

Exploring the nature of mathematical connections in pre-university students' triangle similarity tasks

Elizabeth Santos-Casildo¹, Javier García-García^{1*}, Aloisius Loka Son²,
Gerardo Salgado-Beltrán¹

¹Facultad de Matemáticas, Universidad Autónoma de Guerrero, Guerrero, Mexico

²Department of Mathematics Education, Universitas Timor, East Nusa Tenggara, Indonesia

*Correspondence: jagarcia@uagro.mx

Received: Mar 17, 2026 | Revised: Jun 15, 2026 | Accepted: Jun 25, 2026 | Published Online: Jul 28, 2026

Abstract

Triangle similarity is a mathematical concept with substantial conceptual richness; however, its teaching typically emphasizes procedural approaches over conceptual understanding, leading to a disconnect between concepts and procedures. In this context, the present study aimed to identify the mathematical connections established by a group of Mexican pre-university students while solving tasks related to triangle similarity. A conceptual framework based on the notion of mathematical connections and their typologies was adopted. The study followed a qualitative methodology with a descriptive scope, using a case study approach. Data were collected through task-based interviews. Three tasks were designed and completed by eight students, and the resulting data were analyzed using thematic analysis. The findings revealed six types of mathematical connections: procedural, feature-based, different representations, meaning, implication, and interconceptual; the part-whole connection was not observed. This absence suggests that students tend to focus on isolated properties and procedures rather than recognizing triangle similarity as part of broader mathematical structures. The results suggest the need to design instructional tasks that foster both conceptual understanding and procedural accuracy.

Keywords:

Mathematical connections, Pre-university level, Qualitative research, Triangle similarity

How to Cite:

Santos-Casildo, E., García-García, J., Son, A. L., & Salgado-Beltrán, G. (2026). Exploring the nature of mathematical connections in pre-university students' triangle similarity tasks. *Infinity Journal*, 15(3), 773-786. <https://doi.org/10.22460/infinity.v15i3.p773-786>

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1. INTRODUCTION

Geometry, as a central branch of mathematics, provides a rich context for the development of students' mathematical reasoning, as it requires the integration of both geometric and algebraic knowledge (Agustini et al., 2017; Bila et al., 2024). Within geometry, triangle similarity is a fundamental topic due to its strong links to proportional reasoning and to basic theorems such as Thales' theorem (Climent et al., 2021). According to DeJarnette et

al. (2014), the study of triangle similarity requires the integration of geometric and proportional reasoning. That is, it involves establishing proportional relationships among the corresponding sides of two triangles based on the equality of their corresponding angles (Ubah & Bansilal, 2019).

Although students are expected to understand these ideas, the literature reports that they face difficulties in understanding the characteristics and properties of similar triangles; for example, when applying similarity theorems to pairs of triangles (Haj-Yahya, 2022) and when relating this concept to others, such as proportionality and triangle congruence (Dündar & Güdüz, 2017). These difficulties hinder their ability to move across different representations, for instance, from the geometric register to the algebraic register (Ubah, 2022). This is partly because instruction and learning related to this concept tend to emphasize procedural aspects over conceptual understanding. Godino et al. (2018) note that students apply the rule of three in proportional situations but do not justify why they use it. In this regard, Wijaya et al. (2021) argue that learning this concept entails developing not only visualization skills, but also the ability to establish mathematical connections and to construct theorem proofs.

Making mathematical connections requires students to interrelate mathematical concepts—an essential aspect for meaningful understanding in mathematics (Purnomo et al., 2024; Rahmi et al., 2020)—which in turn fosters deeper understanding (García-García, 2024; Siregar & Siagian, 2019). Mathematical connections have been studied from various perspectives. In the domain of geometry, research has focused on students' mathematical connections in problem-solving and the identification of relationships among geometric ideas (e.g., Agustini et al., 2017; Caviedes et al., 2024; Rahmi et al., 2020). Nevertheless, despite these advances, existing research has mainly concentrated on general educational contexts or on mathematical topics other than geometry, with comparatively limited attention given to concepts such as triangle similarity.

Likewise, research on mathematical connections has been developed from different perspectives, leading to the identification of various connection types that emerge when solving mathematical tasks (e.g., Businskas, 2008; García-García, 2024; García-García & Dolores-Flores, 2018). García-García (2024) reported evidence of interconceptual connections when students solved tasks involving linear equations and functions. In this regard, the present study seeks to provide empirical evidence of this type of connection in the context of triangle similarity, thereby extending the investigation of mathematical connections to the domain of geometry.

