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Integrating the Lesh translation model in e-learning to foster students' numeracy skills

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Abstract

Although the Lesh Translation Model (LTM) has proven effective in addressing students' difficulties in translating mathematical ideas across representations, its application has so far been limited to face-to-face learning and its effectiveness when integrated into structured e-learning environments to improve students' numeracy skills has not been widely studied. This study aims to examine the effectiveness of integrating LTM into an e-learning platform (*E-Numerasi*) in enhancing students' numeracy skills compared to conventional instruction. This research uses a quasi-experimental pretest-posttest control group design and involves 119 eighth-grade students from two public junior high schools in Kuningan Regency, West Java, who were divided into an experimental group (n = 61) and a control group (n = 58). T-tests confirmed the equivalence of initial numeracy skills between the two groups prior to the intervention ($p = 0.832$). After the intervention, the experiment group obtained a significantly higher posttest mean score ($M = 74.46$) than the control group ($M = 63.88$), $t(117) = 4.962$; $p < 0.001$, supported by a moderate Normalized Gain (0.39) and a very large effect size (Cohen's $d = 0.91$). These results indicate that integrating the Lesh Translation Model into e-learning is significantly more effective in improving students' numeracy skills than conventional instruction.

Keywords:
E-learning, Lesh translation model, Mixed-method, Numeracy skills, Representation translation

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1. INTRODUCTION

Numeracy is not merely basic arithmetic ability, but rather a critical life skill for processing, interpreting, and communicating numerical information to solve complex problems in various everyday situations (Asmara et al., 2026; Chan & Scalise, 2022; Dewantara et al., 2023; Fauzan et al., 2024; Nindiasari et al., 2026; Verkerk et al., 2026). As fundamentally emphasized by Steen (2001), numeracy has now transformed into "a new form

of literacy for a society inundated with data," in which the ability to read, write, and reason mathematically in various life contexts has become absolutely essential.

In the 21st century, mastery of numeracy skills serves as the foundation for academic success, smooth transition into the workforce, and adaptive problem-solving (Hidayah et al., 2025; Rahayu & Subekti, 2025; Rosita et al., 2026; Verkerk et al., 2026; Yerizon et al., 2025). However, this urgency clashes with the reality of educational achievement in Indonesia. According to the results of the 2022 Programme for International Student Assessment (PISA), the average mathematics score for Indonesian students declined from 379 (in 2018) to 366, which remains far below the OECD average of 472 (OECD, 2019, 2023; Wijaya et al., 2024). These data are even more concerning given findings that highlight that only 18% of Indonesian students are able to reach the minimum proficiency level (Level 2) and are not yet able to independently model complex mathematical situations (Annajmi et al., 2026; OECD, 2023).

The root cause of these low numeracy skills is not solely due to the abstract nature of mathematics, but rather stems from students' specific difficulties in translating between mathematical representations (Duval, 2006; Lockwood & Ellis, 2022; Scheid et al., 2019). Many students fail to bridge the gap between formal knowledge acquired in school and its application in the real world due to difficulties at the stage of exploring the source of a problem, particularly when they must convert verbal representations into graphical or symbolic ones (Annajmi et al., 2026; Hoogland et al., 2018; Kandemir & Karadeniz, 2021).

On the other hand, although digital technology is becoming increasingly common, the current use of conventional e-learning platforms often fails to integrate an adequate pedagogical framework and still poses a number of challenges, resulting in a lack of deep conceptual understanding (Hidayat & Chao, 2025; Vaillant et al., 2021; Weinhandl et al., 2026). The use of technology without a well-defined framework often fails to facilitate students' cognitive flexibility in translating contextual narratives into mathematical models (Abraham & Prediger, 2025; Hidayat & Chao, 2025; Weingarden et al., 2026).

To address this cognitive deadlock, the Lesh Translation Model (LTM) offers a highly relevant and precise pedagogical framework. This model asserts that a comprehensive understanding of mathematical concepts requires students to be able to operate interactively across five modes of representation: contextual, manipulative, pictorial, verbal, and symbolic (Lesh et al., 1987; Mainali, 2021; Stohlmann et al., 2020). According to this framework, understanding is not formed passively, but rather through an active translation process as students reinterpret one form of representation into another (Cramer et al., 2009; Duval, 2006; Lesh & Doerr, 2003). LTM is a highly relevant solution for numeracy skills because it directly targets students' representational fluency, bridging the gap from text to mathematical models, and then back to real-world contexts.

The power of the representation-translation approach in fostering deep conceptual understanding has been demonstrated by various previous studies in face-to-face classrooms. Research by Bossé et al. (2014) shows that translation, as an active cognitive process, has been shown to help students understand broader mathematical ideas.

These findings are supported by recent studies that emphasize students' flexibility in transforming, shifting, and converting various forms of representation as the most pivotal element in building robust and deep mathematical concept understanding (Abraham &

Prediger, 2025; Jäder & Johansson, 2025; Weingarden et al., 2026). Furthermore, the process of translation connecting these representations enables students to activate their sensorimotor and cognitive resources, making abstract mathematical ideas more tangible, logical, and easier to understand (Kandemir & Karadeniz, 2021; Khatin-Zadeh et al., 2022; Santosa & Filiz, 2025).

