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Designing and enacting mathematics lessons: Insights from pre-service teachers' learning through project-based learning and lesson study

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Abstract

Bridging lesson design and instructional enactment remains a challenge in mathematics teacher education. This qualitative case study explored how Project-Based Learning (PjBL) and lesson study supported pre-service mathematics teachers (PSTs) in addressing this challenge. PjBL provided the macro-pedagogical architecture for a seven-week collaborative project, while LS structured the micro-cycles of Plan, Do, See, and Redesign. Participants were 13 third-year PSTs enrolled in an undergraduate Mathematics Education program and taking an Elementary Mathematics Instruction course at an Indonesian university. Organized into three groups, they selected an elementary mathematics topic, designed a two-meeting lesson sequence, and completed two lesson study cycles through peer teaching. Data consisted of lesson designs and semi-structured group interviews and were analyzed thematically. The findings revealed that participants engaged in pedagogical reasoning, designed tasks intended to promote mathematical reasoning and communication, considered simulated learner responses, and used reflection to redesign their lessons. However, interview data revealed instances of reverting to direct explanation when simulated learners hesitated or required additional time. These findings identify a design–enactment discontinuity in which the capacity to construct student-centered lessons did not automatically translate into responsive enactment. The study suggests that combining sustained project work with iterative professional inquiry can provide a useful structure for lesson design, but PSTs also require repeated and scaffolded opportunities to rehearse responding to learner thinking.

Keywords:
Elementary mathematics instruction, Lesson design, Lesson study, Pre-service teachers, Project-based learning

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1. INTRODUCTION

Preparing pre-service teachers (PSTs) to design and deliver effective instruction in elementary classrooms has become a central concern in teacher education. Beyond content knowledge, novice teachers must develop the capacity to plan instruction that promotes conceptual understanding, addresses student misconceptions, and engages diverse learners (Allmendinger et al., 2023). This includes selecting and sequencing meaningful tasks, anticipating learners' possible responses, and determining appropriate pedagogical support (Sujadi et al., 2025). However, this instructional design competence cannot be developed in isolation from practice. Research has consistently shown that traditional university-based coursework tends to separate theoretical knowledge from the practical demands of real teaching, leaving teacher candidates underprepared for the complexity of the classroom (Christiansen & Erixon, 2024; Hoover & Ball, 2021). A particularly persistent challenge is the design–enactment gap: the presence of student-centered elements in a lesson design does not necessarily ensure that novice teachers can enact those elements responsively in practice. (Farrow et al., 2024; Fitriati & Novita, 2018). In response, there is growing interest in embedding practice-based approaches into teacher education programs, emphasizing teaching as a form of disciplined inquiry and design (Mancenido et al., 2026).

Project-Based Learning (PjBL) has emerged as a promising approach in teacher preparation because it positions pre-service teachers (PSTs) as active designers and decision makers. Through authentic, extended projects, PSTs learn to connect theory with practice, collaborate to solve instructional problems, and create meaningful products such as lesson designs (Farrow et al., 2022; Pratiwi et al., 2025; Qi et al., 2025). In Indonesia, the use of project-oriented learning in higher education has been reinforced through the national Key Performance Indicators for higher education. Under KPI 7, collaborative and participatory learning is recognized through the use of case methods or team-based projects, in which students work collaboratively on authentic problems or complex questions and present a final product for feedback (Directorate General of Higher Education, 2021; Ministry of Education Culture Research and Technology, 2023). Through PjBL, PSTs are encouraged to structure lessons around rich problems, anticipate student strategies, and design learning trajectories that promote inquiry of learning in the 21st century. However, while PjBL strengthens PSTs' conceptual understanding of instructional models, research indicates that novice teachers often struggle to enact these designs in real classrooms without additional guided practice (Liang et al., 2025; Magaji et al., 2024).

Lesson study offers a complementary structure by situating PSTs' learning within cycles of collaborative planning, observation, reflection, and revision (Fitriati et al., 2023; Tan et al., 2024). In lesson study, teachers jointly plan a lesson, often called a research lesson, observe its implementation in a real classroom, and engage in structured reflection based on student learning evidence (Lewis, 2016; Lewis & Hurd, 2011; Murata, 2011). The process enables PSTs to focus deeply on teaching and learning, to test and refine ideas, and to grow professionally through collective inquiry (Dudley, 2014; Lewis, 2016). While lesson study has traditionally been used for in-service teacher development, recent studies have highlighted its potential in teacher education, especially for enhancing PSTs' skills in lesson planning,

pedagogical reasoning, and classroom observation (Fitriati et al., 2025; Tan et al., 2024; Wood et al., 2019).