Consequently, there is a need for further research on mathematical connections, particularly in relation to triangle similarity among pre-university students. This educational level is especially relevant because students are transitioning to higher education, where the ability to establish and coordinate mathematical connections becomes increasingly important for understanding and working with more advanced mathematical concepts. Therefore, this study aims to address the following research question: What mathematical connections do pre-university students make when solving tasks involving triangle similarity? The aim is to identify the mathematical connections made by a group of Mexican pre-university students while solving tasks related to triangle similarity.

This research is relevant because their results may inform classroom practices by supporting instructional designs that address triangle similarity through a connections-based approach.

2. METHOD

This study adopts a qualitative approach with a descriptive scope, specifically employing a case study design (Kothari, 2004). The methodological process followed the next stages.

2.1. Context and Case Study Selection

The study was conducted at a public preuniversity school in Guerrero, Mexico, serving students aged 15 to 18. This school follows a technical education model, offering training in four areas: social work, accounting, office automation, and computer equipment support and maintenance. Instructions are provided in both morning and afternoon shifts.

At the time of data collection, the school operated under the official Mexican curriculum for pre-university education, which specifies that the Geometry and Trigonometry course should be taught in the second semester of the six-semester program. Eight students were selected as case studies based on the following criteria: (a) official enrollment in the school, (b) regular academic status with a minimum grade of 8 on a 10-point scale, and (c) voluntary participation with informed consent for the use of their work in research. To ensure confidentiality, participants are referred to using pseudonyms: E1 through E8.

2.2. Data Collection Instrument

Data were collected through three mathematical tasks and a set of auxiliary interview questions. The tasks, validated by experts, were selected for their clarity, appropriateness to the students' educational level, and potential to promote rich mathematical connections by solving them—aligned with the principles of open-ended problem solving (Goldin, 2000). More specifically, these tasks were chosen because they were aligned with the objectives of the study and were accessible to the participants, allowing them to engage meaningfully with the mathematical ideas involved.

Task 1 was adapted from Aravena Díaz et al. (2016) and consisted of identifying and justifying which of the given figures contained pairs of similar triangles. Task 2, also based on Aravena Díaz et al. (2016), required students to construct a triangle similar to a given triangle using a specified ratio. Students were provided with a ruler, compass, and blank paper to support the geometric construction. Finally, task 3, was adapted from Giacomone et al. (2018), involved applying Thales' theorem to determine the height of the Great Pyramid of Khufu based on proportional reasoning. All interviews were audio- and video-recorded and had an average duration of 80 minutes.

2.3. Data Analysis

The data were analyzed using thematic analysis, a method designed to identify, analyze, and report patterns or themes within qualitative data (Braun & Clarke, 2006). This approach was selected because it supports both inductive and deductive forms of analysis. On

the one hand, it enabled the identification of specific mathematical connections established by the participants while solving tasks involving triangle similarity. These connections emerged inductively from the data and were conceptualized as subthemes. On the other hand, thematic analysis also facilitated a deductive process in which the identified subthemes were related to previously established categories of mathematical connections reported in the literature. Therefore, a hybrid approach to thematic analysis was adopted: an initial inductive phase was used to identify subthemes from the participants' responses, followed by a deductive phase in which these subthemes were interpreted and classified according to existing typologies of mathematical connections. The thematic analysis followed six phases:

Phase 1. Familiarization with the data

In this phase, researchers reviewed students' verbal and written responses to become immersed in the data and to interpret the strategies and reasoning used in solving each task.

Phase 2. Generating initial codes

Relevant excerpts from the task-based interviews were coded based on the mathematical reasoning exhibited by students. These initial codes captured early expressions of meaningful relationships between ideas, procedures, theorems, representations, and definitions (see [Table 1](#)).

Table 1. Generation of initial codes

Excerpt	Codes
I: What does it mean for two figures to be similar? E4: [...] two figures are similar if they have proportionality, their size increases but the shape remains the same. I: For the figures you didn't circle, why do you say they aren't similar? E4: They share one equal angle, but the others aren't equal; their sides aren't proportional. You can see at a glance that [the corresponding sides] aren't in proportion, and they don't have equal angles (points to Figure 1 in the left column of Task 1). This one has one equal angle, but it would need another [pair of] equal angles to use the AA [theorem], and it would need all sides to be proportional to meet the SSS [theorem] (points to Figure 6, right column). This one also has one angle, but it needs to have an equal angle and lacks proportional sides (points to Figure 3, left column).	• When two triangles are similar, their sides are proportional, but their shape remains the same. • When two triangles only share one angle and their sides are not proportional, they are not similar.