On the other hand, other studies confirm that the flexible use of diverse representations is essential for constructing mathematical understanding and activating higher-order metacognitive skills in problem-solving activities (Abdullah et al., 2014; Jäder & Johansson, 2025). Nevertheless, a review of the literature across different fields found that contemporary mathematics e-learning designs, although they have utilized digital tools to enhance mathematical literacy, generally have not explicitly incorporated cross-modal translation activities as a central component (Abraham & Prediger, 2025; Dyrvold & Bergvall, 2024; Hidayat & Chao, 2025).

Based on this literature review, a distinct pattern emerges between two separate research streams: the pedagogical potential of the representation translation model, which has been strongly validated in face-to-face classrooms, and the development of e-learning technology that is not yet fully grounded in cognitive representation theory. Therefore, a research gap in the current literature is clearly evident. However, to date, there have been no studies examining the integration of Lesh's Translation Model as the primary pedagogical foundation in the design of mathematics e-learning to improve the numeracy skills of junior high school students.

Based on the gap described above, the research question posed in this study is explicitly: "How does the integration of Lesh's Translation Model into an e-learning environment affect the improvement of junior high school students' numeracy skills?" Based on this question, this study aims to empirically test and validate the effectiveness of structurally designed mathematics e-learning grounded in the Lesh Translation Model in improving students' numeracy skills.

2. METHOD

2.1. Research Design

This study employs a mixed-methods approach with an exploratory sequential design (Creswell, 2014), as shown in Figure 1. In this design, qualitative data were collected and analyzed first to identify students' specific difficulties; these findings then directly served as the basis for designing an e-learning intervention based on the Lesh Translation Model (LTM) in the subsequent quantitative phase. This sequence was chosen because the pedagogical design and development of numeracy instruments need to be based on empirically identified field needs, rather than mere a priori assumptions.

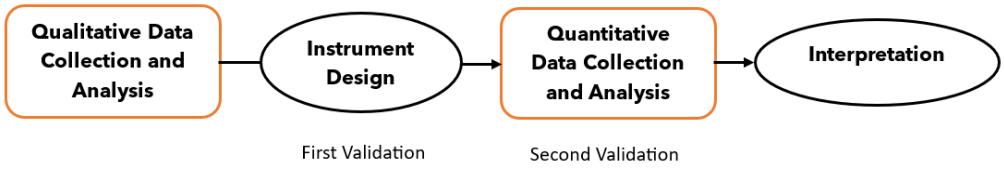


Figure 1. Exploratory sequential design model

Specifically, the quantitative phase adopted a quasi-experimental pretest-posttest design with a non-equivalent control group to compare the effectiveness of the intervention against a control group learning using conventional methods. This is shown in Table 1.

Table 1. Research design

Sampel Groups	Pre-test	Treatment	Post-test
Experiment	Q_1	X_1	Q_3
Control	Q_2	X_2	Q_4

Table 1 shows the research design (Cohen et al., 2002), where Q_1 and Q_2 represent the pre-tests for the experiment and control groups, respectively. Q_3 and Q_4 represent the post-tests for the experiment and control groups, respectively. Furthermore, in the treatment column, X_1 represents the treatment for the experimental group, in which the participants were taught using e-learning based LTM, and X_2 represents the treatment for the control group, in which the participants were taught using a conventional learning model.

2.2. Participants

A total of 119 eighth-grade students from two public junior high schools in Kuningan Regency, West Java (Indonesia), participated in this study. Both schools hold an "A" accreditation and have comparable demographic profiles and socioeconomic backgrounds. Participants were selected using purposive sampling (Creswell, 2014), from four classes recruited in their entirety, which were then designated as the experimental and control groups. These students had all completed the same seventh-grade mathematics curriculum and had never previously received instruction based on the Lesh Translation Model. The selection of schools was based on the equivalence of their characteristics, so that any differences in learning outcomes that emerge later can be more reliably attributed to the treatment administered, rather than to differences in school characteristics. The results of the pre-test analysis also confirmed that the initial numeracy abilities of both groups were at a statistically equivalent level, thus fulfilling the assumption of baseline equivalence in the quasi-experimental design used (Cohen et al., 2002).

2.3. Instrumentations and data analysis technique

Given that this study employs a mixed-methods approach, data collection was conducted using two types of instruments:

a. Qualitative Instruments: During the exploratory phase, data were collected using observation sheets and semi-structured interview guidelines. These instruments were designed to identify the sources of students' cognitive barriers in translating problem narratives into mathematical models and to explore their perceptions of the developed e-learning interface.
b. Quantitative Instrument: Numeracy proficiency is measured using a test instrument consisting of three open-ended (essay) questions focused on the topics of Numbers and Algebra. The test instrument was developed based on numeracy test items created by the Ministry of Education and Culture. These questions were designed in alignment with the PISA 2022 framework for mathematical process competencies, namely formulating,

using, and interpreting situations mathematically (OECD, 2023). The maximum test score is 100 points. Content validity was established through evaluation by two panels of mathematics education experts to ensure the relevance of the questions and their alignment with the LTM construct. This instrument was then pilot-tested on a group of students outside the main sample (n = 30), yielding a Cronbach's α coefficient of 0.78. This value exceeds the 0.70 threshold, indicating a level of internal consistency (reliability) that is highly adequate for educational research (Creswell, 2014).

