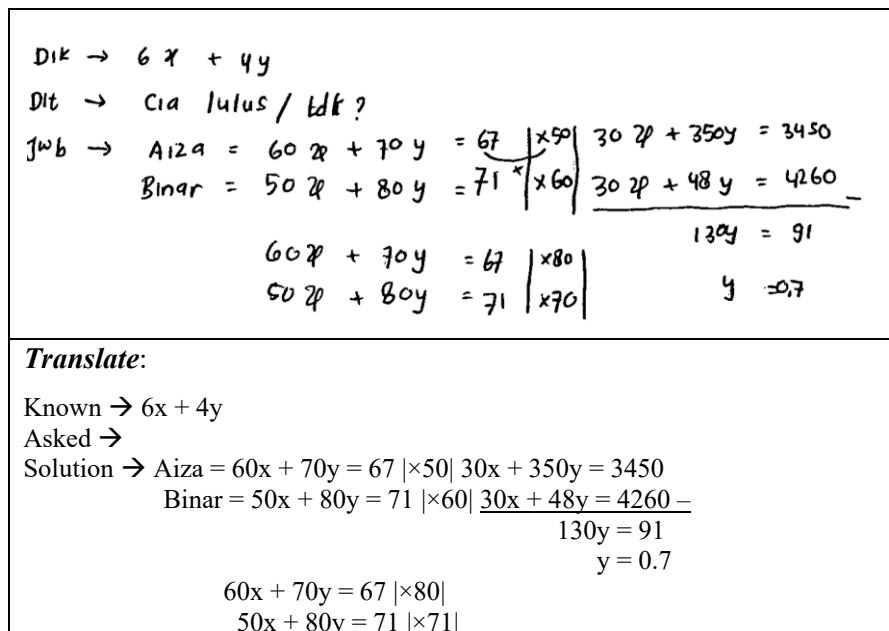


Figure 7. Errors in the MATH (horizontal mathematics) category

Figure 7 shows the student's answer to question no. 2, but the student's answer only comes to finding the value of the variable y while the value of the variable x does not exist. This causes students to not be able to find solutions to the problems given. Students make mistakes when compiling mathematical models. The correct mathematical model based on the problem is $4x+3y = 61,000$ and $3x + 3y = 57,000$. However, students presented a mathematical model of $4s+3k=118,000$ and $3s+p = 12,000$.

The mistakes made by the students are included in the MATH category errors with the horizontal indicator of mathization. Of course, it will have a domino effect when students determine the solution. Mistakes in modeling will of course produce the wrong variable values as well so that the answers asked for the questions will be wrong.

Analysis of student interview results

Based on the results of the trial and interviews with ten students from the independent learner, instruction learner, and frustration learner categories, it was obtained that students experienced various difficulties in working on PISA problem solving problems of change and relationship content. Students are not used to working on PISA-type questions and find it difficult to understand the meaning of the questions, especially in figure questions and story questions with contexts they have never encountered before. The interview transcripts for each category are presented in Table 3.

In question number 2, students have difficulty understanding certain terms such as "weight" and have to read repeatedly to understand the meaning of the question. Question number 3 is the biggest challenge because students have not fully understood the concept of social arithmetic, especially related to the percentage of taxes, discounts, and profits, due to

the incompleteness of previous learning. Story questions are also something that students avoid because they find it difficult to identify the information known and asked, the steps to solve it, and the formula to be used, thus reducing their motivation in working on the contextual problems presented.

Table 3. Key findings and category interview excerpts

Category	Main Difficulties	Interview Excerpts
Independent Learner	<ol style="list-style-type: none"> 1. Translating text into mathematical models 2. Interpreting diagrams 3. Understanding textual information 4. Social arithmetic concepts 	<p>Researcher : "How did you feel when working on Question 1?"</p> <p>Student : "It was difficult because we had to understand the diagram first. Story problems are easier."</p> <p>Researcher : "What about Question 2?"</p> <p>Student : "I was confused by the word '<i>weight</i>'. I had to read it several times before understanding which part became the variable."</p>
Instruction Learner	<ol style="list-style-type: none"> 1. Translating text into mathematical models 2. Interpreting diagrams 3. Understanding textual information 4. Decimal operations 5. Longer completion time 	<p>Researcher : "Which part was the most difficult?"</p> <p>Student : "Question 1a and Question 2. The diagram in 1a was confusing; in Question 2 the decimals and new terminology were difficult."</p> <p>Researcher : "How about Question 3?"</p> <p>Student : "I still don't understand tax percentages, discounts, and profit-loss concepts beyond selling and purchase price."</p>
Frustration Learner	<ol style="list-style-type: none"> 1. Translating text into mathematical models 2. Arithmetic operation errors 3. Understanding variables 4. Understanding textual information 5. Interpreting diagrams 6. Decimal calculations 	<p>Researcher : "Which questions were easy or difficult?"</p> <p>Student : "Question 1 was easy but I still had trouble reading the receipt. Question 2 was the hardest; I didn't understand what the question meant and I can't calculate decimals."</p> <p>Researcher : "How about Questions 3 and 4?"</p> <p>Student : "For Question 3 I just checked randomly, and for Questions 4a and 4b, some were done and some weren't because I ran out of time."</p>

