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Transforming AI chatbots into metacognitive partners: Culturally grounded mathematics learning in LMS-based environments

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Abstract

The utilization of AI chatbots in mathematics education is on the rise, with these bots often serving as the primary providers of answers, potentially limiting learners' engagement in reflective and metacognitive reasoning processes. The present study examines how AI chatbots can function as metacognitive partners within a culturally contextualized mathematics learning framework implemented through a learning management system (LMS). Using an embedded multiple-case study design, the data were collected from 33 preservice teachers enrolled in the Department of Mathematics Education during their seventh semester. The data were collected from several sources: chatbot interaction logs, participatory observations, metacognitive reflective journals, and semi-structured interviews. Four participants were purposively selected for in-depth analysis. Thematic analysis revealed two distinct patterns in the interaction of pre-service mathematics teachers with the AI chatbot when completing assignments: an iterative interaction pattern and a linear interaction pattern. The iterative pattern involves repeated exploration, whereas the linear pattern follows a straightforward path with minimal verification. The two patterns yield different metacognitive outcomes, depending on the degree of regulation. When iterative interaction is well regulated, it fosters metacognitive success; however, when it becomes overly regulated, it leads to a metacognitive failure known as mirage, in which learners overanalyze and begin to question correct reasoning. Similarly, when linear interaction is well regulated, it also facilitates metacognitive success. Yet, when it becomes less regulated, it results in a metacognitive failure, blindness, due to the uncritical acceptance. The findings of the present study suggest that AI-supported learning functions as a process of co-regulation rather than passive information delivery, underscoring the importance of regulatory awareness in culturally mathematics learning.

Keywords:
AI chatbots, Culturally mathematics learning, Learning management system, Metacognitive, Preservice mathematics teachers

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1. INTRODUCTION

The integration of artificial intelligence (AI) chatbots, like ChatGPT and Gemini, into education environments is gaining significant traction. Recent empirical evidence demonstrates that these chatbots are being perceived as highly useful by both students and educators, with impacts across factors including effectiveness, engagement, personalization, and confidence building (Luzano, 2024). These bots may also enhance higher education by providing personalized support, addressing diverse learning needs, improving efficiency, and fostering self-directed learning (Chan & Hu, 2023).

Despite these beneficial impacts, several studies indicate that AI chatbots are used merely as providers of answers rather than as facilitators of cognition and reflection. Students may experience a decline in their engagement with the cognitive processes underlying critical thinking when they primarily utilize AI to obtain immediate answers instead of supporting deep learning (Panit, 2025). Artificial intelligence (AI) systems often provided instant responses, which could limit prolonged cognitive effort and reduce opportunities for learners to construct understanding progressively (Katsenou et al., 2025). Studies have consistently demonstrated that while AI chatbots are highly efficient in producing correct solutions, diagnosing misconceptions, and offering prompt feedback, their dominant usage pattern tends to encourage reliance on automation rather than fostering meaningful cognitive engagement (El Fathi et al., 2025; Harahap et al., 2025; Yunianto et al., 2024). This pattern poses the potential to limit students' intellectual autonomy, particularly in mathematics, where comprehension depends not only on obtaining correct answers but also on evaluating and justifying the process and reasoning to obtain those answers.

Mathematics learning presents unique demands that extend beyond content delivery. Unlike many subjects, mathematics requires advanced reasoning and metacognition, defined as the ability to reflect on one's own cognitive process (Lucangeli et al., 2019). In mathematics learning, students are expected to not only arrive at correct answers but also to understand how solutions work, recognize different solution pathways, and develop transferable problem-solving strategies (Taranikanti & Davidson, 2023; Ullah et al., 2025). Research has indicated that although AI chatbots can efficiently generate correct answers to mathematical problems, they often fail to scaffold the metacognitive and reasoning processes that are fundamental to mathematics learning (Rizki & Kusumah, 2025). It is highly probable that students will accept ChatGPT responses that are mathematically plausible without thoroughly examining the validity of the underlying reasoning. Therefore, there is a concern that students' ability to justify, verify, and critically evaluate AI-generated solutions must be strengthened (Sutrisno AB et al., 2025). This concern is especially relevant in non-routine, context-based mathematical problems, where successful engagement depends not only on content knowledge but also on learners' ability to regulate their thinking throughout the solution process (Kholid et al., 2025).

This concern is further exacerbated by research showing that habitual dependence on AI-generated solutions may compromise students' long-term cognitive and metacognitive development in mathematics learning. The over-reliance on AI may negatively affect cognitive abilities, as individuals increasingly prefer fast, seemingly optimal solutions over slower, more effortful reasoning processes (Zhai et al., 2024). The excessive use of AI tools in

undergraduate education has been observed to potentially reduce critical-thinking skills and memory retention, with the most pronounced cognitive declines reported among humanities students (Rohilla, 2025). While AI tools can enhance short-term outcomes and provide efficient access to explanations when used prudently (Wang et al., 2025; Wong et al., 2025), extant evidence shows that, in the absence of structured pedagogical guidance, students frequently bypass essential problem-solving processes and rely on AI to produce complete answers (Wang et al., 2025). This behavior undermines metacognitive skills, particularly planning, monitoring, and evaluation skills, which strongly predict mathematics achievement (Abidin et al., 2025; Říčan et al., 2022). Furthermore, it may foster an "illusion of competence" wherein students believe that they fully comprehend a content merely by viewing AI responses (Matueny & Nyamai, 2025).

Studies on cognitive offloading have further demonstrated a negative correlation between frequent AI use and critical thinking abilities (Gerlich, 2025; Hassen, 2025), with digital-native learners showing higher dependency and lower critical thinking scores (Gerlich, 2025). Such dependency inhibits the development of core mathematical competencies, including heuristic reasoning, strategic flexibility, and mental model construction (Duffy et al., 2024; Md Nasir & Singh, 2023), ultimately leading to cognitive disengagement (Matueny & Nyamai, 2025). Moreover, AI-generated solutions interrupt the essential stages of mathematical problem-solving, understanding, planning, executing, and evaluating (Bahbah et al., 2023). This hinders the growth of representational, strategic, procedural, and metacognitive skills, whereas active engagement in these processes is known to deepen conceptual understanding, in comparison to the superficial learning that results from passive acceptance of answers (Liljedahl et al., 2016). The long-term consequences include the deterioration of developmental trajectories in mathematical thinking (Sheromova et al., 2020) and the diminution of self-efficacy due to over-reliance on AI (Lee et al., 2025). Despite the existence of empirical evidence suggesting that the use of structured and reflective AI may enhance metacognition and critical thinking (Lee et al., 2024), a growing body of research warns that habitual, unguided reliance on AI tends to engender cognitive passivity, as opposed to the higher-order thinking needed for synthesis, evaluation, and creative mathematical problem-solving (Gonsalves, 2024; Hassen, 2025).

In order to mitigate these undesirable consequences, a structured digital environment such as a learning management system (LMS) can be beneficial by orchestrating the interaction between learners and AI tools in a pedagogically meaningful way. In contrast to the unrestricted use of AI, an LMS facilitates a structured and intentional learning space that allows educators to embed assignments, resources, and scaffolds that systematically guide students toward reflective and metacognitive practices (Azevedo et al., 2022; Fan et al., 2025; Wang et al., 2023). Platforms such as Moodle can function not only as repositories of learning materials but also as metacognitive environments in which learners engage with structured prompts, sequenced tasks, and reflective checkpoints that encourage planning, monitoring, and evaluating their reasoning as they work through mathematical problems (Anupan & Chimmalee, 2024; Astafieva et al., 2023; Chen & Anyanwu, 2025). When integrated with an AI chatbot, LMS allows interaction to unfold within this structured space, enabling the chatbot to deliver hints, reflective questions, and verification prompts aligned with the learning design

rather than offering direct answers. In this sense, the LMS serves as a mediating framework that ensures AI supports students' cognitive processes without replacing them. Instead, it balances cognitive offloading with cognitive regulation and fosters sustained engagement with culturally contextualized mathematics assignments. The employment of AI chatbots has the potential to strengthen this structured support by offering adaptive guidance and responsive feedback, encouraging learners to explore multiple strategies, formulate exploratory questions, and participate in dialogic problem solving rather than merely reproducing routine procedures (Asyura et al., 2026).

Beyond metacognitive structure, mathematics learning also benefits significantly from culturally contextualized experiences that help students' ability to connect abstract concepts to familiar cultural practices, local wisdom, and social knowledge systems. Grounded in insights from ethnomathematics (Pari et al., 2025) and culturally responsive pedagogy (Nolan & Xenofontos, 2023), research has shown that when mathematical ideas are situated within meaningful cultural contexts, learners develop a deeper conceptual understanding and stronger reflective engagement. Within AI-supported environments, the provision of culturally relevant assignments has been shown to encourage students to interpret and apply mathematical reasoning in ways that resonate with their lived experiences, the result of which is a strengthening of both their cognitive and metacognitive involvement (Dankunprasert et al., 2025; Fererde et al., 2024). This approach aligns with constructivist and situated learning theories, which emphasize that knowledge is not transmitted but actively constructed through interaction with meaningful contexts (Kusaeri et al., 2025; Murti, 2023; Urohmah et al., 2025). When AI chatbots mediate mathematical assignments grounded in cultural contexts, they can function as partners that prompt learners to analyze, reflect, and synthesize mathematical ideas emerging from their cultural environment. This ultimately supports preservice teachers in cultivating reflective, autonomous, and culturally grounded mathematical understanding. Cultural contextualization is particularly important because familiar local practices can strengthen the grasp of mathematical concepts and help learners connect abstract representations with experiences encountered in their everyday life (Nindiasari et al., 2026).