Phase 3. Identifying subthemes and themes

To identify subthemes, an exhaustive review of the coded data was conducted, looking for similarities among the statements. According to Braun and Clarke (2006), subthemes may emerge from the data itself. In this context, the codes were grouped into subthemes representing specific mathematical connections established by the participants while solving triangle similarity tasks. Subsequently associated with a particular theme (see [Table 2](#)). In this research, a theme is defined as a type of mathematical connection.

Although different definitions exist, this study considers a mathematical connection as a true relationship that a student establishes between different ideas, definitions, procedures, theorems, representations, and meanings—either within mathematics or in connection with other disciplines and real-world contexts (García-García & Dolores-Flores, 2018). These connections are articulated verbally or in writing while solving a mathematical task. On the other hand, this study draws on models that support the identification of connections relevant to student learning. (Businskis, 2008) categorized five types of mathematical connections: different representations, common features, part-whole, implication, and procedural. García-García and Dolores-Flores (2018) added reversibility and meaning as distinct types of connections. More recently, Garcia-Garcia (2024) introduced the notion of interconceptual mathematical connection. These typologies constitute our preliminary framework for the identification of themes.

Table 2. Construction of subthemes and themes

Code	Subtheme	Theme
1. When two triangles are similar, their sides are proportional, but their shape remains the same.	Definition of triangle similarity	Meaning
2. When two triangles only share one angle and their sides are not proportional, they are not similar.		

Phase 4: Review of subthemes and themes

In this phase, a review of the themes and subthemes is conducted to ensure they relate to the established codes and the data as a whole. If any of them do not clearly capture the response pattern from the data, they may be modified or even eliminated. Subthemes that had the same response pattern were integrated into a single subtheme.

Phase 5. Definition and naming of subthemes and themes

In this phase, it is essential to ensure that the subthemes and themes are unique, specific, and clear. In this regard, the themes relate to the previously defined mathematical connections. Additionally, the subthemes are explained and supported by written or verbal evidence to reinforce the ideas being communicated.

Phase 6. Preparation of the report

The final phase involved writing a comprehensive report that organizes the findings by theme, presenting the types of mathematical connections identified and supported by illustrative data excerpts from the case studies.

Additionally, to ensure the reliability and validity of the results, the study incorporated researcher triangulation in the coding and interpretation phases. The researchers of this paper independently reviewed and coded the data, after which discrepancies were discussed and resolved through consensus. This process helps to mitigate individual biases and deepen the analytic insights by integrating diverse perspectives on the same data set, thus contributing to a more robust and credible interpretation of patterns and themes.

3. RESULTS AND DISCUSSION

3.1. Results

The analysis revealed the use of six types of mathematical connections in the case studies: feature, procedural, different representations, meaning, implication, and interconceptual. However, no evidence of part-whole connection was identified. This suggests that students may not recognize hierarchical relationships involving triangle similarity, such as understanding similar triangles as particular cases of similar geometric figures or viewing specific types of triangles (e.g., equilateral or isosceles triangles) as special cases within broader classes of similar triangles. The frequency and nature of these connections varied across the eight case studies, depending on the task and the student's approach (see Table 3). The reported frequencies in Table 3 indicate the total number of times each type of mathematical connection was established across the three tasks.

Table 3. Mathematical connections made by the students about similar triangles

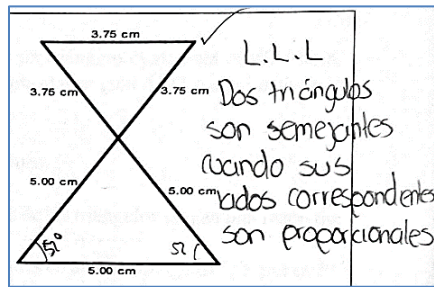
Mathematical connections	Subtheme	Frequency
Feature	Preservation of side ratios.	10
	Preservation of angle measures.	10
	Preservation of shape despite size variation.	8
Different Representations	Geometric interpretation of numerical ratios.	16
Implication	Deductive consequences of similarity.	16
Interconceptual	Conceptual integration in similarity reasoning.	8
	Coordination of concepts in geometric constructions.	8
Procedural	Use of proportionality-based procedures.	13
Meaning	Definition of triangle similarity.	6

3.1.1. Feature

This mathematical connection was the most frequently observed. It occurs when a student describes one or more attributes or properties of a mathematical concept that distinguish it from, or relate it to, another concept. Unlike the meaning connection, this type of connection involves describing only one or a few characteristics rather than providing a complete definition of the concept. Thus, within feature connections, students identify important characteristics of triangle similarity, but these characteristics are considered separately rather than as an integrated whole. In this context, some case studies ($f = 10$) noted the necessity of proportionality among all three pairs of corresponding sides (see Figure 1). This observation contributed to the construction of the subtheme: Preservation of Side Ratios (see Figure 2).