The inclusion of lesson study in initial teacher education aligns with a broader shift toward pedagogies of enactment, which situate professional learning within approximations of teaching practice. As Lewanowski-Breen et al. (2021) argue, such pedagogies provide opportunities that allow PSTs not only to apply knowledge for teaching mathematics but also to develop pedagogical reasoning by connecting instructional purposes, anticipated learner responses, teaching decisions, and reflection on enactment. In mathematics education, pedagogical reasoning is closely related to professional noticing, commonly understood as teachers' capacity to attend to, interpret, and respond to learners' mathematical thinking (Bakker et al., 2022). Research indicates that noticing does not develop automatically; PSTs require structured opportunities to anticipate responses, examine evidence from enactment, and reconsider instructional decisions (Ekawati et al., 2023; Yilmaz & Yetkin Ozdemir, 2023). In this study, pedagogical reasoning and professional noticing therefore provide complementary lenses for examining how PSTs justify lesson-design decisions and interpret simulated learner responses during peer teaching.

Recent research indicates that PSTs benefit from opportunities to design contextually meaningful mathematical tasks, anticipate possible responses, and revise instruction through collaborative inquiry (Sujadi et al., 2025; Yilmaz & Yetkin Ozdemir, 2023). Lesson study supports these processes by connecting planning, enactment, reflection, and redesign (Fitriati et al., 2025; Fitriati et al., 2023; García et al., 2024), while PjBL provides sustained engagement with an authentic professional project. The two approaches therefore contribute at different but complementary levels. PjBL provides the macro-pedagogical architecture that organizes the extended lesson-design project, participant collaboration, project milestones, feedback, and final products. Lesson study operates within that architecture as a micro-cycle of professional inquiry through Plan, Do, See, and Redesign. This nested relationship is consistent with recent mathematics education research showing that PjBL and Lesson study can be combined to support teachers' interpretation and adaptation of instructional resources (Qi et al., 2025). However, evidence of successful lesson design should not be equated with successful enactment, as student-centered features in planned lessons may not transfer directly into instructional practice (Farrow et al., 2024).

Despite growing research on lesson study in teacher education (Baumfield et al., 2022; Larssen et al., 2018; Tan et al., 2024), two issues remain insufficiently examined. First, recent research on lesson study in PSTs education has primarily emphasized collaborative learning, reflective practice, lesson planning, and professional noticing (Almacioglu & Arslan, 2026; Cardoso et al., 2025; Yilmaz & Yetkin Ozdemir, 2023). Although these studies establish the value of lesson study for connecting planning, teaching, and reflection, less explicit attention has been given to discontinuities between PSTs' inquiry-oriented lesson designs and their subsequent enactment. This distinction remains important because emerging evidence indicates that pre-service teachers' pedagogical intentions may exceed what they are able to enact in practice (Farrow et al., 2024; Tsakeni et al., 2026). Second, research explicitly combining PjBL and Lesson study remains limited. A recent study by Qi et al. (2025), for example, examined teachers' curricular noticing and textbook adaptation within PjBL–lesson

study, but did not address the distinct structural functions of the two approaches within a PSTs' education course. Consequently, further research is needed to distinguish PjBL's role in organizing sustained project work from lesson study's role in structuring iterative professional inquiry. These limitations leave uncertainty about how PSTs engage in pedagogical reasoning across lesson design, peer-teaching enactment, reflection, and redesign, and why inquiry-oriented plans may revert to teacher-centered instruction during enactment. Examining this tension is particularly important in peer-teaching settings, where practice is approximated rather than conducted with actual elementary learners.

Accordingly, the present study aimed to examine PSTs' engagement in lesson design and enactment within an integrated PjBL and lesson study framework. Particular attention was given to the distinct but complementary functions of the two approaches: PjBL as the overarching structure for sustained collaborative project work and Lesson study as the professional inquiry process connecting planning, enactment, reflection, and redesign. The study also sought to identify the pedagogical reasoning and professional insights evident in participants' experiences. Equally important, it examined tensions between inquiry-oriented lesson designs and their enactment, thereby avoiding the assumption that well-developed plans necessarily translate into responsive teaching. The study addressed the following research questions: (1) How do PSTs engage in lesson design and enactment within a PjBL-organized course structured through lesson study cycles? and (2) What pedagogical reasoning, professional insights, and design–enactment tensions emerged from their participation in these cycles?