For preservice mathematics teachers, understanding ethnomathematics involves not only the cultivation of culturally grounded mathematical knowledge but also the ability to translate that knowledge into inclusive instructional practices. Preservice mathematics teachers therefore need explicit opportunities to design culturally responsive mathematics lesson plans that are accessible to diverse learners, including students with special educational needs. The integration of ethnomathematics in teacher preparation has the potential to promote meaningful engagement with cultural mathematics and to broaden the perspectives represented in classroom instruction (Presmeg, 1998). However, without structured design experience, positive perceptions of ethnomathematics do not always translate into high-quality, culturally embedded learning materials (Utami & Sagita, 2025). In order to prepare teachers for classrooms with diverse student bodies, there is a necessity to integrate culturally responsive pedagogy, differentiated instruction, and inclusive teaching principles so that the mathematical content can be adapted to learners with varying abilities and support needs (Egara & Mosia, 2025; Tran & Schepers, 2023). These points highlight the importance of examining how preservice mathematics teachers may utilize AI not only to support culturally

contextualized mathematical reasoning, but also to transform such reasoning into culturally inclusive learning lesson plans for students with special needs.

However, despite the growing presence of AI tools in mathematics classrooms, there remains a significant gap in understanding how these tools can be intentionally designed to promote metacognitive engagement rather than mere task completion. Current implementations often prioritize efficiency and accuracy, overlooking the reflective learning processes necessary to cultivate self-regulation and critical thinking. This lack of pedagogical framing poses risks of reinforcing surface-level understanding, where students rely on AI for answers rather than developing the ability to evaluate, justify, and adapt their reasoning to formulate solutions. Investigating this issue for preservice mathematics teachers is particularly important because of their dual role as both learners and future educators. As learners, they need to develop the capacity to critically regulate their use of AI while solving culturally contextualized mathematical problems, including planning strategies, monitoring AI-generated responses, and evaluating the validity of their reasoning. In their future roles as teachers, they must also acquire the skills necesssry to utilize AI to support pedagogical decision-making. This is particularly relevant in the context of designing culturally inclusive learning lesson plans that accommodate diverse learners without undermining their reasoning, autonomy, and metacognitive development. Examining these two contexts is therefore essential, as preservice teachers' interactions with AI influence not only their own mathematical understanding but also their ability to translate culturally grounded knowledge into inclusive instructional practices.

The present study is of both theoretical and practical significance insofar as it addresses the existing gap. Theoretically, the study reframes the interaction between learners and AI chatbots as a process of metacognitive co-regulation. In this framework, AI functions as a cognitive partner that stimulates reflection, monitoring, and strategic reasoning during mathematical problem-solving process. Practically, it provides a framework to guide educators in designing AI-enhanced mathematics learning environments that foster independent and reflective engagement. Accordingly, this study aims to investigate the potential of AI chatbots to evolve from assistive tools into metacognitive partners that would guide preservice mathematics teachers in planning, monitoring, and evaluating their reasoning through culturally contextualized mathematical assignments. In doing so, the study seeks to uncover the underlying learning mechanisms and design principles that enable AI to support deep, autonomous, and transferable mathematical understanding.

2. METHOD

2.1. Research Design

The present study employed an embedded multiple-case study design to investigate how AI chatbots functioned as metacognitive partners in supporting preservice mathematics teachers' metacognitive co-regulation across two related learning contexts. An embedded multiple-case design examines several bounded cases, each containing multiple units of analysis, and enables systematic within-case and cross-case comparison (Al Qur'an, 2025; Yin, 2003). The design was deemed appropriate for the study because multiple-case studies

support the in-depth investigation of complex learning processes in naturalistic mathematics education settings while allowing for the identification of similarities, differences, and recurring patterns across cases (Zulkarnaen & Aziza, 2026). The first case involved the utilization of AI in culturally contextualized mathematical problem solving, whereas the second case focused on the application of AI in the development of a culturally inclusive learning lesson plan. These cases represented metacognitive regulation during mathematical problem solving and instructional design, respectively. Each case was delineated by its specific learning objectives, task characteristics, procedures, and expected outputs within the same LMS-based ethnomathematics course. This clear delineation of the cases and their contexts is consistent with Annamalah et al. (2026), who emphasize that rigorous case study research requires clearly bounded cases to generate context-sensitive and analytically meaningful insights. The decision was made to utilize an embedded multiple-case study design because individual participants and their AI-supported learning processes constituted units of analysis within each case.

2.2. Participants

The participants comprised 33 preservice mathematics teachers who were concurrently seventh-semester students in in the Department of Mathematics Education at a private university in Malang, East Java, Indonesia. The participants constituted the entire cohort that participated in the culturally contextualized mathematics learning in LMS-based environments and were considered eligible for the study because they had completed fundamental mathematical and pedagogical courses and had prior teaching experience. All 33 participants engaged in both cases of the study: AI-supported culturally contextualized mathematical problem solving and AI-supported development of a culturally inclusive learning lesson plan. Across these two cases, the participants engaged in culturally grounded learning activities, reflective problem-solving assignments, and instructional design activities with support from an AI chatbot integrated into Moodle. Within the embedded multiple-case study design, the participants and their AI-supported learning processes constituted the embedded units of analysis within each case.

Four data-rich participants were then purposively selected as the primary embedded units for in-depth analysis across both cases. The selection emphasized the richness, completeness, and variation of the available data rather than statistical representativeness. The process began with the collection of data from all 33 participants, encompassing Moodle chatbot logs, participatory observation notes, metacognitive reflection journals, self-assessment questionnaires, preliminary interviews, mathematical solutions, and lesson-plan products. These data were initially screened to identify meaningful metacognitive episodes, interaction patterns, and possible indicators of metacognitive failure, such as blindness, mirage, vandalism, and stagnation (Alifiani & Faradiba, 2021). The interactions between the participant and chatbot were subsequently coded in order to identify metacognitive processes, strategy revision, response acceptance, clarification, verification, and cultural engagement. The participants were then compared according to the completeness of their data, the diversity of their interaction patterns and metacognitive responses, the depth of their reflection, and the availability of triangulated evidence across data sources. Following this comparative

assessment, four participants, based on their contrasting and analytically rich metacognitive profiles, were selected for detailed within-case and cross-case analysis. The the data from other participants were retained as supporting evidence where relevant.

2.3. Instruments

The selection of instruments were guided by the objective of addressing the research question, with the instruments selected for their ability to capture the participants' metacognitive co-regulation with the AI chatbot across the two cases. Specifically, the study required evidence of the participants' metacognitive processes during culturally contextualized mathematical problem solving and culturally inclusive learning lesson-plan development. The instruments were therefore used to document the interaction process, observable behavior, participants' reflections and explanations, and the products generated in each activity. The instruments used in this study, along with their intended purposes and data sources, are summarized in Table 1.

Table 1. Instrument and data source

No.	Instrument	Purpose	Data Source
1	Chatbot interaction log Analytical rubric	To identify interaction patterns (linear vs. iterative) and AI response types (directive, corrective, reflective)	LMS transcripts and chatbot interaction logs
2	Participatory observation sheets, reflective journals	To assess participants' metacognitive behaviors (awareness, regulation, evaluation)	Observation sheets, participants'reflective journals, field notes
3	Semi-structured interview protocol	To explore participants' metacognitive behavior and perception of AI as a metacognitive partner	Semi-structured interviews

2.4. Research Procedure

The study commenced with the development of a Moodle-based learning management system (LMS) that was integrated with an AI chatbot. The LMS contained instructional materials on ethnomathematics, encompassing various aspects of local culture, including the Javanese primbon, traditional foods, performing arts, traditional architecture, and traditional games. These cultural contexts were selected because they represent varied mathematical ideas, including modular arithmetic, measurement, proportional reasoning, geometry, patterns, probability, combinatorics, and strategic reasoning. The Javanese primbon, for example, is an ancient Javanese compendium of life wisdom that uses culturally embedded numerical calculations to interpret events such as marital compatibility and auspicious timing (Utami et al., 2019). The present study was organized around two cases. In Case 1, the participants were expected to complete culturally contextualized mathematical problem-solving assignment and used the chatbot as a reflective partner for planning, verification, monitoring, and self-evaluation. In Case 2, the participants were expected to utilize the chatbot to support the development of culturally inclusive learning lesson plans for students with special needs,

translating their mathematical and cultural understanding into pedagogical decisions. The study was conducted in four systematic stages as follows: (1) Design and implementation stage, in which the participants engaged with culturally contextualized mathematics assignments within two cases managed through an AI-integrated LMS (Moodle) equipped with a chatbot providing reflective scaffolding; (2) Interaction recording stage, in which the LMS automatically recorded all chatbot–learner exchanges, which were later analyzed for evidence of metacognitive activity; (3) Reflection and interview stage, conducted after each session, in which the participants completed metacognitive reflection journals and participated in follow-up interviews to elaborate on their thought processes and learning strategies; and (4) Data validation stage, in which triangulation and member checking were carried out to confirm the authenticity of interpretations.