In line with the mixed-methods design, data analysis was conducted in two stages. Qualitative data from interviews and observations were analyzed using thematic analysis, which involves the stages of data reduction, data presentation, and drawing conclusions (Miles et al., 2014), to identify the sources of students' translation difficulties. Meanwhile, quantitative data from pre-test and post-test scores were analyzed using statistical software (SPSS). Descriptive statistics were used to present mean values and standard deviations. Improvements in students' numeracy scores were evaluated through N-gain analysis. To determine the effectiveness and the significance of the differences between the class using LTM-based e-learning (experiment) and the conventional class (control), the researcher used the Independent Samples t-test or the Mann-Whitney U test, preceded by prerequisite tests for normality and homogeneity of variances.

2.4. Research Procedures

This study was conducted in three main, continuous phases, in line with an exploratory sequential design. In the first phasequalitative exploration through fieldwork structured classroom observations were conducted over several sessions at the schools serving as research sites. These observations focused on documenting the types of representations used by teachers and students, the frequency and nature of representational transitions, and empirical evidence of students' cognitive barriers in interpreting representations. Next, semi-structured in-depth interviews were conducted with ten mathematics teachers from the participating schools, including the Chair of the Mathematics Subject Teachers' Association, to explore teachers' perspectives on the root causes of students' representational difficulties and unmet pedagogical needs. The collected qualitative data were then analyzed using the interactive model proposed by Miles et al. (2014), which includes the stages of data reduction, data presentation, and drawing conclusions. The credibility and validity of the data were ensured through source triangulation, specifically by cross-checking field notes, interview transcripts, and curriculum documents.

Building on the findings of the first phase, the second phase of this study focused on platform development and instrument design. The *E-Numerasi* e-learning platform was developed as an extension of a previous mobile learning platform (Zaenal et al., 2024), which was redesigned to align with the cognitive characteristics of junior high school students and to integrate the Lesh Translation Model (LTM) modules in a structured manner. This platform operationalizes the five LTM modes of representation as sequential learning pathways: contextual, pictorial/visual, symbolic, manipulative/concrete, and verbal/written. At each stage of this translation process, the platform provides scaffolding features in the form of step-

by-step guidance and immediate formative assessment to ensure that students' representational flexibility is well-established before they move on to the next stage (Setiyani et al., 2025).

The third phase involves quantitative implementation through a quasi-experimental study, conducted after the instruments and platform were validated. At this stage, the experimental class conducts mathematics learning using the LTM-based *E-Numerasi* platform, while the control class is taught using a conventional approach by the teacher. Both groups are given a pre-test before the intervention begins and a post-test after the entire series of materials is completed, to quantitatively measure the level of progress in students' numeracy skills. For further details, see Figure 2.