Difficulties also occur in translating words or sentences into mathematical notation and models, especially when determining the right variables and constants in modeling. In picture problems, students need longer to understand the information provided before they can determine the completion steps. Students' abilities in decimal number calculation operations also still need to be strengthened because there are errors in calculation due to inaccuracy.

Meanwhile, some students admitted that they only learned mathematics in class with their teachers and rarely relearned the material at home, so they were not used to working on contextual problems with medium to high difficulty independently. This difficulty occurs in all three categories with different levels and intensities, where the frustration learner category experiences the most difficulty, takes longer to work, and often runs out of time because it

takes too long to analyze the problem, while the independent learner category has good algebraic skills but still has difficulty understanding PISA problems in an unfamiliar context.

Analysis of Students' Problem-Solving Abilities

Data on students' mathematical problem-solving abilities were obtained from two sources: PISA-type tasks in the Change and Relationship domain and in-depth interviews. The PISA-type tasks consisted of both original PISA items and researcher-developed items that adopted the structure and characteristics of PISA questions to ensure alignment with the research context. Scores from these tasks served as the quantitative measure of students' problem-solving abilities, whereas the interviews provided complementary qualitative insights into students' thinking processes and the difficulties they encountered. Based on the PISA task results, students' problem-solving ability levels were classified into three categories—high, medium, and low.

The average score obtained is 40.07, so the student's problem-solving ability is in the medium category. Meanwhile, the percentage of students' abilities is shown in the [Figure 8](#) and the number of questions answered incorrectly by students is shown in the [Figure 9](#).

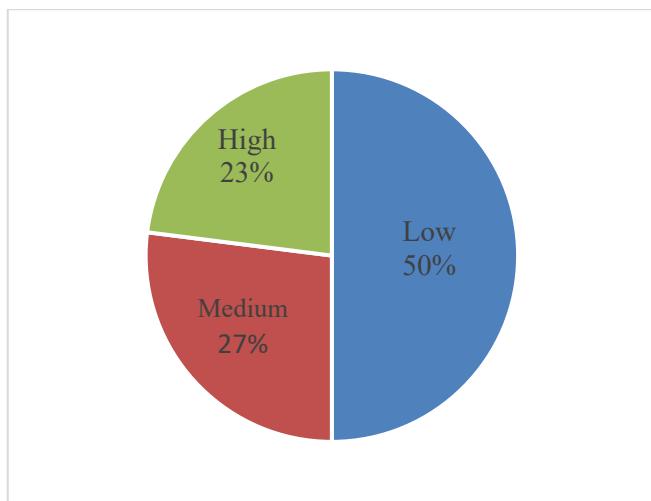


Figure 8. Students' level of problem-solving ability

[Figure 9](#) show the three items with the highest error rates are Item No. 3 (70% incorrect), Item No. 4b (63% incorrect), and Item No. 4a (60% incorrect). The analysis of these error patterns (which covers all item indicators) is essential for identifying trends in students' cognitive obstacles. The high error rates in these three items indicate substantial obstacles associated with higher-level cognitive indicators. Item No. 3 corresponds to the C4 cognitive level, with an indicator requiring students to analyze contextual problems and apply appropriate strategies. The predominant errors suggest that students struggled to analyze contexts beyond mathematics (such as the economic aspect in MP3) and to apply relevant solution strategies.