2.5. Data Analysis

The qualitative data were analyzed using a systematic thematic analysis that integrated the six-step analytic procedure proposed by Naeem et al. (2023) with the setup analysis interpretation structure described by Lochmiller (2021). This combined approach ensured a high degree of analytic transparency, theoretical alignment, and methodological rigor in examining students' metacognitive interactions with an AI chatbot during culturally contextualized mathematics assignments. This approach enabled the analysis to remain grounded in established concepts of metacognitive regulation while also capturing context-specific patterns that emerged directly from participants' interactions with the AI chatbot. Consistent with the embedded multiple-case study design, the two cases—AI-supported culturally contextualized mathematical problem solving and AI-supported development of a Culturally Inclusive Learning lesson plan—were first analyzed separately through within-case analysis and then compared through cross-case analysis. The four selected participants and their AI-supported learning processes served as embedded units within each case.

The analysis proceeded through eight stages: data preparation and familiarization, selection of meaningful quotations, keyword extraction using the 6Rs heuristic, hybrid deductive-inductive coding, within-case theme development, cross-case comparison, conceptual interpretation, and development of the final conceptual model. Moodle chatbot logs, observation notes, reflection journals, mathematical solutions, and lesson-plan products were organized by case and participant and triangulated throughout the analysis. The semi-structured interviews were audio-recorded with the participants' consent and transcribed verbatim. To ensure the accuracy of the transcription, each transcript was reviewed against the original audio recording. Corrections were made where necessary, and the transcripts were then coded. Ambiguous statements were clarified by referring to the recording and, when required, the participant was apprised of the accuracy of the statements. Deductive codes were derived from metacognitive regulation, AI interaction patterns, and metacognitive failure typologies, whereas inductive codes captured emerging processes such as cultural cue activation, clarification loops, response acceptance, strategy revision, and self-regulated recovery. The resulting themes were interpreted through the lenses of metacognitive regulation, learner-AI co-regulation, and culturally grounded reasoning, then synthesized into an interaction-regulation model explaining how different patterns of AI interaction and learner

regulation contributed to metacognitive success or failure across the two cases. The following Figure 1 details the eight stages of the analysis.

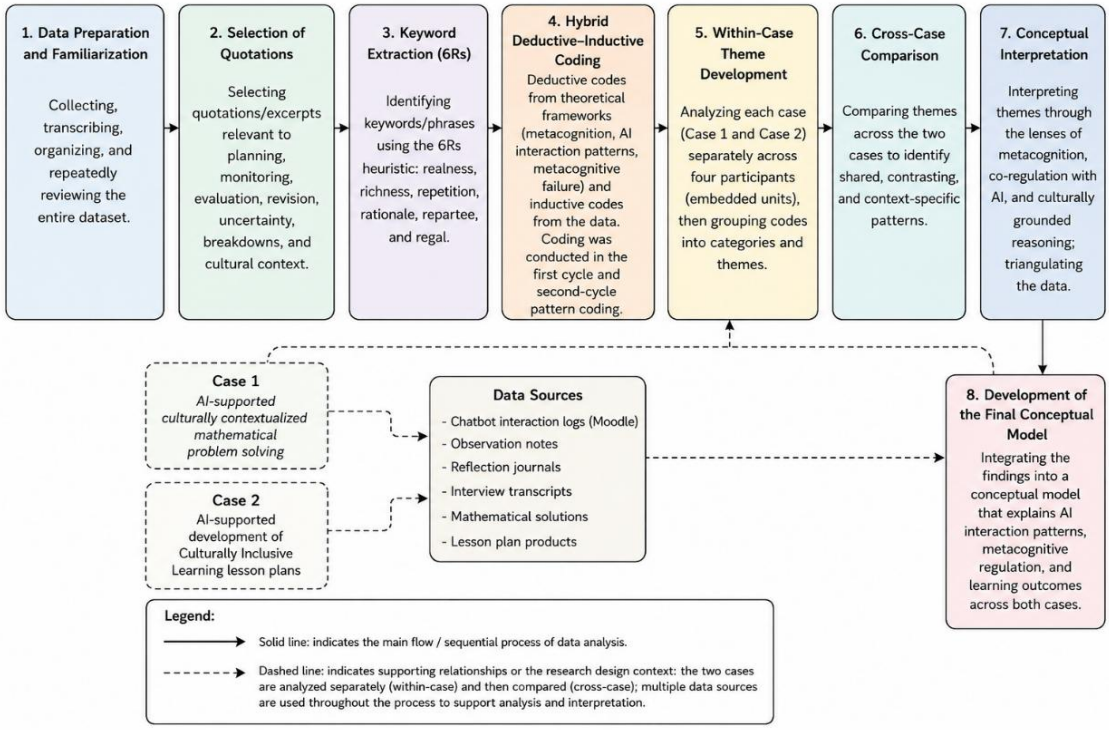


Figure 1. Data analysis procedure

3. RESULTS AND DISCUSSION

3.1. Results

This section expounds on the empirical findings concerning how AI chatbots can function as metacognitive partners in culturally based mathematical learning. The findings are organized around three interrelated themes: (1) the patterns of student interaction with the AI-based chatbot during problem-solving tasks situated in cultural contexts; (2) the metacognitive strategies underlying these interactions, emphasizing students' awareness, regulation, and evaluation processes, including potential metacognitive failures; and (3) the transfer of these metacognitive processes into the design of culturally-inclusive learning (CIL) lesson plans for students with special needs. Collectively, these findings demonstrate the potential of AI chatbots to evolve from assistive tools into reflective cognitive partners that foster culturally grounded, independent, and metacognitively rich learning among preservice mathematics teachers.

3.1.1. Patterns Exhibited by Students in Their Interactions with the AI-Based Chatbot in Moodle

Based on participatory observations, Moodle interaction logs, students' written responses and reflective journals, and in-depth interviews, the present study identified two dominant patterns of interaction between preservice mathematics teachers and the AI chatbot

during culturally contextualized mathematical problem-solving. The cultural problem involved a traditional Javanese context—forecasting of auspicious wedding dates and marital compatibility based on Javanese primbon calculations. These calculations necessitate the utilization of number theory concepts such as division, remainder, least common multiple (LCM), and modulo.

Subject 1 (S1)

Subject 1 demonstrated an iterative pattern through repeated questioning of the chatbot AI, comparison of methods, and requests for alternative calculations. After obtaining a result using modulo 5, she did not immediately conclude the interaction. Instead, she questioned whether the result was sufficient, as evidenced by the following excerpts:

S1 : "Based on the combined neptu calculation of 26, the modulo result is 1, which is "Sri." Are the bride- and groom-to-be compatible?"
S1 : "Use a different method to determine their compatibility."

For additional information, *neptu* is numerical value assigned to specific days and pasaran in Javanese almanac. Subject 1 subsequently continued exploring different calculation procedures and interpretations rather than relying on a single result. In her interaction with the AI chatbot, the subject made several requests for clarification regarding the calculation. She also requested that the chatbot examine alternative methods involving different divisors. For example, she requested that the chatbot explain the procedure more clearly and reconsider the calculation when there were differences in the interpretations. The following excerpts illustrate the requests:

S1 : "Explain the calculation clearly."
S1 : "Based on the calculation of the two neptu values, the total is 26. Use another method to determine the compatibility."

The excerpts show that receiving an answer triggered further questioning rather than closure. Subject 1 repeatedly returned to exploration, compared alternative modulo procedures, and reconsidered the interpretation of the results. This recursive movement between asking, evaluating, and revising indicates an iterative interaction pattern. This cyclical engagement reflects an iterative interaction pattern, whereby evaluation triggers further exploration rather than leading to a closure. The interaction pattern exhibited by Subject 1 (S1) can be illustrated as follows:

> *Reading the problem → Submitting an initial question to the chatbot → Receiving the first response → Conducting a preliminary evaluation (expressing doubt or lack of clarity) → Asking a follow-up question → Trying alternative strategies (mod 5 → mod 7 → mod 4 → mod 9) → Receiving a new response from the chatbot → Comparing with previous results → Deleting or reorganizing steps → Re-submitting the question → (Repeating the spiral process until a plausible result is found) → Determining the final answer.*

Subject 2 (S2)

The interaction between Subject 2 and the AI chatbot was more direct and task-oriented. Subject 2 initially requested a complete solution and then reproduced the chatbot-generated methods for further explanation. The chatbot provided calculations using modulo 5 and modulo 9, including the interpretation that the bride- and groom-to-be were compatible. The subsequent question posed by Subject 2 focused mainly on whether all methods were required, as presented:

S2 : "Do all methods have to be used to determine [an auspicious] wedding date?"

Although this question suggested a procedural curiosity, Subject 2 proceeded from the chatbot's explanation without contesting this computational issue. This limited verification supports the classification of interaction between Subject 2 and AI chatbot as predominantly linear, with the chatbot functioning primarily as a procedural information source. The interaction pattern exhibited by Subject 2 can be delineated as follows:

> *Reading the problem → Asking the chatbot → Receiving the answer → Concluding the interaction.*

Subject 3 (S3)

Subject 3 demonstrated an iterative pattern by detecting inconsistencies in the chatbot's proposed dates and initiating corrective exchanges. In response to the proposition by the chatbot of 14 March 2024 as the next Thursday *Pon*, Subject 3 explicitly rebuffed the proposition, articulating the following:

S3 : "March 14 is not Thursday Pon."