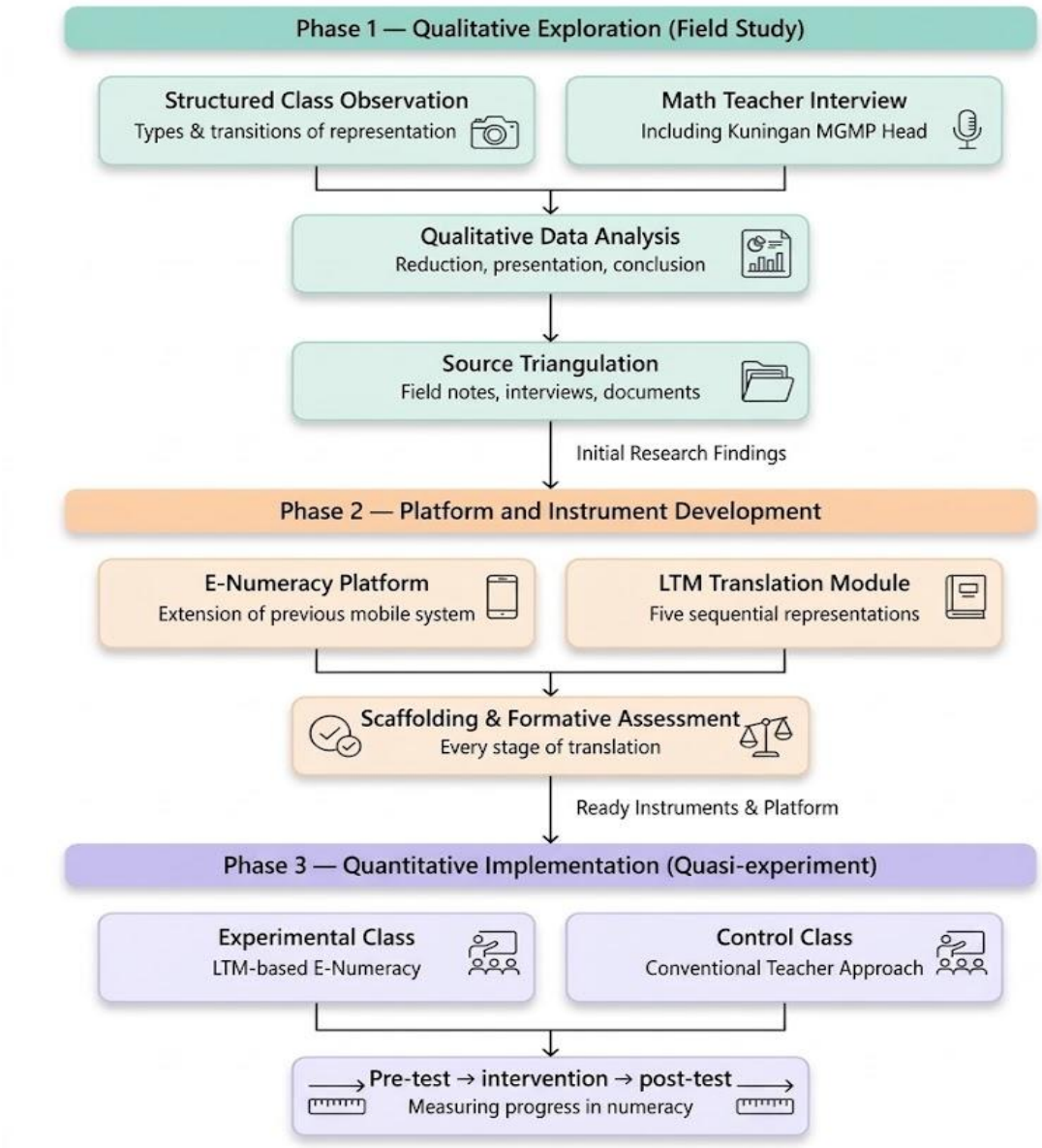


Figure 2. Research procedures

2.5. Hypotheses

Based on the research question and theoretical review outlined above, the statistical hypotheses tested in the quantitative phase of this study are:

H_0 : There is no significant difference in the improvement of numeracy skills between students using e-learning (*E-Numerasi*) based on the Lesh Translation Model and students using conventional learning methods.

H_1 : There is a significant difference in the improvement in numeracy skills between students using e-learning (*E-Numerasi*) based on the Lesh Translation Model and students using conventional learning methods, with the improvement in the experimental group being significantly higher.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1. Phase 1: Qualitative Findings

The qualitative exploration phase revealed significant learning challenges in the field related to numeracy skills. Classroom observations indicated that current mathematics teaching practices are still limited in their use of diverse representations. Students are rarely explicitly trained to translate information from one form of representation to another. As a result, students tend to be passive and do not spontaneously use visual aids, such as diagrams or sketches, to analyze and support problem solving.

Furthermore, in-depth interviews with teachers and students confirmed that around 80% of students experience major difficulties at the stage of understanding and interpreting word problems (verbal representation). When classified according to the stages of representation translation, these cognitive blockages are specifically centered on the phase of unpacking the source. At this stage, students struggle to extract essential information from narrative texts and convert it into mathematical models, even though basic arithmetic concepts have already been taught. These difficulties contribute to a rigid mathematical mindset, where students tend to view mathematics as difficult and are easily discouraged.

In response to these field conditions, teachers expressed an urgent need for an innovative, interactive, and structured e-learning approach. These qualitative findings strongly confirm the urgency of developing an integrative e-learning platform based on the Lesh Translation Model as a potential solution.

3.1.2. Phase 2 : Platform and Instrument Development

Based on the findings from Phase 1, the e-learning platform (*E-Numerasi*) was specifically developed by integrating the Lesh Translation Model (LTM) framework. It is important to note that the basic architecture of this application is based on the Numeracy E-Learning platform, which had already been developed and validated for feasibility in a previous study (Zaenal et al., 2024). Therefore, development in this phase focused exclusively on pedagogical modifications. The conceptual framework illustrated in Figure 3 was developed to guide these modifications, in which the five LTM modes of representation

(verbal, visual, symbolic, concrete, and contextual) are integrated into the platform's digital scaffolding structure to enable the systematic implementation of translation activities.

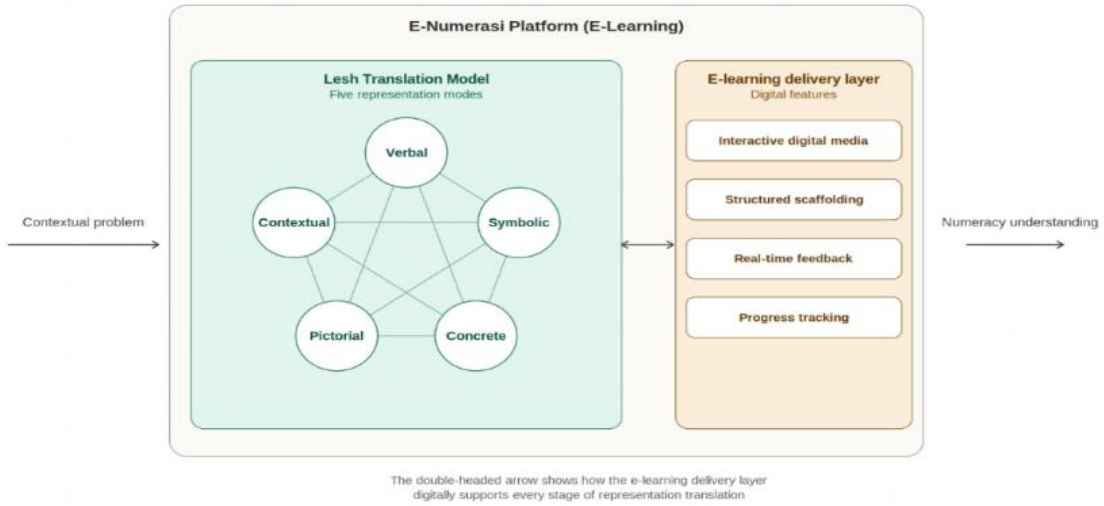


Figure 3. Integrative concept of e-learning and Lesh translation (*E-Numerasi*)