This study advances mathematics teacher education by theoretically distinguishing the complementary functions of PjBL and lesson study. PjBL provides the macro-pedagogical architecture for sustained collaborative project work, while lesson study operates as the micro-cycle of professional inquiry linking planning, enactment, reflection, and redesign. Empirically, the study extends previous research by examining both PSTs' lesson-design processes and the discontinuities between inquiry-oriented plans and their enactment in peer-teaching practice. Furthermore, the study highlights the potential of lesson study to bridge the persistent divide between university coursework and authentic teaching practice by creating a reflective and inquiry-oriented learning environment. The study therefore informs the design of teacher education experiences that combine sustained project work, collaborative professional inquiry, and repeated approximations of practice to prepare PSTs for more deliberate, reflective, and responsive mathematics teaching.

2. METHOD

2.1. Research Design and Case Boundaries

This study employed a qualitative case study design to explore pre-service mathematics teachers' engagement in lesson design and enactment within an integrated PjBL and Lesson study structure. A case study was appropriate because the study investigated a contemporary teacher-learning process within its specific course context and drew on multiple forms of qualitative evidence (Yin, 2003). The bounded case was the PjBL-organized lesson-design project implemented within one 15-meeting Elementary Mathematics Instruction

course at a private Indonesian university. The course began with five theoretical meetings, followed by a project phase in Meetings 6–15. Within this project, the seven-meeting Lesson study component occurred during Meetings 9–15.

The case was bounded by one course, one semester, 13 participating PSTs, and the collaborative production and peer-teaching enactment of elementary mathematics lessons. The three lesson-design groups constituted the embedded units of analysis. Each group selected a different mathematical topic and collectively undertook two Plan–Do–See–Redesign cycles. The case was selected because it provided an information-rich setting in which PSTs moved from studying elementary mathematics pedagogy to designing, enacting, reflecting on, and redesigning instruction within a sustained course project. The analysis examined patterns across the three groups while retaining topic-specific and group-level variations.

2.2. Participants and Course Context

The participants were 13 third-year pre-service mathematics teachers enrolled in a four-year undergraduate Mathematics Education program at a private university in Indonesia. Although they were preparing for professional roles as mathematics teachers, the study was conducted within an Elementary Mathematics Instruction course focusing specifically on the design and enactment of mathematics lessons for elementary-school learners. Before taking this course, the participants had completed foundational coursework in mathematics, mathematics pedagogy, and general educational studies.

The PSTs were organized into three groups of four to five members. Each group selected one topic from the primary mathematics curriculum: fractions, three-dimensional shapes, and measuring angles. During the lesson study cycles, the PSTs assumed complementary roles as lesson designers, model teachers, observers, and peers simulating primary learners. All lesson enactments were conducted through peer teaching at the university.

2.3. Integration of PjBL and Lesson study

PjBL functioned as the macro-pedagogical architecture of the course project. It organized PSTs' work around the authentic professional problem of designing mathematics instruction responsive to difficulties commonly experienced by primary learners. The project involved sustained inquiry, collaborative decision making, selection of mathematical topics and pedagogical approaches, development of instructional media and worksheets, peer feedback, lesson enactment, and production of a finalized lesson design. The groups exercised agency in selecting their topic, distributing responsibilities, designing tasks, choosing representations, and determining the pedagogical support incorporated into their lessons.

Lesson study operated within this broader project as the micro-cycle of professional inquiry. Each group completed two cycles. During Plan, group members collaboratively developed a lesson design containing learning objectives, anticipated difficulties, task sequences, predicted learner responses, and proposed teacher support. During Do, the design was enacted through peer teaching while other PSTs assumed observer or simulated-learner roles. During See, participants and the course instructor discussed the enactment, with attention to learner responses, task accessibility, teacher questioning, and departures from the

plan. During Redesign, each group reconsidered its tasks, prompts, scaffolding, and lesson sequence before the subsequent cycle or final submission. Thus, PjBL organized the overall course project, whereas lesson study provided the recurring process connecting planning, enactment, reflection, and redesign. Table 1 presents an overview of how PjBL and Lesson Study integrate.