Instead of acquiescing to the initial proposition, Subject 3 demanded that the chatbot reevaluate the wetonan cycle. Additionally, she examined the relationship between the proposed auspicious wedding date and a thousandth-day memorial. Her analysis showed that the first proposed date of Thursday *Pon* coincided with or was too close to a memorial day, prompting the search for an alternative date based on the 35-day wetonan cycle. The resulting revision is represented as follows:

S3 : "Thus, the next Thursday Pon is April 11added with 35 days, which is May 16, 2024."

The aforementioned excerpts provide direct evidence that Subject 3 detected a red flag, returned to the calculation, and modified the proposed solution. The sequence of identifying an error, requesting reconsideration, and calculating an alternative date demonstrates a recursive and corrective interaction pattern. The following is an illustration of the sequence:

> *Reading the problem → Submitting an initial question to the chatbot → Receiving the first response → Conducting a preliminary evaluation (noticing something unclear) → Rephrasing and resubmitting the question → Trying alternative strategies (calculating neptu → 35-day wetonan cycle → determining the auspicious Thursday Pon) → Receiving a new response → Comparing the response with previous results → Reorganizing steps (adjusting dates due to an*

overlap with the 1,000th-day memorial day) → Repeating the question → Iterating the cycle (until the result is accurate) → Determining the final answer and drawing a conclusion.

Subject 4 (S4)

Subject 4 began by inputting the entire problem, then narrowed the interaction with the AI chatbot to specific mathematical procedures. Instead of merely requesting for a final interpretation, she made a specific request for detailed calculations, as demonstrated by the excerpts:

S4 : "For the compatibility of the bride- and groom-to-be, provide the mathematical calculation. Do not provide the interpretation directly."
S4 : "For the calculation, use each type of neptu calculation: Type 1 uses modulo 5; Type 2 uses modulo 7; Type 3 uses modulo 4; and Type 4 uses the sum of the two neptu values with modulo 9."
S4 : "Okay, we will proceed slowly. Using the combined neptu total, determine the modulo value for each type that I provided."

These prompts show that Subject 4 supplied explicit procedural criteria and used the chatbot to verify calculations according to a predetermined structure. In contrast to Subject 1 and Subject 3, her prompts did not repeatedly shift between competing interpretations after the required procedure had been established. The interaction between the subject and the AI chatbot was therefore more structured and goal-directed, with the chatbot functioning primarily as a calculation and confirmation tool. The interaction pattern between Subject 4 and the AI chatbot can be illustrated as follows:

Reading → Asking a question → Receiving an answer → Requesting confirmation → Accepting the answer.

From the analysis of all four subjects, two distinct interaction modes emerged:

a. Iterative pattern (Subject 1, Subject 3): The pattern is characterized by recursive questioning, alternative strategy testing, and reflective evaluation. The students used the chatbot as a thinking partner, engaging in multiple cycles of exploration and self-correction.
b. Linear pattern (Subject 2, Subject 4): The pattern is characterized by direct inquiry, immediate acceptance, and minimal verification. The students treated the chatbot as an answer provider or validator, resulting in efficient but shallow engagement.

Table 2 presents the complete characteristics of each interaction pattern identified in the study, distinguishing between the linear and iterative patterns in students' engagement with the AI chatbot.

Table 2. Patterns of interaction between the students and the AI chatbot

Aspect	Linear Pattern	Iterative Pattern
Thinking flow	Straightforward: reading → understanding → implementing → evaluating → concluding	Recursive: alternating between exploration, evaluation, and revision
Efficiency	Quick and result-oriented	Time-consuming but reflective
Student focus	Outcome-focused	Process- and strategy-focused
Role of evaluation	Being conducted once at the end of the interaction	Being conducted iteratively following the response of the chatbot
Function of chatbot	Serving as a single source of answer	Serving as a reflective exploration partner
Cultural context engagement	Following a fixed interpretation	Open to alternative interpretations and strategies, though more prone to ambiguity

3.1.2. Metacognitive Strategies Emerging in the Problem-Solving Process Assisted by AI Chatbot

The present study also analyzed the metacognitive strategies employed by students during problem-solving processes involving interaction with an AI chatbot. The analysis adopted the framework from Magiera and Zawojewski (2011), which includes three key dimensions: (1) metacognitive awareness, which refers to an individual's understanding of their own knowledge state, the strategies available to solve a task, and the relationship between their knowledge and the strategies they select; (2) metacognitive regulating, which involves the ability to reflect on and adjust one's thinking in order to plan and set appropriate goals; and (3) metacognitive evaluating, which concerns awareness of the limitations of one's knowledge, the effectiveness of selected strategies, and the quality of outcomes.

Furthermore, the analysis incorporated the concept of metacognitive failure, as proposed by Goos (2002). This failure was identified through the emergence of three types of red flags: lack of progress (when the process stagnates), error detection (when inconsistencies or mistakes are identified), and anomalous results (when outcomes appear unexpected or contradictory to prior understanding). These red flags may lead to different forms of metacognitive failure, depending on how learners respond to them. Four categories of metacognitive failure are distinguished: (1) metacognitive blindness, which occurs when learners fail to recognize the presence of a red flag; (2) metacognitive mirage, which occurs when learners perceive a red flag even though none actually exists; (3) metacognitive vandalism, which occurs when learners recognize a red flag but their attempted correction violates conceptual or procedural rules; and (4) metacognitive stagnation, which occurs when learners notice a red flag but take no corrective action (Alifiani & Faradiba, 2021; Goos, 2002). Collectively, these constructs provide a comprehensive lens to understand how learners monitor, regulate, and evaluate their thinking when interacting with AI tools during culturally contextualized mathematical problem-solving.

Subject 1 (S1)

During the problem-solving process, Subject 1 exhibited an iterative interaction pattern characterized by cycles of exploration, regulation, and evaluation. Subject 1 initiated by carefully reading the mathematical problem and re-entering it into the AI chatbot to request an AI-generated solution. The chatbot proposed several possible solutions, each of which led to different interpretations. For instance, using mod 7 resulted in a negative solution, while mod 5 yielded a positive one. To clarify this discrepancy, Subject 1 engaged in metacognitive regulation by asking the chatbot to confirm the correct solution. Upon receiving the initial response, Subject 1 immediately evaluated it against prior knowledge. However, this did not result in a sense of certainty. Instead, it elicited a sense of doubt, prompting further questioning and strategic adjustment. Demonstrating flexibility, Subject 1 explored alternative strategies by experimenting with different methods (mod 5, mod 7, mod 4, and mod 9), as shown in Figure 2. Taken from the interview section, the following excerpts elucidate the perspective of the subject on the process.

R (Researcher) : *What made you attempt multiple modulo calculations instead of following the chatbot's first response?*

S1 : *At first, I followed [the response of] the chatbot and used mod 5, but I wasn't sure if it was the only correct method. I remembered that Javanese primbon sometimes uses different modulo systems, so I tried mod 7, mod 4, and mod 9 to compare the results.*

After experimenting with several calculation methods, Subject 1 carefully compared the answers and reflected on the meanings associated with each modulo system. Rather than resolving her uncertainty, the different interpretations prompted further uncertainty regarding the most suitable solution, as stated by the subject in the following interview excerpts:

R : *What did you find after comparing them?*

S1 : *Each solution produced a different interpretation. Some [interpretations] were positive and others were negative, so I found myself in a state of confusion instead of clarity. In the end, I wrote down all possible answers because I couldn't decide which one should be prioritized.*

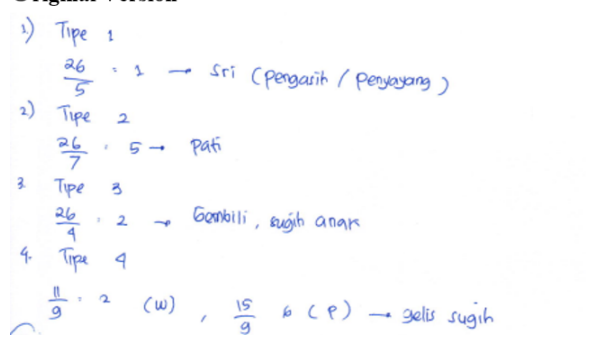
Original Version	**Translation**
	1) Type 1 $\frac{26}{5} = 1 \rightarrow$ *sri* (compassionate/ affectionate) 2) Type 2 $\frac{26}{7} = 5 \rightarrow$ *pati* (harbinger of death) 3) Type 3 $\frac{26}{4} = 2 \rightarrow$ *gembili*, philoprogenitive 4) Type 4 $\frac{11}{9} = 2\ (F), \frac{15}{9} = 6\ (M) \rightarrow windfall$

Figure 2. Answer provided by Subject 1 to culturally based mathematics problem

The statement discloses a dynamic regulatory cycle of modifying, repeating, and testing strategies in response to uncertainty. As metacognitive awareness of Subject 1

strengthened, Subject 1 recognized multiple possible approaches and continuously compared chatbot responses, even deleting or revising steps when confusion persisted. This recursive loop—asking, receiving, evaluating, and reorganizing—illustrates an exploratory but unstable regulation process. After several trials and evaluations, Subject 1 arrived at a final answer, although residual uncertainty persisted due to differing interpretations of each modulo type. This reflects both the depth and complexity of Subject 1's metacognitive engagement and the presence of a metacognitive mirage, in which S1 stopped at an answer she deemed unreliable. This was considered a red flag, despite the answer's actual validity. This uncertainty occurred because the foreknowledge derived from the Javanese primbon can be inherently inconsistent. However, this inconsistency is not inherently a logical flaw; rather, it reflects the flexible and interpretive nature of cultural knowledge. Such foreknowledge represents a form of personal estimation—an interpretive belief that may not necessarily align with an objective truth. This phenomenon illustrates how numbers carry a mystical and symbolic significance in Javanese culture.