The E-Numeracy Platform (accessible at https://e-numerasi.homeofmath.id) was developed to implement this framework. As shown in Figure 4, the platform provides a structured learning sequence that guides students in translating their understanding across various modes of representation, supported by real-time formative feedback features. Concurrent with the platform's development, the research test instrument was also validated and pilot-tested. The results of the pilot test of the numeracy skills instrument yielded a Cronbach's Alpha coefficient of 0.78, confirming that the instrument has an adequate level of internal consistency and is ready for use in subsequent measurement phases.

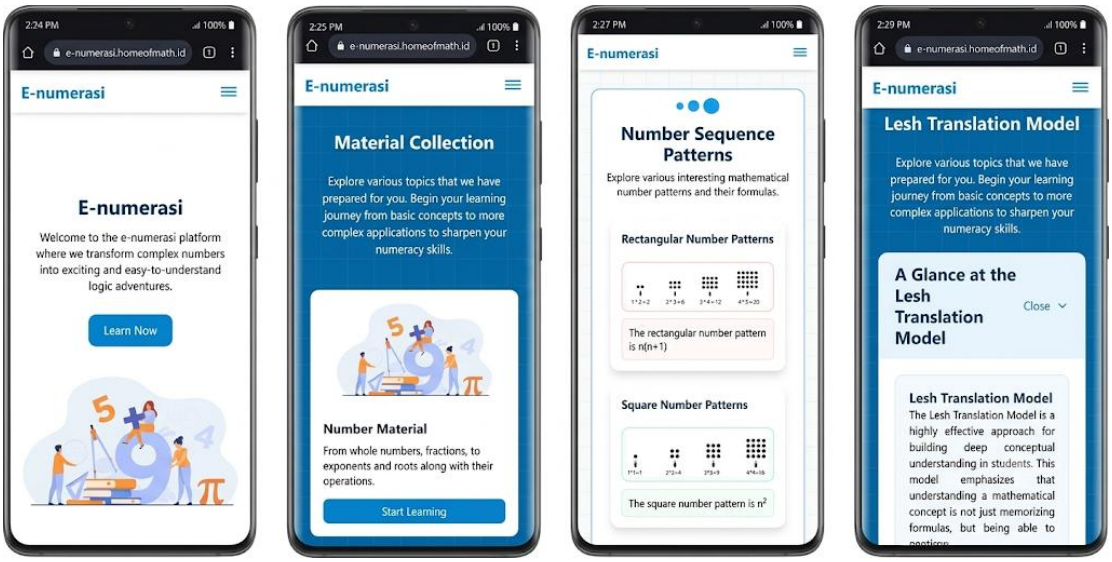


Figure 4. *E-Numerasi* platform display

3.1.3. Pre-Test Analysis: Initial Group Equivalence

Before the intervention was administered, it was necessary to ensure that the experimental and control groups had equivalent initial numeracy abilities. Without this equivalence, any difference found after the treatment could not be attributed with confidence to the intervention itself. For this reason, the pretest data were analyzed in four steps: a descriptive analysis of the pretest scores (see Table 2), a normality test (see Table 3), a test of homogeneity of variances (see Table 4), and an Independent Samples t-Test to compare the mean pretest scores of the two groups (see Table 5). The results of each step are presented in the tables below, followed by a brief explanation.

Table 2. Descriptive statistical analysis for pre-test data

Sample Class	N	Mean	Std. Deviation	Std. Error Mean
Experiment	61	58.43	11.802	1.511
Control	58	57.98	10.867	1.427

As shown in Table 2, the experimental group (n = 61) achieved a mean pretest score of 58.43 (SD = 11.802; SE = 1.511), while the control group (n = 58) achieved a mean pretest score of 57.98 (SD = 10.867; SE = 1.427). The difference in means between the two groups was only 0.45 points, which is very small from a descriptive standpoint. In addition, the standard deviations of both groups were relatively similar, indicating that the spread of scores around the mean was also comparable. These descriptive results indicate that the students' initial numeracy skills in both groups were at a relatively equivalent level before the intervention was administered. However, since similarity in means alone is not sufficient to draw a scientific conclusion, this equivalence was then tested statistically through a series of prerequisite tests, beginning with the normality test.

Table 3. Result of Kolmogorov-Smirnov normality test for pre-test data

Sample Class	Statistic	df	Sig.	Criteria
Experiment	0.074	58	0.200*	Normal
Control	0.087	58	0.200*	Normal

Table 3 presents the results of the Kolmogorov-Smirnov normality test for the pretest data. The significance value was 0.200 for both the experimental group (statistic = 0.074) and the control group (statistic = 0.087). Because both significance values are greater than 0.05, the null hypothesis that the data are normally distributed is not rejected, so the pretest scores of both groups can be considered normally distributed. This result satisfies the first assumption required for parametric analysis and allows the analysis to proceed to the test of homogeneity of variances.