Table 1. Course outline based on project-based learning and lesson study

Meeting	Main activity	PjBL Phase	LS Phase
1–5	Reviewing curriculum, mathematics content, pedagogy, and instructional media	Theoretical preparation	–
6–8	Identifying learning problems, forming three groups, selecting topics, and planning the project	Problem identification and project planning	–
9–11	Designing, peer-teaching, reflecting on, and revising the first lesson	Product development and revision	Cycle 1: Plan, Do, See, Redesign
12–14	Revising, re-enacting, reflecting on, and finalizing the lesson	Product testing and refinement	Cycle 2: Plan Do, See, Redesign
15	Submitting final lesson designs and conducting group interviews	Final product and project reflection	Cycle completion

2.4. Data Sources and Data Collection

The analyzed data comprised two complementary sources including final lesson designs and semi-structured group interviews. The three final lesson designs, one from each group, were submitted after completion of the lesson study cycles. Each document included learning objectives, anticipated content-specific difficulties, task sequences, predicted learner responses, proposed teacher support, group activities, and assessment or reflection components. The documents provided evidence of the pedagogical decisions represented in the groups' final plans; because earlier versions were not included in the analyzed corpus, they were not used to establish developmental change across cycles.

Three semi-structured group interviews were conducted after completion of the lesson study, with one interview for each group. Each interview lasted approximately 20–30 minutes and addressed collaborative planning, peer-teaching enactment, reflection, redesign, pedagogical insights, and difficulties in translating inquiry-oriented plans into practice. All 13 PSTs participated in their respective interviews. The interviews were conducted by the research team in Indonesian, audio-recorded with permission, and transcribed verbatim. Quotations selected for reporting were translated into English while preserving their intended meaning and checked against the Indonesian transcripts.

2.5. Data Analysis

Data were analyzed using a thematic analysis approach within the case study framework. The analysis began with open coding of interview transcripts and lesson plans, followed by axial coding to identify themes related to pedagogical reasoning, collaborative practices, and professional insights. Analysis began with repeated reading of the three lesson

designs and interview transcripts to establish familiarity with each group's account. The primary researcher then conducted open coding across both data sources. Initial codes captured content-specific anticipation, task selection and sequencing, predicted learner responses, instructional scaffolding, collaborative decision making, reflective reconsideration, reported redesign, and departures from the lesson design.

The analysis proceeded in two stages. First, the lesson design and interview for each group were examined as an embedded unit to identify alignments and tensions between planned instruction and participants' retrospective accounts of enactment. Second, codes were compared across the three groups to identify recurring patterns, topic-specific variations, and discrepant evidence. Related codes were clustered into provisional categories and refined into four themes: developing pedagogical reasoning in planning, designing mathematical tasks for reasoning and communication, attending to simulated learner responses, and using reflection to inform lesson redesign. A design–enactment discontinuity was retained as a cross-cutting pattern because it appeared across all three group interviews and challenged a uniformly developmental interpretation of the project.

The primary researcher developed the initial coding and thematic organization. A second researcher familiar with Lesson study and mathematics teacher education reviewed the coded evidence, theme boundaries, and interpretations. Differences were discussed and resolved through re-examination of the original lesson designs and Indonesian interview transcripts.

2.6. Trustworthiness, Reflexivity, and Ethical Considerations

Several strategies were used to strengthen the credibility of the findings. Comparison between the final lesson designs and group interviews enabled source triangulation between planned pedagogical representations and participants' interpretations of their experiences. Member checking was conducted by inviting participants to review summaries of their interview responses for accuracy. Peer debriefing with a researcher familiar with Lesson study and mathematics education was used to examine the plausibility of the codes and themes. Disconfirming evidence was retained in the analysis, particularly participants' reported reversion to direct instruction despite inquiry-oriented lesson designs.

The research team also recognized that the course-based setting and the instructor's pedagogical role could influence participants' responses. Participation in course activities was part of the course, but participation in the research was voluntary. Participants received information about the study and provided written consent before their data were used. They were assured that participation or withdrawal would not affect their academic evaluation. All data were anonymized during analysis, and quotations were reported using group and participant identifiers, such as G1-M1.

3. RESULTS AND DISCUSSION

3.1. Results

This section presents the findings in relation to the two research questions: (1) How do PSTs engage in lesson design and enactment within a PjBL-organized course structured through lesson study cycles? and (2) What pedagogical reasoning, professional insights, and

design–enactment tensions emerged from their participation in these cycles? The discussion is organized around these questions and grounded in both participant data and relevant literature.