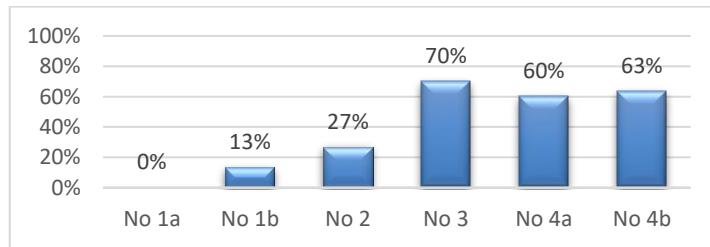


Figure 9. Questions answered correctly and incorrectly

Item No. 4a falls under the C5 cognitive level, which involves formulating mathematical problems or constructing mathematical models. The high error rate indicates students' weaknesses in the stage of mathematical modeling (horizontal mathematization), particularly in converting complex narratives into accurate systems of linear equations in two variables (SPLDV). Meanwhile, Item No. 4b also corresponds to the C4 cognitive level, with an indicator requiring students to use mathematics meaningfully. Errors in this item reflect students' difficulties in applying the concepts already modeled to make logical and meaningful decisions within real-world contexts.

Taken together, this pattern indicates that students' primary obstacles have shifted from basic computational skills toward higher-order processes involving analysis, modeling, and conceptual application (C4–C5 indicators). These error patterns are strongly supported by the interview findings, which revealed difficulties in interpreting unfamiliar contexts and incomplete mastery of social arithmetic concepts. These findings should also be linked to the previously presented data on the percentage distribution of students' problem-solving ability categories.

3.2. Discussion

Analysis of Student Errors

There are several student mistakes found in working on PISA problem solving problems for Change and relationship content based on the AVAEM error category. After searching, based on the results of the research, there are several factors that cause students to have difficulty in working on PISA problem solving problems for Change and relationship content.

ARITH Category Errors

ARITH category errors occur in all categories of students, both independent learners, instruction learners, and frustration learners. Where the error occurs in arithmetic calculation operations, namely errors in decimal calculations. This results in students not being precise in concluding the answers that have been done. Another error of the ARITH category error is related to the related rule where the student gives the wrong x value when determining the set of solutions. Both of the above errors are caused by ontogenetic obstacles. The first mistake in decimal number operations is caused by a conceptual ontogenetic obstacle where students' understanding of the prerequisite material for algebraic form operations is still limited

(Wulandari, 2022). This causes students to be unable to determine the results of decimal multiplication (Emanuel et al., 2021; Kusuma & Retnawati, 2019; Lai & Murray, 2015).

In addition, Kusuma and Adna (2021) also stated that one of the factors that causes students to have difficulty in working on problems with the two-variable linear equation system that causes concept errors is the lack of understanding of concepts in calculations. Meanwhile, calculation errors occur because students do not master the prerequisite material (Cheah, 2020; Khasanah & Sutama, 2015). Meanwhile, the second mistake occurred because the conceptual ontogenetic obstacles related to the prerequisite material for algebraic form operations are still limited (Wulandari, 2022). The prerequisite material is calculation operations on numbers as well as on algebra. The lack of concepts in understanding the material that is a prerequisite-results in students not being able to accept the next material so that students will manipulate concepts in order to solve a problem and result in misconceptions (Arfada & Hia, 2022; Jankvist & Niss, 2018; Ojose, 2015).

In addition to conceptual ontogenetic obstacles, students' difficulties also come from psychological ontogenetic obstacles, namely learning interests. Students' interest in learning algebra causes students to make mistakes in algebraic concepts. The mistake is due to epistemological obstacles, namely that students have difficulty developing the concept of SPLDV. Students have not fully understood the concept of decimal numbers and algebraic operations, as well as the inverse rules of algebraic multiplication. The reasons why students make decimal operation errors are: (1) Students make mistakes in performing subtraction operations; (2) Students are wrong in carrying out division operations; (3) Students are wrong in calculating the % (percent) value into decimal form; (4) Students still find it difficult to subtract integers and decimals; (5) Students are in a hurry to work; and (6) Students are less thorough in doing (Pournara, 2020).

Errors from algebraic operations are mistakes that students often make. One of the mistakes students make in doing algebra is that students have difficulty in distinguishing similar and non-similar syllables in algebra and difficulty simplifying algebraic forms (Maharani & Subanji, 2018; Pournara, 2020; Setianingrum et al., 2020). Mistakes due to didactical obstacles occur due to students' misconceptions regarding the teacher's explanation. Students do not understand the concept of algebra in its entirety, so students make mistakes in determining number and algebraic calculation operations. Errors in addition and subtraction operations of algebraic forms are the most common mistakes and are caused by incomplete/incorrect reasoning, as well as students' lack of interest in learning mathematics (Marpa, 2019).