Subject 2 (S2)

Subject 2 began the problem-solving process by carefully reading the question and promptly identifying key information, such as the days, in both the seven-day Gregorian week and the five-day pasaran cycle, as well as other contextual elements provided. These elements served as the foundation for determining the appropriate *neptu* values. At this initial stage, Subject 2 demonstrated metacognitive awareness by distinguishing between her prior knowledge and the aspects that remained uncertain, particularly regarding the type of modulo to be used in the calculation. Recognizing the uncertainty, Subject 2 immediately sought assistance from the AI chatbot as an external source of guidance. The interaction between Subject 2 and the chatbot was concise and task-oriented, consisting of direct requests for solutions or final answers rather than exploratory questions. The chatbot responded with step-by-step explanations and numerical answers, which Subject 2 briefly evaluated for logical consistency. However, the evaluation was limited to a surface-level confirmation rather than a deeper verification or comparison of alternatives. Upon perceiving the chatbot's response as both logic and sufficiently detailed, Subject 2 accepted the result without further inquiry and concluded the problem-solving process. The following interview excerpts illustrate the process.

R : After identifying the information needed, what did you do next?
S2 : I knew the problem required a neptu [value] calculation, but I was uncertain which modulo to use. I asked the AI chatbot because I thought it would quickly show me the correct solution and answer.

After receiving the chatbot's solution, Subject 2 briefly reviewed it. Because the solution proposed by the AI chatbot appeared to be clear and consistent with her expectations, Subject 2 accepted the proposed solution without comparing it to alternative solutions or questioning its correctness. This is evidenced in the excerpts of the interview.

R : Why didn't you verify the calculation further?
S2 : The answer looked logical and corresponded to my expectation. I didn't feel it necessasry to ask [any] further questions or to explore other methods.

Original Version	***Translation***
Perhitungan Kecocokan Neptu: Metode 1: Dibagi 5 (Sisa): (Neptu Wanita + Neptu Pria) / 5 (11 + 15) / 5 = 26 / 5 = 5 sisa 1 Sisa 1: Sri (berarti keberuntungan, kemuliaan, dan rezeki yang melimpah).	**Method 1: Divided by 5 (Remainder):** (*Neptu* of bride-to-be + *Neptu* of groom-to-be) / 5 (11 + 15) / 5 = 26 / 5 → remainder 1 Remainder 1: *Sri* (which means luck, nobility, and abundant fortune)
Metode 2: Dibagi 9 (Sisa): (Neptu Wanita + Neptu Pria) / 9 (11 + 15) / 9 = 26 / 9 = 2 sisa 8 Sisa 8: Tinari (berarti menemukan kebahagiaan, kemudahan dalam mencari rezeki, dan dihormati).	**Method 2: Divided by 9 (Remainder):** (*Neptu* of bride-to-be + *Neptu* of groom-to-be) / 9 (11 + 15) / 9 = 26 / 9 → remainder 8 Remainder 8: *Tinari* (which is believed to mean that if the couple enters into matrimony, they are likely to find happiness, earn a living easily, and be esteemed)

Figure 3. Transcript overview of the interaction between Subject 2 and the AI chatbot

Figure 3 presents a transcript of the chatbot's response to the question proposed by Subject 2. As illustrated in Figure 3, the calculation using mod 5 is correct, but the computation using mod 9 is incorrect. The *neptu* values of the bride and groom should be calculated separately under mod 9, rather than being directly summed. However, this error went unnoticed by Subject 2 because the answer appeared to be both logical and consistent with her expectations. Subject 2 then copied the chatbot's response exactly as it was presented, as shown in Figure 4. This lack of awareness regarding the computational error, despite clear inconsistency, suggests a form of metacognitive blindness. Subject 2 failed to detect a red flag in the chatbot's reasoning due to her reliance on the perceived plausibility of its answer.

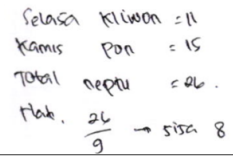
Original Version	***Translation***
	*Tuesday* Kliwon = *11* *Thursday* Pon = *15* *Total* neptu *value* = *26* *Then,* $\frac{26}{9}$ → *remainder 8*

Figure 4. Answer provided by Subject 2 to culturally based mathematics problem

Subject 3 (S3): Full Metacognitive Cycle and Adaptive Regulation

Subject 3 initiated by carefully reading and understanding the problem, identifying key information such as weton of both the bride- and groom-to-be and the date of the bride-to-be's parent's memorial. This stage reflected early metacognitive awareness, as Subject 3 recognized the information required for an accurate solution. Afterward, Subject 3 analyzed and structured the problem, realizing that the *neptu* values had to be calculated, summed, and interpreted according to the precepts of Javanese primbon. The subject also understood that the selected

wedding date must not coincide with the thousandth-day memorial. This process was explained by Subject 3 during the interview.

R : Before posing your question to the AI chatbot, what did you do to understand the problem?
S3 : I began by thoroughly reading the problem and organizing the pertinent information, including the weton of both the bride- and groom-to-be and the date of the parent's death. I wanted to ensure that I understood the necessary calculation before querying the chatbot so that its response would be relevant.

In the exploration stage, Subject 3 employed the chatbot to verify calculations and seek further elucidation. Subsequently, the subject, collaboratively with the chatbot, developed a plan for solving the problem. During the implementation stage, Subject 3 calculated the *neptu* totals and *pancasuda* values and then verified the chatbot's answers. It was discovered that the date proposed by the chatbot coincided with the thousandth-day memorial (see Figure 5). This served as a red flag.

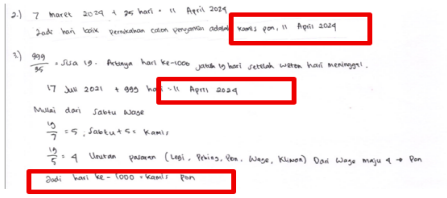
Original Version	Translation
	2. March 7, 2024 + 35 days = April 11, 2024 So, the auspicious wedding day for the couple is Thursday Pon, April 11, 2024. 3. 999 ÷ 35 = remainder 19. This means that the 1000th day falls 19 days after the day of death. July 17, 2021 + 999 days = April 11, 2024. Starting from Saturday Wage: 7 ÷ 5 = 5 → Saturday + 5 = Thursday 19 ÷ 5 = 4 → Sequence of pasaran (Legi, Pahing, Pon, Wage, Kliwon). From Wage, move forward 4 → Pon. So, the 1000th day = Thursday Pon.

Figure 5. Answer provided by Subject 3 to culturally based mathematics problem

Subject 3 addressed this inconsistency exhibited by the chatbot as a signal to re-examine her strategy and identify an alternative solution. This approach was further elaborated upon during the interview, as outlined on the following excerpts:

R : What did you do upon realizing that the auspicious day coincides with the 1,000th-day memorial?
S3 : I realized the suggested date could not be used because it coincided with the thousandth-day memorial. So I asked the chatbot to calculate the next Thursday Pon instead of simply changing the date myself, and then I checked the new result.

As stated in the excerpts, Subject 3 adaptively returned to the exploration stage and asked the chatbot to calculate the next Thursday *Pon* to avoid two overlapping events (see Figure 6).

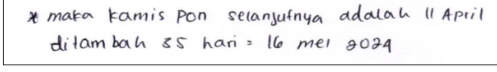
Original Version	Translation
	*thus, the next Thursday *Pon* is April 11 plus 35 days = May 16, 2024

Figure 6. Final answer provided by Subject 3 to culturally based mathematics problem

The ability of Subject 3 to promptly adapt the strategy and seek an alternative solution suggests the presence of effective metacognitive regulation. The cycle of exploration–

implementation–verification was repeated several times. Even when Subject 3 considered a quick fix by simply adding seven days, the subject recognized that such a shortcut violated traditional rules and recalculated correctly. This process showed that each red flag became an opportunity for reflection and refinement, leading to a more accurate and culturally coherent result. Overall, the process taken by Subject 3 was highly iterative and self-corrective, with no signs of metacognitive failures.

Subject 4 (S4): Verification-Oriented Linear Strategy

Subject 4 initiated the problem-solving process by carefully reading and identifying essential information, including weton of both the bride- and the groom-to-be, as well as the date of the bride's parent's passsing. In this stage, Subject 4 demonstrated metacognitive awareness, recognizing the knowledge and strategies necessary to reach an accurate solution. She independently prepared the necessary data, recalling *neptu* values of Gregorian-calendar days and pasaran days, summing them, and interpreting the results according to Javanese primbon rules. Subject 4 demonstrated metacognitive regulation by systematically planning her problem-solving steps. She calculated the *neptu* value of the bride- and groom-to-be, determined their total, interpreted it into categories such as "Sri," "Lungguh," or "Gedong," and performed a 35-day cycle calculation to find an auspicious day. In addition, she maticulously arranged her notes, ensuring the availability of her calculations for re-examination if needed. Subject 4 articulated the following in reflecting of the preparation process:

R : Before using the AI chatbot, how did you prepare to solve the problem?
S4 : I already knew the calculation steps—from determining neptu values, summing them, and then finding the auspicious Thursday Pon. I first organized all the information and then completed the calculation before posing the question to the chatbot.