The data corpus comprised three final lesson designs addressing fractions, three-dimensional shapes, and angle measurement, together with three semi-structured group interviews. The lesson designs represented the participants' planned pedagogical decisions, including anticipated learning difficulties, task sequences, predicted learner responses, and proposed teacher scaffolding. The interviews provided retrospective accounts of collaborative planning, peer-teaching enactment, reflection, and intended redesign. Across data sources, four themes consistently emerged: (1) developing pedagogical reasoning in planning, (2) designing mathematical tasks for reasoning and communication, (3) attending to simulated learner responses, and (4) using reflection to inform lesson redesign. A design–enactment discontinuity also emerged across all three groups as a cross-cutting pattern. Table 2 summarizes the four themes and highlights representative evidence from each data source.

Table 2. Summary of themes and corroborating evidence across data sources

Theme	Plan: evidence from final lesson designs	Do–See–Redesign: retrospective interview evidence
Developing pedagogical reasoning in planning	Identification of content-specific difficulties; sequencing from contextual and concrete representations toward formal mathematics; anticipation of responses and corresponding teacher support	PSTs described analyzing learning objectives, selecting contexts appropriate for elementary learners, and reconsidering design decisions through collaborative feedback
Designing mathematical tasks for reasoning and communication	Contextual images, manipulatives, group worksheets, problem-solving activities, and presentations were incorporated, although the degree of openness varied across topics	PSTs associated effective tasks with opportunities for discussion and recognized that overly difficult or tightly directed questions restricted learner participation
Attending to simulated learner responses	The designs predicted silence, misconceptions, incorrect strategies, and difficulties with representations, procedures, and mathematical language	PSTs reported becoming more attentive to waiting time, learner passivity, the accessibility of their questions, and the distinction between silence and lack of understanding
Using reflection to inform lesson redesign	The final designs contained anticipated scaffolds, follow-up prompts, representations, and guidance for predicted difficulties	PSTs reported reconsidering worksheets, questions, scaffolding, and time allocation after feedback and post-lesson reflection
Cross-cutting pattern: design–enactment discontinuity	The three designs incorporated inquiry-oriented intentions and planned group problem solving	All groups reported reverting to direct explanation during peer teaching because of time pressure, uncertainty, limited probing skills, perceived learner passivity, or familiar teacher-centered routines

3.1.1. Developing Pedagogical Reasoning in Planning

Across three mathematics lessons (fractions, three-dimensional shapes and the angle-measurement), pedagogical reasoning was evident in the connections PSTs established among mathematical goals, anticipated difficulties, representational choices, and instructional support. The fraction design (G1) identified difficulties involving unlike denominators, mixed numbers, and fraction operations, and moved from familiar food representations toward symbolic procedures. The three-dimensional-shapes design (G2) anticipated weak understanding of spatial properties and volume and sequenced familiar objects, comparisons between cubes and rectangular prisms, unit-cube exploration, and formal volume formulas. The angle-measurement design (G3) similarly anticipated content-specific errors, including failure to align one ray with the zero line of the protractor, confusion between its two scales, and inaccurate classification of angles. These anticipated difficulties were paired with proposed prompts or representations rather than being listed independently from the instructional sequence. Interview accounts supported this interpretation. Participants described beginning with curriculum objectives, selecting contexts close to elementary learners' experiences, and reconsidering their ideas through group and lecturer feedback. As one PST explained, "We began by analyzing the learning outcomes and objectives and then looked for a context close to learners' everyday experiences" (G1-M1). Collectively, the data show the presence of pedagogical reasoning in the final designs, particularly in the alignment of content, anticipated difficulty, task sequence, and teacher support.

3.1.2. Designing Mathematical Tasks for Reasoning and Communication

The final lesson designs incorporated tasks intended to elicit and extend mathematical reasoning. The three groups attempted to move beyond procedural exercises by incorporating contextual prompts, manipulatives, group work, and opportunities to communicate mathematical ideas, although the intellectual openness of the tasks varied. In the fraction design (G1), images of pizza and cakes introduced part–whole relationships before learners addressed unlike denominators and mixed numbers through worksheets and group presentations. The three-dimensional-shapes design (G2) provided the strongest opportunity for conceptual investigation: learners were expected to identify spatial forms in everyday objects, compare cubes and rectangular prisms, fill a larger cube with unit cubes, and explain the relationship between repeated addition and multiplication in determining volume. The angle design (G3) asked learners to identify angles in familiar objects, measure and draw angles with a protractor, compare results, and present group solutions. Nevertheless, several sequences, particularly in the fraction and angle designs, contained tightly directed teacher questions followed by explanations, limiting the space available for multiple strategies. This tension was recognized in the interviews. One PST stated that becoming a teacher involved "creating opportunities for learners to think and discover the concept, not merely explaining the material" (G1-M3), while another proposed designing problems closer to learners' experiences so that they could enter the discussion more readily (G3-M2). Thus, the evidence indicates an emerging orientation toward reasoning and communication, accompanied by continuing difficulty in designing sufficiently open and accessible tasks.