VAR Category Errors

The category of VAR errors only occurs in the frustration learner category. The mistake that students make in this category is that students do not interpret variables correctly. Another mistake made by students is that students do not interpret variables correctly and determine the results of algebraic operations with different variables. These two errors show that students' understanding of variables is still lacking and students have not understood that one variable means an object or something else. These errors are caused by ontogenetic obstacles and didactical obstacles. Ontogenetic obstacles are conceptual in nature where students'

understanding of the prerequisites for algebraic form operations is still limited. Students who make mistakes in understanding variable notation, students interpret symbols only as a single value rather than a single value unit is a form of conceptual ontogenetic obstacles, because students do not have a formal definition related to the meaning of variables (Zulfa et al., 2020).

Meanwhile, didactical obstacles are the result of students' misunderstanding of the concepts given by the teacher because the language used by the teacher is not understood by the students. This is in line with Djadir et al. (2021) that external factors include uncondusive classroom situations, lack of variation in examples/practice questions given by teachers, teacher language and package books that are difficult to understand. These mistakes show that students' understanding of variables is still lacking. In line with the error in the ARTH category, the error shows that students have not fully understood the concept of algebraic operations.

AE Category Errors

AE category errors occur in the instruction learner category, and frustration learners. The mistake made is in the form of ignoring the importance of negative signs from the algebraic form (the lack of gestalt view). Students give positive results for numbers that should have a negative value. This happens because of conceptual ontogenetic obstacles where students' understanding of the prerequisites for algebraic form operations is still limited so they make mistakes (Cesaria & Herman, 2019; Siagian et al., 2022; Sulistiawati & Surgandini, 2019). The mistakes that students often make are the type of Operation errors, namely: 1) errors in the calculation of addition operations. 2) errors in the calculation of reduction operations (Cahyani & Sutriyono, 2018).

EQS Category Errors

EQS errors are commonly classified as procedural or symbolic notation errors in algebra. When an EQS error occurs, the subsequent problem-solving process—even if the arithmetic calculations are correct—will inevitably lead to an incorrect solution. In this study, no errors in the EQS category were identified, resulting in a percentage of 0%. Theoretically, this finding indicates that students possess sufficiently strong procedural understanding of the basic rules for constructing and organizing elements within an equation (Nurikawai et al., 2021). This result further suggests that students are able to avoid fundamental syntactic errors in algebra (Rahayu & Setiyadi, 2023). Therefore, their primary difficulties are concentrated in the stages of variable modeling (VAR category), which reflect conceptual understanding (Horizontal Mathematization), and in performing calculations (ARITH/AE categories), rather than in structural or syntactic aspects of the equation itself.

MATH Category Errors

This MATH category error is the category of error with the highest number of cases committed by students and occurs in all categories of students (independent learner, instruction learner, and frustration learner). The first error is that students only answer wrong and correct grades without including a reason. These errors are included in the category of Mathematization (MATH) with the Vertical Mathematics indicator. This shows that students are not yet able to combine, integrate, or use the information either given in the assignment or

given as a result of calculations in solving symbolic algebraic problems and applying solving equations when simplifying algebraic expressions.

The cause of errors in the MATH category of vertical mathematic indicators experienced by students is caused by Ontogenetic obstacles, epistemological obstacles and didactical obstacles. Errors due to Ontogenetic psychological obstacles occur because students lack self-motivation to work on problems related to social arithmetic material and use the language used in PISA questions. Therefore, when working on questions, students' motivation decreases because students consider difficult questions and difficult language to understand. This is in line with the opinion of Tririnika et al. (2024) that one of the factors that affect motivation in learning, namely dynamic elements in learning in the form of questions given to students.

Errors due to epistemological obstacles occur due to students' limited understanding of the context of various questions. Students are not able to process the information in the questions related to social arithmetic and students also lack understanding the language of the questions presented. This is strengthened by the results of other studies that state that the cause of misconception can come from the students themselves because of students' associations with everyday terms that cause misconceptions and students do not understand the problems or problems that exist (Dachi & Sarumaha, 2021; Khalid & Embong, 2020).

Based on the research, the average student's problem-solving ability was 40.07 with 23% of students in the high category, 27% of students in the medium category, and 50% of students in the low category. Meanwhile, the indicators of problem-solving ability that have not been completed in this study are 1) students are less skilled in applying strategies to solve various problems (similar and new problems) in or outside mathematics, 2) formulate mathematical problems or compile mathematical models, and 3) use mathematics meaningfully. This causes students to make mistakes and find it difficult when working on PISA problem solving problems for Change and relationship content.