During the manual computation process, Subject 4 continued metacognitive monitoring, verifying her additions, checking the correctness of the 35-day cycle, and ensuring the resulting day matched the intended pasaran day. In the evaluation stage, she examined whether the chosen date overlapped with the thousandth-day memorial and whether her result made sense from the cultural perspective.

Having confidence in her solution, Subject 4 decided to use the AI chatbot solely for verification purposes, not for generating an answer. Her exploration stage was therefore concise and goal-oriented—she only asked the chatbot to confirm the solution she had obtained manually. The planning and implementation stages focused on the entry of identical data into the chatbot for confirmation. During the verification process, Subject 4 discovered that her solution matched perfectly with that of the chatbot, reinforcing her confidence, as she subsequently explained:

R : Why did you still use the AI chatbot after completing the calculation yourself?
S4 : I was already confident with my calculation, but I wanted to ensure I had correctly determined the [auspicious] day and completed the calculation. When the chatbot produced the same result, I became more confident that my answer was correct.

No red flags emerged, as there were no discrepancies between her solution and the chatbot's solution. The subject's process followed a linear pattern—reading, understanding, analyzing, exploring (albeit briefly for verification), implementing, and verifying—without the occurrence of repeated cycles of exploration or planning. The interaction pattern exhibited by the subject with the AI chatbot reflected effective metacognitive engagement, with the chatbot functioning primarily as a validator rather than a problem solver.

Table 3 summarizes the interaction patterns exhibited by the four subjects with the AI chatbot during the process of solving culturally grounded mathematical problems. Each subject demonstrated distinct metacognitive behaviors depending on their approach to exploration, regulation, and evaluation. As described in Table 3, these interactions have revealed varying levels of metacognitive engagement—from iterative cycles of reflection and adjustment to linear, verification-oriented reasoning—along with potential metacognitive failures that emerged or were successfully avoided.

Table 3. Metacognitive behavior exhibited by the subjects during culturally based problem-solving supported by AI chatbot

Subject	Interaction Pattern	Metacognitive Activities	Metacognitive Failure
1	Iterative	Demonstrating deep metacognitive awareness through repeated checking and reflection; showing regulation and self-correction during exploration with the chatbot	**Metacognitive Mirage**—doubting a correct result due to an overreliance on perceived inconsistency.
2	Linear	Displaying surface-level evaluation and quick acceptance of the chatbot responses; focusing on efficiency rather than exploration	**Metacognitive Blindness**—uncritical acceptance of answers without deeper verification.
3	Iterative	Showing a high level of metacognitive awareness, regulation, and adaptability; detecteing and responding to *red flags* effectively; engaging in iterative exploration–verification cycle	**None**—consistently self-corrective without metacognitive breakdown
4	Linear	Displaying strong self-regulation and confidence; utilizing the chatbot only for confirmation purposes; being characterized by organized and systematic thinking	**None**—maintaining control and accuracy without overreliance or doubt

The findings indicate that the interaction pattern strongly mediates the quality and depth of metacognitive processes that emerge during AI-assisted problem-solving process. In iterative engagement (as shown by Subjects 1 and 3), learners perpetually traverse the stages of exploration, regulation, and evaluation. This recursive process encourages deeper awareness and reflection, allowing learners to detect errors, reevaluate their strategies, and refine their reasoning. However, this openness to revision can also increase cognitive load and the potential for metacognitive mirage, where learners become entrapped in cycles of doubt or over-analysis, as observed in Subject 1, who expressed uncertainty regarding a correct answer due to conflicting interpretations.

In contrast, linear interaction patterns (as observed in Subjects 2 and 4) reflect a more structured and goal-oriented approach. Learners are able to plan, implement, and verify in a straightforward sequence with minimal backtracking. This pattern promotes efficiency and confidence, particularly in instances where learners have already acquired adequate prior knowledge, as evidenced in Subject 4. Yet, it can also engender metacognitive blindness, where learners prematurely accept results without conducting critical verification, as demonstrated by S2.

Overall, the analysis suggests that neither pattern is inherently superior. On the one hand, iterative engagement nurtures deep reflection but requires careful regulation to prevent confusion. On the other hand, linear engagement ensures efficiency but benefits from deliberate evaluative checkpoints. The optimal metacognitive functioning of learners arises when they can flexibly shift between these modes—engaging iteratively when uncertainty emerges and employing linear verification when confidence is founded on prior understanding.

3.1.3. Metacognitive Strategies in Designing Culturally Inclusive Learning (CIL) Lesson Plan

After completing culturally contextualized mathematical problem-solving assignments, the subjects, in their roles as preservice mathematics teachers, were assigned to design culturally inclusive learning (CIL) lesson plan intended to support students with disabilities. In the course of the design process, the subjects interacted with the AI-based chatbot to assist their reflection and regulation processes while designing lesson plans that integrated cultural contexts and inclusive pedagogies. The following sections provide an overview of the processes.

Subject 1 (S1)

Subject 1 initiated the process of designing a lesson plan on the topic of circles within a culturally inclusive learning framework for a class that includes students with hearing impairments. Subject 1 incorporated *kawung* batik motifs as cultural representations of circular shapes. From the beginning, she showed metacognitive awareness, recognizing the potential challenges faced by deaf students in comprehending abstract definitions of radius and diameter and the necessity of a strong visual approach to facilitate learning. Utilizing an AI chatbot, she explored various strategies, such as the incorporation of colored patterns, digital images, and real fabric to illustrate circle properties. Each response of the AI chatbot triggered a regulation process. Subject 1 compared different alternatives, reorganized the lesson steps, and refined her plan to include visual coloring activities and sign language integration.The evaluation process done by Subject 1 was reflective, yet inconclusive. Despite converging on a valid design, she expressed uncertainty about the optimal medium.

The subject's expressed uncertainty indicated the presence of a metacognitive mirage, where uncertainty persisted despite the attainment of a correct and feasible solution. Subject 1 began to doubt as to whether students with hearing impairments were capable of understanding and solving problems that involve overlapping circular areas (see Figure 7). This skepticism was rooted in the assumption that their disability might affect their ability to engage in abstract reasoning. In reality, students with hearing impairments have cognitive capacities equal to

their hearing peers. Their challenges lie only in accessing verbal or auditory information, not in mathematical reasoning itself.



Figure 7. Culture-based mathematics problems designed by Subject 1 in CIL design

Subject 2 (S2)

Subject 2 designed a lesson plan on curved solid geometry intended for students with visual impairments. During the lesson design process, Subject 2 requested that the AI chatbot design the lesson plan. Subject 2 employed problem-based learning (PBL) model, featuring a contextual case about *bonang*, a traditional Javanese musical instrument crafted from bronze. Subject 2 accepted the chatbot's proposition regarding a problem involving *bonang* as the contextual case without further reflection. Subject 2 presented the mathematical problem in the same form as that proposed by the chatbot (see Figure 8).

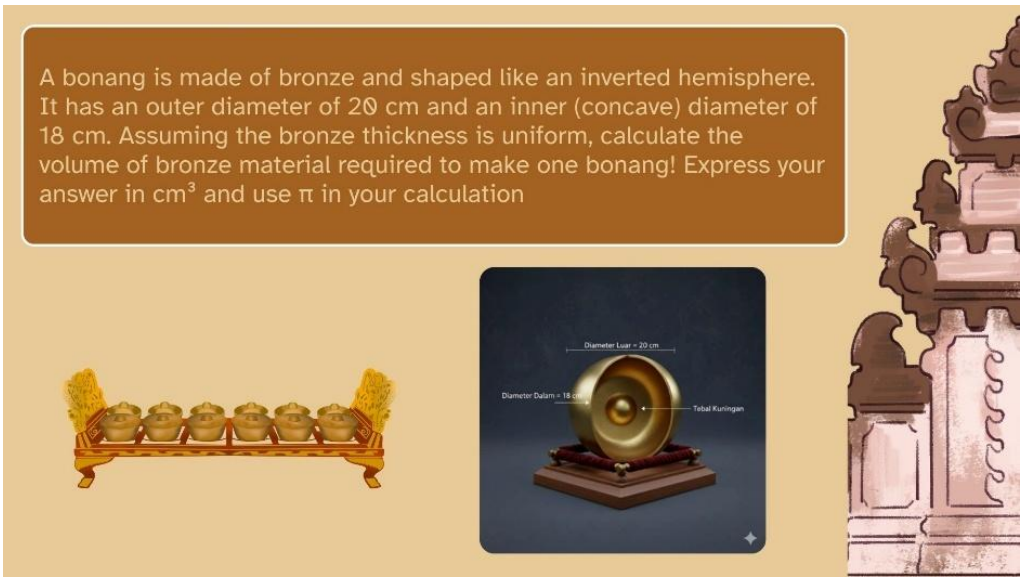


Figure 8. Culture-based mathematics problems designed by Subject 2 in CIL design

The problem designed required students to determine the material volume of a *bonang* based on the geometric model provided. In the initial stage, Subject 2 demonstrated metacognitive awareness by recognizing the necessity to design a lesson plan for students with visual impairment. Subject 2 realized that a mere verbal explanation of the formulas would suffice, as blind students need tactile or auditory representations of geometric forms. However, during her interaction with the AI chatbot, her process was brief and linear. Subject 2 directly asked the AI chatbot about how to design the lesson plan, adhered to the chatbot's recommendations, and quickly accepted the resultant plan. There was minimal metacognitive regulation, as the subject did not evaluate whether the geometric assumption matched the actual shape of a *bonang*. In reality, the shape of a *bonang* is slightly flattened and does not perfectly align with the mathematical definition of a hemisphere. This oversight risked leading students to form an incorrect geometric conception. The subject's inability to detect this conceptual inaccuracy was indicative of metacognitive blindness, a failure to recognize a red flag due to overtrust in the AI-generated explanation.