3.1.3. Attending to Simulated Learner Responses

Attention to simulated learner responses appeared first as anticipation within the lesson designs and later as retrospective interpretation in the interviews. Each design included a dedicated column predicting possible responses and specifying corresponding teacher support. The fraction group (G1) anticipated silence, uncertainty, incorrect fraction representations, and questions about converting between improper and mixed fractions. The three-dimensional-shapes group (G2) predicted confusion between two- and three-dimensional forms, uncertainty about whether a sphere belonged to flat-faced solids, and the use of addition rather than multiplication to calculate volume. The angle group (G3) anticipated inaccurate measurements, difficulty selecting the appropriate protractor scale, and groups being unable to obtain an answer. Interview reflections extended this attention beyond correctness. Participants reported recognizing that learner silence could indicate a need for additional thinking time rather than an absence of understanding. A member of Group 3 reflected, "We only gave them about one or two minutes before the teacher began explaining. We were not accustomed to giving learners space to explore the problem" (G3-M3). Another explained that Lesson study shifted attention from the teacher's performance toward the learning process (G1-M2). These accounts indicate anticipatory and reflective noticing; however, because the enactments involved peer teaching, the findings concern attention to simulated responses rather than authentic elementary learners' mathematical thinking.

3.1.4. Using Evidence to Revise the Lesson

The interviews showed that post-lesson reflection encouraged participants to reconsider the relationship among task difficulty, teacher questioning, waiting time, and learner participation. Participants reported reviewing feedback, discussing points at which enactment departed from the plan, and reconsidering questions that had not elicited discussion. Group 1 described using feedback to prepare more productive follow-up questions and allocating a larger proportion of lesson time to learner discussion. Group 2 reported identifying sections in which the teacher had taken over the reasoning process and expressed the need for greater rehearsal and stronger models of inquiry-oriented teaching. Group 3 reconsidered the accessibility of its problems, the amount of waiting time provided, and the need for guiding questions that supported thinking without revealing procedures. As one PST explained, "For the next practice, we tried to improve the questions given to learners so that they would participate more actively in the discussion" (G1-M3). The final lesson designs contained scaffolds consistent with these concerns, including predicted responses, follow-up prompts, representations, and group presentations. Nevertheless, because earlier design versions were not included in the analyzed corpus, these features cannot be treated as documentary evidence of improvement. The data instead support a more bounded conclusion: reflection generated participants' reported intentions and rationales for lesson redesign.

3.1.5. Cross-cutting pattern: design–enactment discontinuity

While the data indicated PSTs' attention to anticipating learner responses, designing reasoning-oriented tasks, and reconsidering instructional decisions, they also revealed notable challenges in enacting these pedagogical intentions during peer teaching. Across the three

groups, inquiry-oriented planning did not consistently translate into inquiry-oriented enactment. All groups reported that the model teacher reverted to direct explanation when simulated learners remained silent, appeared confused, or failed to begin discussing the task. The immediate decisions were not planned in the lesson designs. Group 1 attributed the shift to time pressure, limited confidence, and difficulty formulating probing questions. Group 2 emphasized the influence of an apprenticeship of observation: despite understanding PBL conceptually, members reproduced the explain–demonstrate–practice sequence they had repeatedly experienced as school students. As one participant stated, "When I had to make decisions quickly, I returned to the teaching pattern that was most familiar to me" (G2-M1). Group 3 attributed the shift to interpreting silence too quickly as inability and providing only limited waiting time before taking over the solution process. Reflection enabled participants to recognize that these responses reduced opportunities for learner exploration. The cross-group consistency of this pattern indicates that the central challenge was not simply producing inquiry-oriented lesson designs, but sustaining their pedagogical intentions under the uncertainty of enactment. This discontinuity therefore constitutes a substantive professional learning issue rather than an isolated implementation failure.