Table 4. Result test of homogeneity of variances for pre-test data

Levene Statistic	df1	df2	Sig.	Criteria
1.280	1	117	0.260	Homogenous

The results of Levene's test for homogeneity of variances are presented in Table 4. The Levene statistic was 1.280 with a significance value of 0.260. Since this significance value is

above 0.05, the variances of the pretest scores of the experimental and control groups are deemed homogeneous. Together with the normality result in Table 3, this means that both assumptions were met, so the comparison of the two groups' means could be carried out using the Independent Samples T-Test under the assumption of equal variances.

Table 5. Result of independent samples test for pre-test data

	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	1.280	0.260	0.213	117	0.832	0.443	2.083
Equal variances not assumed			0.213	116.883	0.831	0.443	2.078

The results of the t-test in Table 5 show a t-value of 0.213 with 117 degrees of freedom (df) and a significance level (two-tailed) of 0.832 under the equal variances assumed condition. The mean difference between the two groups was only 0.443 with a standard error of 2.083. Since the significance value is far above 0.05, there is no statistically significant difference in mean pretest scores between the two groups. This finding reinforces the previous descriptive results: both groups started from an equivalent baseline level of numeracy ability before receiving the intervention. Consequently, if a significant difference is found in the posttest phase, that difference can be more confidently attributed to the effect of the intervention, namely the integration of the Lesh Translation Model into the *E-Numerasi* e-learning platform, rather than to differences in baseline ability between the groups.

3.1.4. Post-Test Analysis

After the treatment was administered, the posttest data were analyzed to determine whether the integration of the Lesh Translation Model into the *E-Numerasi* e-learning platform produced different numeracy outcomes compared to conventional instruction. The analysis followed the same sequence as the pretest: a descriptive analysis of the posttest scores (see Table 6), a normality test (see Table 7), a test of homogeneity of variances (see Table 8), and an Independent Samples t-Test to examine the difference in mean posttest scores between the experimental and control groups (see Table 9). Each table is followed by an explanation of its results.

Table 6. Descriptive statistical analysis for post-test data

Sample Class	N	Mean	Std. Deviation	Std. Error Mean
Experiment	61	74.46	12.286	1.573
Control	58	63.88	10.885	1.429

After the treatment was administered to each group, the posttest data were analyzed to assess the effectiveness of the intervention. As shown in Table 6, the experimental group (n = 61) achieved an average posttest score of 74.46 (SD = 12.286; SE = 1.573), while the control group (n = 58) achieved an average of 63.88 (SD = 10.885; SE = 1.429). In contrast to the nearly equivalent pretest scores, the posttest revealed a difference in mean scores of 10.58 points between the two groups, with the experimental group outperforming the control group.

When compared to each group's initial performance at the pretest stage, the experimental group showed an average increase of 16.03 points (from 58.43 to 74.46), which was far greater than the increase in the control group, which was only 5.90 points (from 57.98 to 63.88). This pattern descriptively indicates that the intervention administered to the experimental group had a greater impact on improving students' numeracy skills than the conventional instruction received by the control group.

Table 7. Result of Kolmogorov-Smirnov normality test for post-test data

Sample Class	Statistic	df	Sig.	Criteria
Experiment	0.099	58	0.200*	Normal
Control	0.070	58	0.200*	Normal

As in the pretest phase, the difference in mean values alone is not sufficient to draw scientific conclusions; therefore, the posttest data were again tested through a series of prerequisite tests before the mean difference test was conducted. The results of the Kolmogorov-Smirnov normality test in Table 7 show a significance value of 0.200 for the experimental group (statistic = 0.099) and 0.200 for the control group (statistic = 0.070), both of which are above 0.05. Thus, the posttest data for both groups can be considered normally distributed.

Table 8. Result test of homogeneity of variances for post-test data

Levene Statistic	df1	df2	Sig.	Criteria
2.213	1	117	0.140	Homogenous

The test for homogeneity of variances using Levene's test, presented in Table 8, shows a statistic value of 2.213 with a significance level of 0.140. Since this significance level is greater than 0.05, the variances of the posttest data between the two groups are deemed homogeneous. With both the normality and homogeneity assumptions met, the analysis of the difference in means could proceed using the Independent Samples T-Test under the "equal variances assumed" condition.

Table 9. Result of independent samples test for post-test data

	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	2.213	0.140	4.962	117	0.000	10.580	2.132
Equal variances not assumed			4.978	116.431	0.000	10.580	2.125

The results of the t-test in Table 9 show findings that differ markedly from the pretest results. The t-value obtained was 4.962 with 117 degrees of freedom (df) and a significance level (two-tailed) of 0.000. Since this significance level is far smaller than 0.05, it can be concluded that there is a statistically significant difference in means between the posttest scores of the experimental group and the control group. The mean difference between the two groups was 10.580, with a standard error of 2.132. The magnitude of the t-value, coupled with

a significance level of $p < 0.001$, indicates that this difference is not merely a coincidence or sampling variation but reflects a real effect of the treatment administered.