Other cases ($f = 10$) emphasized the equality of corresponding angles in similar triangles. For instance, E1 stated: *Even though the measurements are different, the angles would be equal. [...] When two figures are similar, their angles are equal, but their sizes can vary.* This reasoning supports the subtheme: Preservation of angle measures. Finally, some participants ($f = 8$) distinguished similarity from congruence by stating that: *Congruence means that they are equal and of the same size [...]. Similarity means different sizes but the*

same shape. This interpretation led to the identification of the subtheme: Preservation of shape despite size variation.



Translation:

S.S.S.

Two triangles are similar when their corresponding sides are proportional.

Figure 1. Proportionality of corresponding sides in similar triangles in task 1

3.1.2. Different representations

This mathematical connection is made when a student moves between representations within the same register (e.g., from one geometric figure to another) or across different registers (e.g., from a geometric to an algebraic representation). This was the second most frequently identified mathematical connection, appearing 16 times across the case studies. For instance, in Task 2, E7 constructed triangle $\triangle ABC$ with sides of 3, 4, and 6 cm, and then doubled those lengths to form triangle $\triangle DEF$ with sides 6, 8, and 12 cm. Conversely, E2 reduced the side lengths by a factor of 2 (see Figure 2). These cases support the subtheme: Geometric interpretation of numerical ratios. This form of conversion was also observed in Task 3.

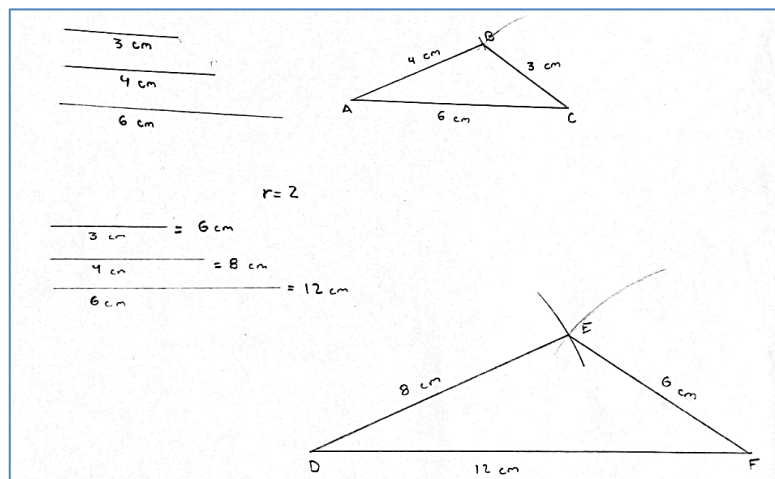


Figure 2. Numerical-geometric representation in the construction of similar triangles

3.1.3. Implication

This mathematical connection involves a logical relationship between two ideas or statements—if A, then B. This connection was also significant across case studies ($f = 16$). Students logically related geometric concepts and employed them in deductive and inductive reasoning. For instance, in Task 1, E6 explicitly identified the criteria required for triangle similarity. The student reasoned that if the sum of two angles is known, the third can be

deduced, satisfying the AA theorem (see excerpt from E6; Task 1). This logical reasoning led to the subtheme: Deductive consequences of similarity.

E6 : This one is similar (pointing to one triangle) to this one (pointing to another triangle) because they have the same shape in their sides, although their sizes are different.

I : Why?

E6 : I'm sure this figure is similar [pointing to the triangles in Figure 2, then writing SSS. Yes, that's it.

I : What another triangle is similar?

E6 : This triangle is also similar [pointing to the triangles in Figure 4] because it has two equal angles, or rather, all three. [...] We know that the sum of the interior angles of a triangle is 180 degrees, and subtracting the sum of the corresponding angles will give us the third angle we are looking for.