3.2. Discussion

The findings indicate that the PjBL and Lesson study arrangement created opportunities for PSTs to engage with lesson design as a reasoned and collaborative activity rather than merely as the completion of a planning template. The planning evidence reflects an emerging form of pedagogical reasoning in which mathematical content was transformed into instructional sequences responsive to anticipated learner difficulties. For example, the groups planned transitions from concrete or visual representations to more formal mathematical ideas and incorporated sharing and jumping tasks to extend reasoning. This finding is consistent with Shulman (1987) conception of pedagogical reasoning as the transformation of subject matter for teaching. It also complements recent research showing that lesson study can support PSTs in anticipating student strategies, selecting mathematically productive tasks, and preparing classroom discussions (Cardoso et al., 2025; Clivaz & Ni Shuilleabhain, 2019; Fitriati et al., 2025; Nguyen & Tran, 2023). The present study adds that these forms of reasoning were visible across three distinct elementary mathematics topics such as fractions, three-dimensional shapes, and angle measurement, although the available data do not establish a longitudinal increase in competence.

The PSTs' attention to possible misconceptions and solution strategies can also be interpreted through professional noticing (Qi et al., 2025; Yilmaz & Yetkin Ozdemir, 2023). Their lesson designs showed evidence of preparing to attend to learner responses, while the interviews demonstrated retrospective efforts to interpret why particular tasks or representations did not function as expected. Lesson study has similarly been found to support PSTs' attention to and interpretation of mathematical thinking through collaborative planning and reflection. However, the current findings also expose an important limitation. Because the enactments involved peers simulating primary students, the observed responses cannot be equated with authentic students' mathematical thinking. Consequently, the evidence supports emerging attention to simulated learner responses, not fully developed professional noticing

in actual primary classrooms. Moreover, noticing requires more than anticipating or recognizing a response, it also involves selecting an appropriate instructional response in the moment. The PSTs' difficulty at this responding stage became particularly visible during enactment.

The cross-cutting design–enactment discontinuity provides the study's central theoretical contribution. Although all three groups designed lessons intended to promote exploration, explanation, and comparison of strategies, their interview accounts indicated that they sometimes returned to direct explanation when peer learners hesitated, remained silent, or required more time. The groups associated this reversion with time pressure, limited confidence in probing learner thinking, short wait time, and familiarity with teacher-centered instruction. Thus, producing an inquiry-oriented design did not ensure responsive enactment. From the perspective of pedagogical reasoning and action, PSTs could formulate pedagogical intentions during planning but found it more difficult to adapt those intentions under the uncertainty of interaction. From the perspective of professional noticing, they could anticipate possible responses but did not consistently translate attending and interpreting into productive responding. This distinction is consistent with recent evidence that instructional intentions may exceed PSTs' capacity to enact inquiry-oriented teaching when contextual uncertainty and limited opportunities for supported practice intervene (Tsakeni et al., 2026).

The collaborative and inquiry-oriented nature of PjBL and lesson study is consistent with a social constructivist perspective, in which professional knowledge is constructed through participation, dialogue, and shared examination of practice (Schunk, 2014). Through PjBL and lesson study activities, PSTs construct their knowledge by working collaboratively with peers and experts to plan the lesson, implement and observe the lesson and reflect on it. Interaction with peers and experts is seen as an effective way for teachers as learners to construct new knowledge and skills. Similar findings appear in contemporary lesson study research, where collaborative cycles have been shown to support shared professional judgment and the examination of mathematics teaching practices (Fitriati et al., 2026; Fitriati et al., 2023; Nguyen & Tran, 2023). In the present study, this collaborative process was particularly evident in participants' discussions of task accessibility, learner participation, and the need to provide opportunities for mathematical explanation rather than merely correct answers.

The findings also clarify the distinct functions of PjBL and lesson study within the course. PjBL served as the macro-pedagogical architecture as it organized the extended project through which the PSTs identified a problem of practice, selected a mathematical topic, and produced a final lesson design. This function is consistent with research positioning projects as structures for sustained investigation and the production of meaningful instructional artifacts (Farrow et al., 2024). Lesson study operated within that architecture as the micro-cycle of professional inquiry through which each design was planned, enacted through peer teaching, examined, and redesigned. In this sense, PjBL provided continuity and a shared project outcome, whereas lesson study supplied the iterative mechanism for examining instructional decisions. Their interaction is reflected in the movement from problem identification to repeated lesson-design cycles. However, because the analyzed corpus comprised final lesson designs and retrospective group interviews, the study cannot isolate the causal effect of PjBL or determine whether the same outcomes would have occurred through

lesson study alone. The contribution is therefore conceptual and interpretive: the case demonstrates a feasible division of pedagogical functions rather than comparative evidence of effectiveness. This interpretation is compatible with recent work showing that PjBL and Lesson study may be coordinated while retaining analytically distinct roles (Qi et al., 2025).