This finding becomes particularly meaningful when considered in light of the pretest results. At the initial stage, the two groups were found to be statistically equivalent ($t(117) = 0.213$; $p = 0.832$), indicating that neither group had an initial advantage. However, after receiving the intervention, the experimental group demonstrated a much greater and statistically significant improvement in numeracy skills compared to the control group. This shift from "not significantly different" on the pretest to "significantly different" on the posttest provides strong evidence that the difference in numeracy achievement on the posttest can be validly attributed to the intervention, namely the integration of the Lesh Translation Model into the *E-Numerasi* e-learning platform, and is not due to differences in initial ability between the groups or other chance factors. Thus, these findings support the research hypothesis that integrating the Lesh Translation Model into e-learning is effective in improving students' numeracy skills compared to conventional learning.

3.1.5. Normalized Gain Analysis

To measure the effectiveness of the intervention more specifically beyond mere statistical significance, a Normalized Gain (N-Gain) score analysis was also conducted to assess the extent of improvement in students' numeracy skills in each group following the intervention (Hake, 1998). The N-Gain calculation takes into account the difference between pretest and posttest scores relative to the maximum score a student could achieve, thereby providing a more precise picture of the extent to which the available room for improvement was actually utilized.

As presented in Table 10, the analysis results show that the experimental group achieved an N-Gain score of 0.39, which, according to Hake (1998) categorization, falls into the moderate category. Meanwhile, the control group achieved an N-Gain score of only 0.14, which falls into the low category. This difference in categories reinforces previous findings that the implementation of the e-learning platform based on the Lesh Translation Model (*E-Numerasi*) in the experimental group not only resulted in higher final achievement but also improved numeracy skills in a qualitatively more meaningful way compared to the conventional learning received by the control group.

The findings in Table 10 are consistent with and mutually reinforce the results of the Independent Samples t-Test on the posttest data (see Table 9), which showed a significant difference in means between the two groups ($t(117) = 4.962$; $p < 0.001$). While the t-test indicates that this difference is statistically significant, the N-Gain analysis complements these findings by demonstrating that the improvement in the experimental group is also practically meaningful, not merely statistically significant, but truly reflecting a more optimal realization of students' potential for numeracy skill improvement. Thus, the integration of the Lesh Translation Model into the *E-Numerasi* e-learning platform has proven effective in improving students' numeracy skills, both in terms of statistical significance (see Table 9) and the magnitude of the improvement achieved (see Table 10).

Table 10. Result normalized gain analysis

Sampel Group	Pre-Test Mean	Post-Test Mean	N-Gain	Category
Experiment	58.43	74.46	0.39	Moderate
Control	57.98	63.88	0.14	Low

As an additional layer of evidence, a Cohen's d effect size analysis was also conducted to measure the magnitude of the effect of the treatment administered, beyond mere statistical significance (Cohen et al., 2002). The results showed a Cohen's d value of 0.91. This finding complements the body of evidence that has been presented incrementally previously. The Independent Samples t-Test (see Table 9) shows that the difference in posttest scores between the two groups is statistically significant; the N-Gain analysis (see Table 10) indicates that the improvement in the experimental group is practically meaningful compared to the control group; and the Cohen's d analysis confirms that the effect size of the treatment is very large. These three lines of evidence consistently point to the same conclusion: the integration of the Lesh Translation Model into the *E-Numerasi* e-learning platform not only succeeded in producing statistically significant differences but also had a substantial and meaningful impact on improving students' numeracy skills skills compared to conventional learning.

3.2. Discussion

The results of this study show that students who learn using an e-learning platform based on the Lesh Translation Model (*E-Numerasi*) experience a much greater and statistically significant improvement in numeracy skills compared to students who learn using conventional methods. To understand why this happens, it is important to return to the core theory underlying this model. The Lesh Translation Model states that deep mathematical understanding is formed not merely because students are familiar with a single form of representation, but because students can shift between and translate the same mathematical ideas into various forms of representation: concrete objects, pictures, real-life situations, words, and written symbols (Kandemir & Karadeniz, 2021; Lesh et al., 1987; Stohlmann et al., 2020).

When students are trained using only a single form of representation, for example, by solving problems in the form of mathematical symbols without ever connecting them to real-life situations, the understanding they develop tends to be superficial and easily forgotten (Hidayat & Chao, 2025). Conversely, it is this ability to translate between representations that is a key factor in students' success in learning mathematics and solving problems. The E-Numeracy platform used in this study was specifically designed to train this translation ability in a structured manner, so that students are not merely given problems. Still, they are also guided through a series of interconnected representations before arriving at the symbolic form (Weinhandl et al., 2026). This approach theoretically explains why the experimental group demonstrated a significantly greater improvement in numeracy skills: students did not merely memorize procedures. Still, they truly built understanding by translating meaning from one form of representation to another.

These findings also help explain why the control group, although it still showed a slight improvement (low-category N-Gain), lagged far behind the experimental group. Conventional

instruction in the control group tended to present material in a one-way manner. It focused on only one form of representation, typically symbolic representation, without giving students explicit opportunities to practice translating between representations. As a result, students in this group likely only memorized problem-solving procedures without truly understanding the meaning behind them, leading to superficial and limited improvements (Hidayat et al., 2018; Jonsson et al., 2016; Kusaeri et al., 2022; Lithner, 2017; Mac an Bhaird et al., 2017). This pattern is consistent with the view that a single representation, no matter how often repeated, is insufficient for building deep mathematical understanding, and that translation between representations is the mechanism that truly distinguishes meaningful learning from mere rote memorization (Duval, 2006; Lesh et al., 1987).