3.1.4. Interconceptual

This mathematical connection emerges when a student relates two or more mathematical concepts or uses a theorem to solve a specific task. This connection emerged in all case studies across one or more tasks. For instance, in Task 1, participants connected similarity theorems (AA, SAS, SSS) with the concepts of ratio and proportion to describe properties and identify similar triangles. On the other hand, in Task 3, all students solved the problem related to the height of the Great Pyramid of Khufu by applying the concepts of ratio, proportion, and similarity theorems (SSS or SAS). The integration of these concepts played a key role in identifying similar triangles in Task 1 and solving a triangle similarity problem in Task 3. This coordination of mathematical ideas contributed to the emergence of the subtheme: Conceptual integration in similarity reasoning (see Figure 3).

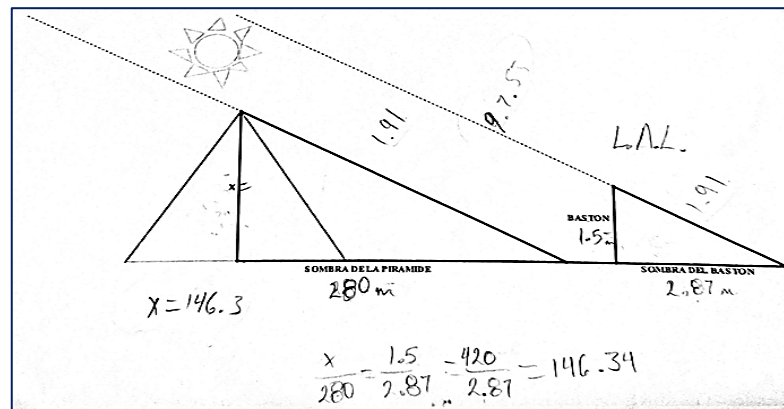


Figure 3. Ratio, proportion, and the SAS similarity theorem used to solve the task

Other participants ($f = 8$), in Task 2, coordinated the concepts of ratio, proportion, and similarity theorems while constructing similar triangles. This indicates that the geometric construction process requires the integration of interconnected mathematical ideas rather than the isolated application of a single concept. Consequently, this evidence contributed to the identification of the subtheme: Coordination of concepts in geometric constructions.

3.1.5. Procedural

This mathematical connection is established when a student applies known mathematical rules, formulas, or algorithms to obtain a solution. In this study, this connection was observed in all cases ($f = 13$), particularly in Tasks 2 and 3, where participants employed multiple procedures to solve the problems. Analysis of the participants' responses revealed two distinct procedures: they used the given ratio to calculate the side lengths of the second triangle, ensuring its similarity to the original and, they calculated the height of the Great Pyramid using proportional reasoning (see Figure 4). These procedural strategies were grouped under the subtheme: Use of proportionality-based procedures.

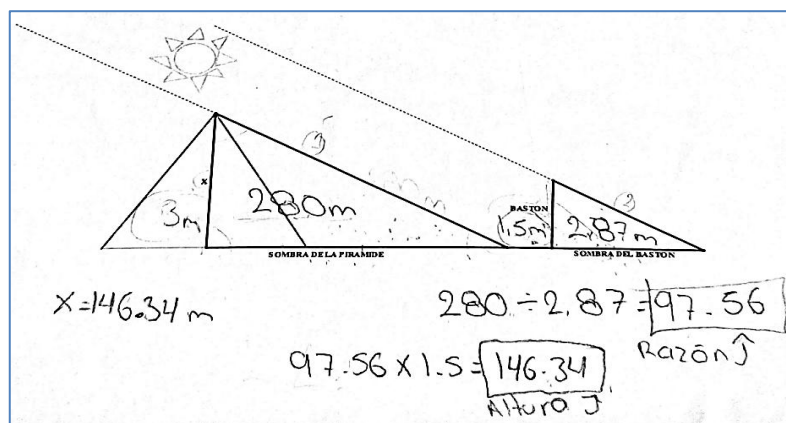


Figure 4. Using ratio to find the value of the fourth proportional

3.1.6. Meaning

This mathematical connection is established when a student assigns conceptual significance to a mathematical idea, articulating what it is and what it represents based on its definition and context. Unlike the feature-type, which emphasizes attributes, the meaning-type focuses on the personal understanding or interpretation of the concept. This mathematical connection was established by the study cases ($f = 6$) when the students not only described the properties of similar triangles but also articulated what those properties meant in context. For instance, in Task 2, E1 stated that the constructed triangles were similar because their corresponding sides were proportional and their corresponding angles were equal (see the excerpt from E1 in Task 2). This understanding reflects the subtheme: Definition of triangle similarity.

I : What are those triangles you constructed like?

E1 : They are similar.

I : Why are they similar?

E1 : Because they have the same shape, their sides are proportional, and their corresponding angles are equal.

E6 : [...] it would be SAS.