A key indicator of teacher learning is the ability to revise instruction based on evidence rather than habit. In this study, PSTs revised tasks, scaffolds, and representations based on mismatches between the lesson designs and peer-teaching enactments. This aligns with research indicating that lesson study supports adaptive instructional decision-making by making learner responses visible and providing structured opportunities for collaborative reflection and evidence-based refinement (García et al., 2024; Lewis, 2016; Nguyen & Tran, 2023). In this study, PSTs reconsidered representations used to introduce volume, clarified prompts related to fraction denominators, and adjusted task guidance and waiting time. These revisions reflected emerging attention to responsive teaching, although the available data did not establish developmental change across the two cycles.

The findings highlight several actionable implications for strengthening mathematics teacher preparation. First, integrating structured lesson design should become a core component of coursework and practicum. Explicit attention to constructing didactic situations, anticipating student responses, and planning teacher moves enables PSTs to engage in deeper pedagogical reasoning. Embedding these elements in methods courses can better prepare PSTs for the complexities of classroom instruction since designing effective mathematics instruction is not an easy task for them (Fitriati & Novita, 2018). Second, lesson study offers a productive practice-based framework for developing these competencies. Its iterative cycles create authentic opportunities for PSTs to plan tasks for reasoning and communication, test predictions about student thinking, and revise designs using evidence from peer-teaching enactments and collaborative reflection. Embedding Lesson study within school–university partnerships can enrich practicum experiences by connecting coursework with real student learning. Finally, teacher educators should foreground the practice of noticing and evidence-based revision as essential habits of mind. Structured observation tools, guided reflection, and collaborative debrief can help PSTs learn to interpret student thinking and make principled instructional decisions. By cultivating these practices, teacher education programs can better equip future teachers to design mathematically rich and responsive learning experiences.

Finally, this study is bounded by its small sample and single institutional context, which may limit transferability of the findings. Each group designed and enacted lessons for only one mathematical topic; therefore, the identified patterns may partly reflect topic-specific features and do not demonstrate whether the same PSTs would make similar planning and enactment decisions across different mathematical domains. Moreover, the lessons were enacted through peer teaching rather than with primary pupils, and the analyzed corpus was limited to final lesson designs and retrospective group interviews. Future research should examine repeated lesson study cycles across multiple topics and authentic classroom settings.

4. CONCLUSION

This study explored prospective mathematics teachers' engagement in lesson design within a course that combined PjBL as its macro-pedagogical architecture with lesson study as a form of professional inquiry. The findings suggest that this arrangement provided a collaborative structure for engaging PSTs in lesson design. Across three elementary mathematics topics, their final lesson designs incorporated anticipated learning difficulties, sequenced representations, sharing and jumping tasks, and prompts intended to support mathematical reasoning and communication. Their interview accounts also indicated attention to the rationale underlying task selection, teacher guidance, and lesson revision.

At the same time, the findings revealed a cross-cutting discontinuity between design and enactment. Although the three groups designed student-centered lessons, they reported instances of reverting to direct explanation during peer teaching when simulated learners hesitated or required more time. This pattern indicates that design competence does not automatically translate into enactment competence. The capacity to plan reasoning-oriented instruction must therefore be complemented by repeated and scaffolded opportunities to elicit, interpret, and respond to learner thinking under the uncertainty of instructional interaction. Within this process, PjBL organized the sustained lesson-design project, while lesson study provided the iterative structure for examining and reconsidering instructional decisions.

Finally, the study illustrates that PjBL and lesson study can serve as a bridge between theory and practice in teacher education. This arrangement supports PSTs in learning to reason from the student's perspective, design mathematically purposeful tasks, and revise instruction based on evidence. It also highlights the importance of explicit modeling, structured reflection, and repeated practice to help PSTs translate innovative lesson designs into equally innovative classroom enactment. For teacher education programs, these insights reaffirm the value of collaborative, practice-based learning experiences that cultivate both the vision and the skills needed for responsive, high-quality mathematics teaching.

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