Subject 3 (S3)

Subject 3 initiated the design of the lesson plan on the topic of circles with an awareness that the concept could be abstract and challenging for students with mild intellectual disabilities. Demonstrating strong metacognitive awareness, she identified the need for concrete cultural representations and selected *gong* from Javanese gamelan ensemble as a tangible example of a circle. This awareness guided her to seek an appropriate instructional strategy that could connect geometry with sensory and cultural experiences.

During her interaction with the AI chatbot, Subject 3 entered the regulation stage by asking how to teach the concept of radius to students with cognitive impairments, using a *gong* as a medium for learning. The chatbot proferred various suggestions, such as using color cues, stickers, and strings. However, Subject 3 did not adopt these ideas directly. Instead, she regulated her thinking by analyzing the feasibility and practicality of each suggestion. In doing so, she considered the cultural sensitivity of a gamelan ensemble and the students' cognitive abilities. She then revised the plan by replacing the original *gong* with safe and accessible circular objects (e.g., plastic plates or cardboard circles) to ensure hands-on learning while maintaining the cultural link.

Through cycles of reflection and revision, Subject 3 exhibited iterative metacognitive engagement—oscillating between exploration, regulation, and evaluation. She engaged in mental rehearsals of classroom scenarios, evaluating the comprehensibility and manageability of the proposed sequence of activities for her students. Whenever she identified potential difficulties, she returned to the chatbot for further clarification or alternative strategies. This behavior can be interpreted as active self-monitoring and adaptive regulation processes.

The process demonstrated by Subject 3 manifested a rich metacognitive pattern, characterized by an awareness of the needs of the learners, strategic regulation based on contextual and cultural considerations, and evaluative reasoning to ensure instructional clarity. The iterative dialogue with the chatbot not only supported her decision-making process but also strengthened the inclusivity and practicality of her final lesson design.

Subject 4 (S4)

Subject 4 developed a lesson plan on plane geometry, using *joglo* as the cultural context. *Joglo* is a traditional Javanese house known for its distinct roof structure, which comprises trapezoidal and rectangular shapes. She selected this context for its cultural relevance and its potential to teach area and perimeter while introducing students to local architectural heritage. The lesson was designed for students with mild visual impairments, requiring the use of multisensory media to support tactile learning.

In the metacognitive awareness stage, Subject 4 realized that visual representations alone were insufficient for visually impaired students. She recognized the need for tangible materials and thus decided to devise a textured geometry board equipped with thin wooden pieces in the shapes of triangles, trapezoids, and rectangles that could be attached with adhesive. The intention was that students would be able to feel the sides and shapes through touch and relate them to the actual structure of *joglo*.

During the regulation stage, Subject 4 systematically organized the learning sequence: introducing students to a short story about *joglo*, guiding them as they explore tactile models, performing simple calculations of area and perimeter with teacher assistance, and comparing the size of the roof (triangular) with the main room (rectangular). The subject consulted the AI chatbot to confirm the suitability of her approach using textured boards and to explore ways to enhance multisensory engagement. The chatbot suggested the integration of verbal counting in conjunction with tactile manipulation of the shapes. Subject 4 reflected on the suggestion, made minor adjustments, and incorporated the idea to enrich the sensory experience.

In the implementation stage, Subject 4 documented clear instructional steps for teachers, including narrative descriptions, guided tactile exploration, and simplified formula use. Her evaluation process involved comparing the chatbot's feedback with her own simulated classroom plan, ensuring the activities were realistic and suited to her students' needs. This reflective comparison strengthened her confidence in her design.

Overall, the metacognitive process exhibited by Subject 4 was found to be linear, yet highly reflective. She demonstrated a strong awareness of the needs of the learners, systematic regulation of multisensory strategies, and critical evaluation through thoughtful validation rather than repetitive revision. There was no indication of metacognitive failure or false red flags. The chatbot was employed as a validator to reinforce the subject's confidence and to ensure the inclusivity and practicality of her culturally grounded lesson plan. The cross-case analysis of metacognitive processes and interaction patterns observed among the four subjects during the development of culturally-inclusive lesson plans with the assistance of an AI chatbot is summarized in Table 4.

Table 4. Metacognitive behavior exhibited by the subjects during CIL lesson plan designing process supported by AI chatbot

Subject	Interaction Pattern	Dominant Metacognitive Traits	Metacognitive Failure
1	Iterative	Strong awareness, active regulation, and repeated evaluation; using chatbot dialogically for exploration	**Metacognitive Mirage**—doubting the lesson plan's suitability for deaf students despite its pedagogical soundness

Subject	Interaction Pattern	Dominant Metacognitive Traits	Metacognitive Failure
2	Linear	Awareness of student needs, limited regulation, and minimal evaluation; brief, procedural interaction	**Metacognitive Blindness**—accepting chatbot suggestions for geometry problems using *bonang* as a contextual case without realizing that the shape of a *bonang* is not a perfect hemisphere, resulting in a risk of conceptual misunderstanding
3	Iterative	Continuous awareness–regulation–evaluation cycles; adaptive strategy modification	**None**—refining and simplifying the lesson plan, which employed *gong* as a contextual case, for students with mild intellectual disabilities through repeated evaluation and adjustment
4	Linear	Systematic regulation and reflective evaluation; using chatbot for confirmation, not exploration	**None**—employing the chatbot to validate that her lesson plan for low-vision students, with the roof of *joglo* as a contextual case and the integration of tactile media, is effective, accompanied by confident self-evaluation

The analysis indicated that iterative engagement facilitated deeper metacognitive reflection, though it was susceptible to over-analysis (metacognitive mirage), while linear engagement promoted efficiency but could lead to uncritical acceptance (metacognitive blindness). When used for validation purposes, however, linear interactions—as seen in Subject 4—fostered reflective confidence without failure. Moreover, the cultural contexts selected by the subjects (*kawung* batik, *bonang*, *gong*, and *joglo* architecture) influenced their reasoning by linking mathematical abstraction with inclusive pedagogy. These contexts not only grounded learning in cultural meaning but also heightened awareness of students' diverse needs. This, in turn, demonstrates that AI-assisted reflection can strengthen both the cognitive and ethical dimensions of culturally responsive mathematics teaching.

3.2. Discussion

The present study demonstrates that when AI chatbots are intentionally embedded within culturally contextualized mathematics assignments, they can shift from functioning primarily as answer providers, as previously critiqued in other works (El Fathi et al., 2025; Harahap et al., 2025; Yunianto et al., 2024), to acting as metacognitive partners that support planning, monitoring, and evaluation. This shift directly addresses the pedagogical problem identified in the introduction, wherein unregulated chatbot use often leads to surface-level learning, cognitive dependency, and diminished reflective engagement (Wu, 2024). By situating an AI chatbot within a structured LMS and assignments grounded in ethnomathematical contexts, the present study provides empirical evidence that purposeful design can mitigate these risks and activate deeper metacognitive processes.

Two distinct interaction patterns emerged during the students' engagement with problem-solving and culturally inclusive lesson (CIL) design assigments: iterative and linear. These patterns align with the introduction's conceptualization of human-AI interaction as a form of metacognitive co-regulation rather than unidirectional assistance (Awi et al., 2025; Mulholland et al., 2025). The iterative pattern, characterized by cyclical questioning,

reflection, and strategy refinement, reflects the scaffolded engagement emphasized in constructivist and multimodal mathematical learning theories (Fitria et al., 2025; Wachira & Mburu, 2019). Subjects 1 and 3 exhibited this pattern. Subject 3 exhibited productive iterative engagement, using the chatbot to validate reasoning and sustain structured reflection—consistent with theories that conceptualize metacognition as involving planning, monitoring, and evaluation processes (Lucangeli et al., 2019; Taranikanti & Davidson, 2023). In contrast, Subject 1 experienced a metacognitive failure termed "metacognitive mirage," a form of over-analysis that accords with concerns raised in the introduction about the potential for AI to distort reasoning when regulation is weak (Watts, 2025).

The linear pattern exhibited by Subjects 2 and 4 corresponds ro the widespread tendency among learners to rely on AI as a generator of answers (Kim et al., 2024; Wu, 2024). Subject 2's uncritical acceptance of AI outputs exemplifies metacognitive blindness and reflects the risks associated with cognitive offloading without sufficient regulation, echoing the concept of "cognitive debt" described in the introduction (Tankelevitch et al., 2024). However, Subject 4 has demonstrated that linear interaction is not inherently detrimental. When learners deliberately use a chatbot for validation rather than substitution, linear engagement can foster reflective confidence. This finding reinforces the introduction's argument that cognitive offloading is productive only when it is paired with evaluative regulation (Florean et al., 2025). The two interaction patterns are further depicted in the scenario in Figure 9.