3.2. Discussion

The findings of this study suggest that students mobilize a variety of mathematical connections when reasoning about triangle similarity; however, these connections do not develop uniformly. Operational connections—such as feature, procedural, interconceptual, and implication connections—were considerably more frequent than meaning connections, indicating that students' understanding was primarily grounded in operational and relational processes rather than in a fully articulated conceptual understanding of similarity.

The predominance of feature connections suggests that students' understanding of triangle similarity was largely attribute-based rather than conceptually integrated. Although participants recognized important properties such as proportional corresponding sides and equal corresponding angles, these properties were seldom coordinated into a coherent definition of similarity. Furthermore, the absence of part-whole connection reinforces this interpretation. Students did not appear to recognize hierarchical relationships in which triangle similarity could be understood as a particular case of broader classes of similar geometric figures, nor did they relate specific types of triangles to more general categories within similarity structures. Together, these findings suggest that students' reasoning remained focused on local properties rather than on broader mathematical structures.

This interpretation is consistent with Haj-Yahya (2022), who argues that understanding the defining attributes of similarity is fundamental for grasping similarity theorems. However, the present study shows that recognizing such attributes alone is insufficient for developing a formal understanding of similarity. From the perspective of mathematical connections, feature connections appear to represent a more elementary level of understanding than meaning connections, which is consistent with the levels of understanding proposed by Garcia-Garcia (2024). The limited presence of meaning connections also supports Bila et al. (2024), who argue that meaning serves as a foundation for the development of other mathematical connections. Consequently, students may successfully solve similarity tasks while relying on isolated properties and procedures without necessarily constructing a coherent conceptual understanding of similarity.

The prevalence of procedural connections helps explain how students were able to solve similarity tasks despite their limited conceptual understanding of the concept. Participants frequently relied on ratios, proportions, and the rule of three to determine unknown measures and solve similarity problems. These procedures often produced correct solutions; however, they rarely justified why the procedures were mathematically valid in a given situation. This finding is consistent with Godino et al. (2018), who note that students often apply the rule of three in proportional situations without explaining the reasoning that justifies its use. Likewise, Arican et al. (2018) and DeJarnette et al. (2014) report that proportional procedures play a central role in reasoning about corresponding sides in similar figures. Although such strategies are mathematically useful, their predominance in this study suggests that students' success was frequently procedural rather than conceptual.

Interestingly, despite this reliance on procedural strategies, students were able to coordinate concepts such as ratio, proportion, Thales' theorem, and similarity criteria even when their explicit definitions of similarity remained incomplete. This suggests that interconceptual connections may emerge from task-oriented activity before students develop

a fully articulated conceptual meaning. In other words, students can mobilize networks of related concepts for problem solving without necessarily possessing a formal understanding of the central concept that links them. This is consistent with Climent et al. (2021), who argue that Thales' theorem plays a key role in connecting various geometric concepts. However, these results contrast with those of Wulandari et al. (2020) and Agustini et al. (2017), who report that students often struggle to establish such relationships. This discrepancy may be explained by the nature of the tasks used in this study, which appear to have encouraged the simultaneous activation of multiple concepts.

The emergence of different representation connections appears to be closely linked to interconceptual connections. Students' ability to interpret numerical ratios as geometric transformations required them to coordinate proportional reasoning with geometric visualization. Thus, representational flexibility did not emerge in isolation but as a consequence of connecting multiple mathematical ideas. However, the systematic omission of the similarity symbol ($\sim$) suggests a disconnect between operational and formal aspects of mathematical knowledge. So, although students successfully performed procedures and interpreted geometric relationships, they did not consistently employ the symbolic conventions that formally communicate these relationships. This may indicate that their understanding remained largely action-oriented rather than mathematically formalized. These findings are consistent with Mbatha and Bansilal (2023), and Ubah and Bansilal (2019), who argue that students may manipulate representations without fully understanding their structural relationships or formal meaning. Therefore, as suggested by Wijaya et al. (2021), it is necessary to promote both visualization skills and relational thinking to strengthen these connections.

The role of implication connections seemed primarily pragmatic rather than deductive. Students used similarity criteria to justify conclusions already reached through calculations or visual inspection, instead of employing them as components of a systematic deductive argument. This finding suggests that logical reasoning was often retrospective, serving to confirm solutions rather than to generate them. This type of reasoning, associated with deductive and inductive processes, is consistent with Mhlolo (2012), who emphasizes its importance in developing geometric reasoning.