In addition to the representational aspect, the experimental group's superior performance can also be explained by the role of formative feedback inherent in the *E-Numerasi* platform. Feedback provided immediately after students complete each translation step allows them to identify their errors and correct them on the spot, rather than waiting for an end-of-lesson evaluation as in conventional learning. This aligns with an effective feedback model that emphasizes answering three fundamental questions for students: where should I be heading, how far have I progressed so far, and what steps should I take next A meta-analysis on the effectiveness of scaffolding in online learning also found that mathematics actually benefits more from scaffolding than most other subject areas, with an effect size exceeding the moderate threshold. These findings reinforce the argument that the scaffolding features and formative feedback integrated into the representation translation stage are not merely technical add-ons, but components that substantially contribute to the effectiveness of the *E-Numerasi* platform in improving students' numeracy skills (Abraham & Prediger, 2025; Weinhandl et al., 2026).

The effect sizes obtained in this study also need to be interpreted within the context of broader findings from similar research. The Cohen's d value of 0.91 (Very Large category) found in this study exceeds the average effect size from a large meta-analysis of computer-assisted instruction studies in mathematics education, which reported an average effect size of $d = 0.52$. Thus, the superiority of the *E-Numerasi* platform does not appear to stem solely from the factor of "learning using a computer or mobile device," but rather from its specific pedagogical design namely, the sequential and structured arrangement of the five LTM representation modes, which provides added value beyond the general effects of digitizing learning (Dyrvold & Bergvall, 2024). This comparison is important because it underscores that simply moving mathematics instruction to digital media does not automatically result in significant improvements (Cirneanu & Moldoveanu, 2024; Drijvers & Sinclair, 2024; Engelbrecht & Borba, 2024); rather, it is how representations and feedback are organized within that medium determines the magnitude of the learning impact (Vaillant et al., 2021).

These findings hold particular relevance when considered in light of the long-standing issue of numeracy skills in Indonesia, as reflected in the low numeracy achievement of Indonesian students in international assessments. The root of the problem, as outlined in the introduction namely, students' difficulty in translating real-world problems into mathematical forms and vice versa does not appear to be an issue that can be resolved simply by increasing the number of practice problems or by introducing learning technology in general. An

approach is needed that specifically trains students' ability to translate between these representations, and this study demonstrates that the Lesh Translation Model, when implemented in a structured manner on an e-learning platform, can serve as one solution to this problem.

Theoretically, this finding makes an important contribution because most previous research on the Lesh Translation Model has been conducted in the context of face-to-face classroom learning. In contrast, its application in a structured e-learning environment has rarely been empirically examined. This study demonstrates that the five LTM modes of representation can not only be implemented digitally but also benefit from the interactive features of e-learning platforms, such as immediate feedback and students' ability to repeat translation steps at their own pace. These findings expand the scope of LTM application previously discussed primarily in the context of conventional learning while addressing the research gap identified in the introduction. For math teachers, these findings indicate that improving students' numeracy skills requires more than simply moving content to a digital platform; what matters most is how the sequence and structure of representations are organized within the platform, as well as how feedback is provided throughout the learning process. Teachers who wish to design or select digital numeracy learning media should ensure that the media facilitates students in explicitly practicing the translation of one form of representation into another, accompanied by prompt and targeted feedback, rather than merely presenting one-way practice problems.

Nevertheless, several limitations of this study should be noted. This study involved only eighth-grade students at two schools with relatively similar demographic characteristics; therefore, generalizing the findings to other educational levels or different socioeconomic contexts should be done with caution. Furthermore, the limited duration of the intervention in this study did not allow for the measurement of the long-term impact of the *E-Numerasi* platform on the retention of students' numeracy skills. Future research should consider a longitudinal design and involve a more diverse population to test the extent to which these findings can be generalized more broadly.

4. CONCLUSION

This study aims to examine the effectiveness of integrating the Lesh Translation Model into an e-learning platform (*E-Numerasi*) in enhancing students' numeracy skills compared to conventional learning methods. The analysis of the pretest results confirmed that the experimental and control groups had equivalent initial numeracy abilities before the intervention, ensuring that any subsequent differences could be validly attributed to the treatment. After the intervention, t-test results on the posttest data showed that the experimental group experienced a statistically significant improvement in numeracy skills compared to the control group. This finding is further supported by the N-Gain value of the experimental group, which was in the moderate category and far exceeded the control group, which was only in the low category, as well as by the effect size (Cohen's d), which was considered very large. Thus, these three analyses consistently confirm that the integration of the Lesh Translation Model into the e-learning platform is proven effective in improving

students' numeracy skills, as evidenced by statistical significance, the magnitude of improvement, and the effect size achieved.

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Declarations

Author Contribution	: RMZ: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Visualization, Writing - original draft, and Writing - review & editing; NAM: Data curation, Formal analysis, Investigation, and Writing - review & editing; LDH: Data curation, Formal analysis, Investigation, and Writing - review & editing.
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