Improving students' problem-solving skills in PISA's Change and relationship content.

The inability of students to complete indicators to apply strategies to solve various problems (similar and new problems) in or outside mathematics is due to the inability of students to master mathematical concepts in SPLDV material. Students tend to forget what has been learned and find it difficult to interpret the meaning of the question, so that when doing the process of connecting, students cannot do it (Nurdianti et al., 2019).

Meanwhile, students' inability to apply strategies when facing problems results in students making mistakes in answering problems including not being able to do modeling, not being able to connect concepts in mathematics, and even making mistakes in using rules related to algebraic operations. This is according to Santoso et al. (2019) because in general, students have not been able to master the basic concepts of SPLDV material, so it is difficult to understand the data in the story of the implementation of SPLDV. These errors by the AVAEM error category are included in the ARITH, AE, VAR and MATH (Vertical Mathematization) categories with the cause of the error being the learning obstacle (Anindiy, 2019).

Formulating mathematical problems or compiling mathematical models

The difficulty of formulating a mathematical model is indeed a problem that is often encountered when students work on problem solving problems, especially story problems. This is in line with research conducted by (Latifah & Afriansyah, 2021; Sonia et al., 2022). The incompleteness of this indicator causes students to make mistakes in the categories of VAR and MATH (Horizontal Mathematization). These errors are errors in translating words or sentences into notation and mathematical models correctly.

The causes of these errors include (1) students have not understood the steps in doing mathematical modeling, (2) students have not been able to assume the sentence of the problem in verbal mathematical sentences, especially in the form of linear equations, (3) students have not been able to explain why they chose the symbols contained in mathematical equations or mathematical models, and (4) students have not been able to sort out important sentences in the problem that can be used as mathematical modeling (Lestari & Afrilianto, 2021). These difficulties lead to errors in determining mathematical solutions, which are categorized as ARITH and MATH (vertical mathematization) errors (Aini et al., 2023; Gazali, 2016). Notably, ARITH errors may occur across all problem-solving indicators, reflecting gaps in students' procedural understanding and the accuracy of their mathematical representations.

There are two factors that cause the incompleteness of the indicators that have been mentioned. The first is the condition of students who lack student concentration and lack of interest in learning. This is in line with Ardila and Hartanto (2017) who stated that there are 4 (four) factors that affect the low learning outcomes of students, namely, lack of interest in mathematics lessons, lack of student concentration during the learning process, low understanding of students' concepts, and lack of student discipline. Not only the factors of student conditions and student interests, but also students' time management skills also influence. Students' ability to do time management also affects student learning outcomes (Haruna & Fajar, 2021).

4. CONCLUSION

The findings of this study indicate that eighth-grade students experienced significant difficulties in solving PISA-type problems, as evidenced by the high frequency of errors in the Mathematization (MATH) category (74.87%) and in the Arithmetic (ARITH) category. Errors within the MATH category reflect substantial shortcomings in horizontal mathematization, particularly students' inability to transform contextual problems into appropriate algebraic models. Interestingly, no EQS errors (related to understanding the different meanings of the equal sign) were identified across all student categories. The absence of EQS errors suggests that students possessed a solid understanding of variable assignment and basic equation formulation, indicating that no major didactic obstacles were associated with the foundational concept of algebraic equality.

Nevertheless, the remaining error types observed can be attributed to three forms of learning obstacles: ontogenetic obstacles (conceptual and psychological misconceptions), didactical obstacles, and epistemological obstacles. The integration of AVAEM error analysis with these learning obstacle classifications reveals that students' difficulties in formulating

mathematical models were predominantly caused by ontogenetic obstacles related to fundamental algebraic concepts (ARITH and VAR), as well as epistemological obstacles arising during the modeling process. These findings underscore the importance of identifying learning obstacles as a foundational step in designing targeted instructional interventions within the Didactical Design Research (DDR) framework, with the ultimate aim of improving students' mathematical problem-solving abilities.

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Author Contribution	: BYGP: Conceptualization, Methodology, Writing - original draft, and Writing - review & editing; PY: Conceptualization, Validation, and Writing - review & editing; NF: Conceptualization, Data curation, Investigation, Project administration, and Writing - original draft; SS: Writing - review & editing; IIS: Writing - review & editing; AM: Formal analysis, and Writing - review & editing.
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