4. CONCLUSION

The findings of this study show that pre-university students establish a variety of mathematical connections when solving tasks involving triangle similarity. However, these connections did not develop in a balanced manner. Operational connections were considerably more frequent than meaning connections, suggesting that students' understanding of triangle similarity was primarily grounded in the use of properties, procedures, and relationships among concepts rather than in a fully articulated conceptual understanding. Furthermore, the absence of part-whole connection indicates that students rarely related triangle similarity to broader mathematical structures, which may limit the development of a more integrated understanding of the concept.

This study also contributes to the literature on mathematical connections by providing empirical evidence of interconceptual connections in the domain of geometry. These findings

support the view that mathematical understanding involves the integration of multiple interconnected concepts and provide further evidence of the relevance of interconceptual connections in mathematical activity.

This study has some limitations that should be acknowledged. First, the analysis focused exclusively on pre-university students' mathematical connections when solving tasks involving triangle similarity. Therefore, the findings do not provide information about how these connections develop over time or how they might differ across educational levels. Second, the study examined students' written responses and problem-solving processes within a specific set of tasks, which may have influenced the types of connections that emerged. Different tasks or instructional contexts could potentially promote other forms of mathematical connections. Future research could explore the development of mathematical connections through longitudinal studies, compare connections across different educational levels, or examine the impact of instructional interventions designed to deeper mathematical understanding. Additionally, further studies could investigate mathematical connections in other geometric topics to determine whether the patterns identified in this study are specific to triangle similarity or reflect broader tendencies in students' geometric reasoning.

Acknowledgments

The authors express their sincere gratitude to the students who voluntarily participated and contributed to the data collection process, significantly supporting the success of this study.

Declarations

Author Contribution	: ES-C: Data curation, Formal analysis, Visualization, Writing - original draft, and Writing - review & editing; JG-G: Conceptualization, Formal analysis, Methodology, and Writing - review & editing; ALS: Validation, Supervision, and Writing - review & editing; GS-B: Formal analysis, and Writing - review & editing.
Funding Statement	: This research did not receive public or private funding.
Conflict of Interest	: The authors declare no conflict of interest.
Additional Information	: Additional information is available for this paper.

REFERENCES

- Agustini, R. Y., Suryadi, D., & Jupri, A. (2017). Construction of open-ended problems for assessing elementary student mathematical connection ability on plane geometry. *Journal of Physics: Conference Series*, 895, 012148. <https://doi.org/10.1088/1742-6596/895/1/012148>
- Aravena Díaz, M. d. I. M., Gutiérrez Rodríguez, Á., & Jaime Pastor, A. (2016). Estudio de los niveles de razonamiento de Van Hiele en alumnos de centros de enseñanza vulnerables de educación media en Chile [Study of Van Hiele levels of reasoning in students from

- vulnerable secondary schools in Chile]. *Ensenanza De Las Ciencias*, 34(1), 107–128. <https://doi.org/10.5565/rev/ensciencias.1664>
- Arıcan, M., Koklu, O., Olmez, I. B., & Baltacı, S. (2018). Preservice middle grades mathematics teachers' strategies for solving geometric similarity problems. *International Journal of Research in Education and Science*, 4(2), 502–516. <https://doi.org/10.21890/ijres.428297>
- Bila, H., Maphutha, K., & Mutodi, P. (2024). Learners' algebraic and geometric connections when solving Euclidean geometry riders. *Pythagoras*, 45(1), 1–9. <https://doi.org/10.4102/PYTHAGORAS.v45i1.810>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Businskas, A. M. (2008). *Conversations about connections: how secondary mathematics teachers conceptualize and contend with mathematical connections*. Doctoral dissertation. Simon Fraser University. <https://summit.sfu.ca/item/9245>
- Caviedes, S., De Gamboa, G., & Badillo, E. (2024). Mathematical connections involved in area measurement processes. *Research in Mathematics Education*, 26(2), 237–257. <https://doi.org/10.1080/14794802.2024.2370333>
- Climent, N., Espinoza-Vásquez, G., Carrillo, J., Henríquez-Rivas, C., & Ponce, R. (2021). Una lección sobre el teorema de Thales, vista desde el conocimiento especializado del profesor [A lesson on Thales' theorem, viewed from the professor's specialized knowledge]. *Educación matemática*, 33(1), 98–124. <https://doi.org/10.24844/em3301.04>
- DeJarnette, A. F., Walczak, M., & González, G. (2014). Students' concepts- and theorems-in-action on a novel task about similarity. *School Science and Mathematics*, 114(8), 405–414. <https://doi.org/10.1111/ssm.12092>
- Dündar, S., & Güdüz, N. (2017). Justification for the Subject of Congruence and Similarity in the Context of Daily Life and Conceptual Knowledge. *Journal on Mathematics Education*, 8(1), 35–54. <https://doi.org/10.22342/jme.8.1.3256.35-54>
- García-García, J. (2024). Mathematical understanding based on the mathematical connections made by Mexican high school students regarding linear equations and functions. *The Mathematics Enthusiast*, 21(3), 673–718. <https://doi.org/10.54870/1551-3440.1646>
- García-García, J., & Dolores-Flores, C. (2018). Intra-mathematical connections made by high school students in performing calculus tasks. *International Journal of Mathematical Education in Science and Technology*, 49(2), 227–252. <https://doi.org/10.1080/0020739x.2017.1355994>
- Giacomone, B., Godino, J. D., & Beltrán-Pellicer, P. (2018). Desarrollo de la competencia de análisis de la idoneidad didáctica en futuros profesores de matemáticas [Development of the competence of analyzing didactic suitability in future mathematics teachers]. *Educação e Pesquisa*, 44, 1–21.
- Godino, J. D., Giacomone, B., Font, V., & Pino-Fan, L. (2018). Conocimientos profesionales en el diseño y gestión de una clase sobre semejanza de triángulos: Análisis con herramientas del modelo CCDM [Professional knowledge in the design and management of a lesson on triangle similarity: Analysis using CCDM model tools.].