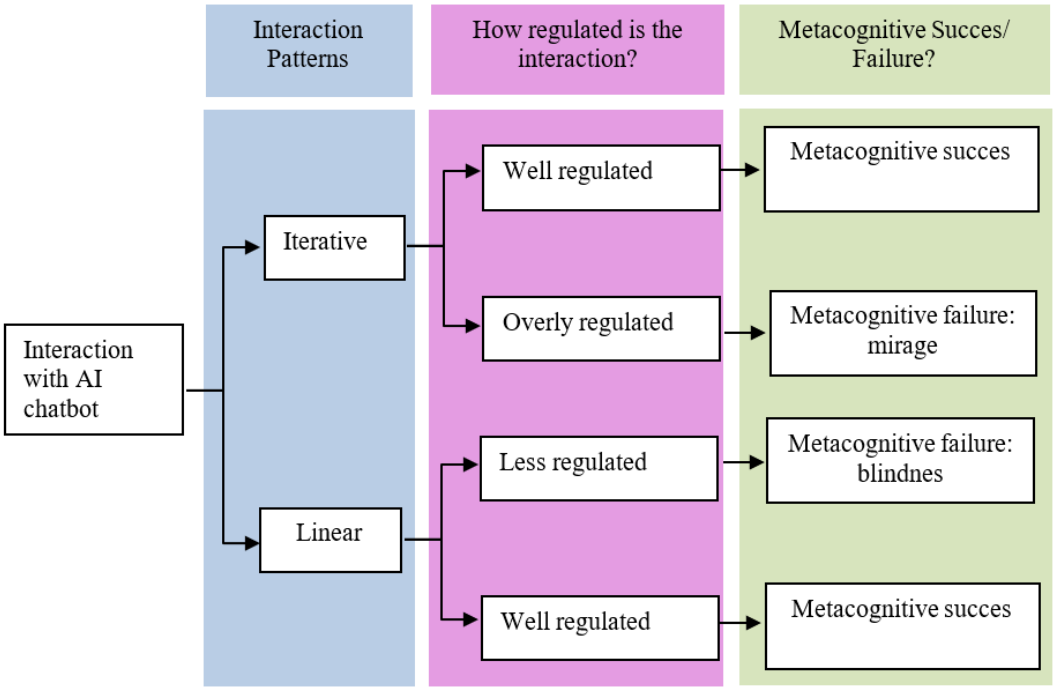


Figure 9. Metacognitive scenario in interaction with AI chatbots

Figure 9 illustrates the central mechanism of AI-learner interaction identified in the present study, showing how interaction patterns with the chatbot—iterative and linear—lead to different metacognitive outcomes depending on the degree of regulation enacted by the learner. As visualized in Figure 9, iterative interaction may evolve into a well-regulated

process that fosters metacognitive success or into an overly regulated process that culminates in metacognitive mirage, a form of over-analysis in which learners doubt correct reasoning—an outcome anticipated in the introduction through discussions of excessive cognitive offloading and uncertainty in AI use. In a similar vein, a linear pattern may become less regulated, leading to metacognitive blindness, where learners accept AI-generated information uncritically, aligning with the concerns in the introduction regarding passive answer-seeking behavior (Kim et al., 2024). However, when linear interaction remains well-regulated, it also yields metacognitive success, particularly when learners use the chatbot as a verification tool rather than as a substitute for reasoning. By demonstrating how iterative and linear pathways can be integrated within a unified regulatory framework, the present study lends support to the theoretical position previously introduced that AI-supported learning operates as a process of co-regulation as opposed to passive information delivery (Awi et al., 2025; Mulholland et al., 2025).

A notable development was observed as the learners transitioned from mathematical problem solving to designing culturally inclusive lessons. While the diagram captures the core cognitive mechanism, the findings have shown that this mechanism expands during lesson design into what this study identifies as meta-pedagogical cognition—an extended form of metacognitive regulation applied to instructional decision-making. Embedding mathematical assignments within culturally meaningful contexts has been demonstrated to deepen both conceptual and metacognitive engagement by establishing a connection between mathematical ideas and individuals' lived experiences, as well as social knowledge (Howlader & Sarkar, 2025; Jojo, 2024; Kusaeri et al., 2025). It was found that iterative users engaged in dialogic co-regulation with the chatbot, revising tasks to preserve mathematical coherence, cultural authenticity, and inclusivity in ways consistent with culturally responsive pedagogical principles (Nolan & Xenofontos, 2023). In contrast, linear users demonstrated a tendency to accept AI-generated instructional components with minimal contextual refinement, indicative of a more procedural engagement with ethnomathematical elements. These findings extend the regulatory mechanism illustrated in the diagram by showing that well-, over-, and less-regulated profiles manifest not only in individual reasoning but also in pedagogical judgment.

Taken as a whole, the findings reaffirm the introduction's call to reconfigure AI-supported mathematics environments, orienting them toward reflective, autonomous, and culturally grounded learning. By demonstrating how interaction patterns and regulatory profiles shape both cognitive and pedagogical outcomes, the present study provides empirical nuance to the cognitive offloading and regulation balance discussed in earlier works (Tankelevitch et al., 2024; Watts, 2025). Furthermore, it offers evidence that AI can support both cognitive and cultural dimensions of mathematics learning when grounded in meaningful contexts (León-Domínguez, 2024). The findings, when considered collectively, strengthen the argument that AI should be repositioned from a role that merely provides answers to that of a metacognitive partners. This repositioning is essential in order to cultivate deep, autonomous, and culturally responsive mathematical learning. The present study examined participants' interactions within a specific Moodle-based, culturally contextualized learning environment. It did not assess whether the identified metacognitive patterns were sustained over time or transferred into actual classroom practice. Therefore, future studies are recommended to

validate the proposed framework with larger and more diverse populations, mathematical topics, and cultural contexts.

4. CONCLUSION

The present study has demonstrated that preservice mathematics teachers' interactions with AI chatbots unfold as processes of metacognitive co-regulation shaped by their monitoring, management, and evaluation of their own reasoning within culturally contextualized tasks. The iterative-exploratory and linear-reflective patterns that emerged have revealed that learning with AI chatbots is not determined by the chatbots' accuracy but by how learners regulate their engagement. The metacognitive scenario in interaction with AI chatbots synthesizes this mechanism by illustrating how each interaction pattern can lead to either productive or problematic metacognitive outcomes depending on the degree of regulation. Well-regulated iterative or linear engagement produces metacognitive success. Over-regulated iterative engagement results in metacognitive mirage, and under-regulated linear engagement leads to metacognitive blindness. This summary aligns with the concerns raised in the introduction regarding excessive cognitive offloading and passive answer acceptance. Additionally, it shows the conditions under which AI chatbots can support rather than undermine reflective reasoning.

Culturally grounded problems have been shown to further strengthen these regulatory processes by anchoring abstract mathematical reasoning in familiar cultural referents and by enabling AI to function less as an answer generator and more as a cognitive partner that stimulates strategy refinement, monitoring, and evaluative judgment. The findings underscore the importance of designing AI systems with metacognitive scaffolding—features that prompt justification, surface inconsistencies, and encourage the reconsideration of alternative strategies. Moreover, the findings highlight the essential mediating role of teachers in ensuring that AI complements human reasoning and culturally informed understanding.

In its entirety, the present study clarifies that the metacognitive outcomes of AI-supported mathematics learning depend not only on the interaction pattern but also on the manner in which that interaction is regulated. As illustrated in Figure 9, iterative interaction can yield metacognitive success when it is well regulated, but may result in metacognitive mirage when learners become over-regulated and begin to question acceptable reasoning. In a similar vein, linear interaction may facilitate metacognitive success when used in a well-regulated manner, particularly for verification. However, it can also lead to metacognitive blindness when learners adopt a less-regulated and uncritical acceptance of AI-generated responses. The integrated mechanism proposed in Figure 9 provides a conceptual foundation for the development adaptive AI systems capable of detecting different regulatory conditions and responding to emerging metacognitive failures while supporting broader cross-cultural applications that promote reflective, autonomous, and culturally responsive mathematical thinking.

It is recommended that future studies be conducted to validate the proposed interaction-regulation framework with larger and more diverse populations, mathematical topics, and cultural contexts. Longitudinal and mixed-method research is needed to examine whether metacognitively scaffolded chatbot use leads to sustained improvements in learners'

planning, monitoring, evaluation, and independence from AI. Moreover, further studies may endeavor to develop adaptive chatbot features that are capable of detecting under-regulation, over-regulation, and emerging metacognitive failures and providing context-sensitive prompts in real time. Comparative studies involving culturally contextualized and non-contextualized tasks may clarify how cultural familiarity and interpretive ambiguity shape learners' regulation of AI-supported reasoning. Finally, the investigation of the potential transfer of these metacognitive interaction patterns from problem solving to authentic teaching practice and culturally inclusive instructional decision-making is recommended for future studies.

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Declarations

Author Contribution : A: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Visualization, Writing - original draft, and Writing - review & editing; SSF: Data curation, Formal analysis, Investigation, Methodology, Validation, and Writing - review & editing; GFK: Data curation, Investigation, Validation, Visualization, and Writing - review & editing.

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