- Avances de investigación en Educación Matemática*(13), 63–83.
<https://doi.org/10.35763/aie.v0i13.224>
- Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517–545). Routledge.
- Haj-Yahya, A. (2022). Students' conceptions of the definitions of congruent and similar triangles. *International Journal of Mathematical Education in Science and Technology*, 53(10), 2703–2727. <https://doi.org/10.1080/0020739x.2021.1902008>
- Kothari, C. R. (2004). *Research methodology: Methods and techniques*. New Age International.
- Mbatha, M., & Bansilal, S. (2023). Using the Van Hiele theory to explain pre-service teachers' understanding of similarity in euclidean geometry. *Education Sciences*, 13(9), 861. <https://doi.org/10.3390/educsci13090861>
- Mhlolo, M. K. (2012). Mathematical connections of a higher cognitive level: A tool we may use to identify these in practice. *African Journal of Research in Mathematics, Science and Technology Education*, 16(2), 176–191. <https://doi.org/10.1080/10288457.2012.10740738>
- Purnomo, Y. W., Nabillah, R., Aziz, T. A., & Widodo, S. A. (2024). Fostering mathematical connections and habits of mind: A problem-based learning module for elementary education. *Infinity Journal*, 13(2), 333–348. <https://doi.org/10.22460/infinity.v13i2.p333-348>
- Rahmi, M., Usman, U., & Subianto, M. (2020). First-grade junior high school students' mathematical connection ability. *Journal of Physics: Conference Series*, 1460(1), 012003. <https://doi.org/10.1088/1742-6596/1460/1/012003>
- Siregar, R., & Siagian, M. D. (2019). Mathematical connection ability: teacher's perception and experience in learning. *Journal of Physics: Conference Series*, 1315(1), 012041. <https://doi.org/10.1088/1742-6596/1315/1/012041>
- Ubah, I. (2022). Pre-service mathematics teachers' semiotic transformation of similar triangles: Euclidean geometry. *International Journal of Mathematical Education in Science and Technology*, 53(8), 2004–2025. <https://doi.org/10.1080/0020739x.2020.1857858>
- Ubah, I., & Bansilal, S. (2019). The use of semiotic representations in reasoning about similar triangles in Euclidean geometry. *Pythagoras*, 40(1), a480. <https://doi.org/10.4102/pythagoras.v40i1.480>
- Wijaya, T. T., Mutmainah, I. I., Suryani, N., Azizah, D., Fitri, A., Hermita, N., & Tohir, M. (2021). Ninth grade students mistakes when solving congruence and similarity problem. *Journal of Physics: Conference Series*, 2049(1), 012066. <https://doi.org/10.1088/1742-6596/2049/1/012066>
- Wulandari, A. N., Usodo, B., Sutopo, S., Setiawan, R., Kurniawati, I., Kuswardi, Y., & Aulia, I. I. (2020). Fragmentation of thinking structure: concept construction and problem solving in geometry of junior high school students. *Journal of Physics: Conference Series*, 1567(3), 032007. <https://doi.org/10.1088/1742-6596/1567/3/032007>